32nd URSI GASS, Montreal, 19–26 August 2017

A New Compressed Sensing Based Approach for Null Steering of Linear Arrays by Perturbing Minimum Number of Elements

Magdy A. Abdelhay*(1), and Said E. El-Khamy(2)
(1) Alexandria Institute of Engineering and Technology, Alexandria 21311, Egypt.
(2) Department of Electrical Engineering, Alexandria University, Alexandria 21544, Egypt.

Abstract

In this paper, a compressive sensing approach for wide null steering in partially adaptive arrays is introduced. The proposed method imposes the wide nulls by canceling the whole sidelobes where the interference signals arrive. The problem is first formulated and relaxed to obtain a convex programming problem. The relaxed problem is then solved using iterative re-weighted ℓ_1 minimization algorithm to enhance the sparsity in the final solution. Simulations were conducted for one and two wide nulls steering in small and large linear arrays. Results show that the proposed algorithm is capable of steering the required nulls with small number of perturbed array elements

1 Introduction

Interference mitigation in phased antenna array systems using null steering is essential in many applications such as radar [1], and cognitive radio [2]. The nulls are steered towards the directions of interference signals by controlling the antenna elements positions [3], the complex (magnitude and phase) excitations [4], amplitude only [5] or phase only excitations [6] of the antenna elements. Usually null steering is achieved by changing the excitations or locations of all the array elements which leads to increased complexity and cost. These arrays are referred to as fully adaptive arrays.

In [7, 8] algorithms for wide null steering which cancel the whole sidelobe at the interference angle by controlling the edge elements of a partially adaptive array was presented. The authors exploited that the grating lobes of the edge elements are closely matched in width to the sidelobes of the equally spaced array. To cancel a side lobe, its peak angle is determined, then the weights of the edge elements are adjusted so that their pattern is equal in magnitude to and in antiphase with the array pattern at the desired angle.

Recently, an algorithm for wide null steering using two-step smoothed ℓ_0 (SL0) sparse decomposition technique [9] was proposed in [10]. The algorithm uses SLO to minimize the number of perturbed array elements. But, the number of perturbed array elements is chosen arbitrary which may result in using more than the required number of elements. Also, the algorithm was tested only for large arrays and a single wide null.

In this paper, the compressive sensing theory [11] is used to solve the problem of wide null steering by controlling minimum number of antenna elements. First the problem is formulated as a sparse recovery problem, then relaxed to a convex form [12]. Finally, the relaxed problem is solved using the iterative re-weighted ℓ_1 algorithm [13]. The proposed algorithm is capable of canceling one or more sidelobes of the array pattern to achieve the desired single or multiple wide nulls.

The rest of the paper is organized as follows. Section 2 contains the problem formulation and the proposed algorithm. In section 3, the simulation results are presented. Finally, conclusions are given in section 4.

2 Formulation and Algorithm

The array factor of linear array with equally spaced N isotropic radiating elements placed along horizontal axis x as a function of the elevation angle θ, measured from the array boresight, can be expressed as

$$AF(\theta) = \sum_{n=1}^{N} I_n e^{-jkd_n \sin(\theta)}, \tag{1}$$

where I_n and d_n are the complex excitation and the position of n-th array element, respectively. $k = 2\pi/\lambda$ is the wave number and λ is the wavelength. To steer the main beam towards the desired angle, θ_s, the conventional excitation for the n-th array element is given by

$$I_{c_n} = \alpha_n e^{jkd_n \sin(\theta_s)}, \tag{2}$$

where α_n is the excitation amplitude of n-th element. The conventional pattern in this case is given by

$$AF_c(\theta) = \sum_{n=1}^{N} I_{c_n} e^{-jkd_n \sin(\theta)} = \sum_{n=1}^{N} \alpha_n e^{-jkd_n (\sin(\theta) - \sin(\theta_s))}. \tag{3}$$

Generating K nulls at the array pattern requires perturbing the array elements from their conventional excitations. The new complex excitation for the n-th element is given by

$$I_n = I_{c_n} + x_n. \tag{4}$$

Then, the new pattern is given by

$$AF(\theta) = \sum_{n=1}^{N} I_n e^{-jkd_n \sin(\theta)} = AF_c(\theta) + \sum_{n=1}^{N} x_n e^{-jkd_n \sin(\theta)}. \tag{5}$$

Ideally, the antenna array forms a null in the directions of the interference signals. Then,

$$AF(\theta_k) = AF_c(\theta_k) + \sum_{n=1}^{N} x_n e^{-jkd_n \sin(\theta_k)} = 0, \tag{6}$$

where $\theta_k, \quad k = 1, 2, ..., K$ are the null angles. Equation (6) can be written as

$$\sum_{n=1}^{N} x_n e^{-jkd_n \sin(\theta_k)} = -AF_c(\theta_k). \tag{7}$$

Equation (7) is a set of linear equations with K unknowns which can be represented in a matrix form as

$$\mathbf{AX} = \mathbf{y}, \tag{8}$$

where

$$\mathbf{y} = \begin{bmatrix} -AF_c(\theta_1) & -AF_c(\theta_2) & \dots & -AF_c(\theta_K) \end{bmatrix}^{\mathrm{T}}, \tag{9}$$

$$\mathbf{X} = \begin{bmatrix} x_1 & x_2 & \dots & x_N \end{bmatrix}^{\mathrm{T}}, \tag{10}$$

and

$$\mathbf{A} = \begin{bmatrix} e^{-jkd_1 \sin(\theta_1)} & e^{-jkd_2 \sin(\theta_1)} & \dots & e^{-jkd_N \sin(\theta_1)} \\ e^{-jkd_1 \sin(\theta_2)} & e^{-jkd_2 \sin(\theta_2)} & \dots & e^{-jkd_N \sin(\theta_2)} \\ \vdots & \vdots & \ddots & \vdots \\ e^{-jkd_1 \sin(\theta_K)} & e^{-jkd_2 \sin(\theta_K)} & \dots & e^{-jkd_N \sin(\theta_K)} \end{bmatrix}. \tag{11}$$

Since the number of array elements, N, is usually much larger than the number of interference signals, K, the system in (8) is an under-determined system and has infinitely many solutions. Minimum mean square error (MMSE) estimator can be used to find the array perturbations that minimize the mean square error in pattern due to the weight perturbations. In this case, the array perturbations are gives by [14]

$$\mathbf{X} = \mathbf{A}^H (\mathbf{A}\mathbf{A}^H)^{-1} \mathbf{y}, \tag{12}$$

where $\mathbf{A}^H$ is the Hermitian transpose of matrix $\mathbf{A}$. The solution in (12) doesn't provide a localized solution, so a fully adaptive array is required to achive the desired nulls.

But, we are interested in generating the K nulls by perturbing the complex weights of a minimum number of array elements, while the vast majority of elements stay at their conventional weights. The aforementioned problem can be formulated as

$$\begin{gathered} \min \|\mathbf{X}\|_0 \\ \text{subject to } \mathbf{AX} = \mathbf{y} \end{gathered} \tag{13}$$

where the l_0-quasi norm $\|.\|_0$ denotes the number of nonzero elements. The optimization problem in (13) is an NP-optimization problem due to the non-convex objective function. To relax the problem to a convex form, the ℓ_0-quasi norm is replaced with ℓ_1 norm. The optimization problem can then be rewritten as

$$\begin{gathered} \min \|\mathbf{X}\|_1 \\ \text{subject to } \mathbf{AX} - \mathbf{y} \leq \varepsilon \end{gathered} \tag{14}$$

where $\|.\|_1$ is the ℓ_1 norm which is the sum of the absolute values of its argument, and ε is a fidelity factor to control the error or the allowed difference between the desired and designed values. The problem in (14) is a convex optimization problem and can be solved using CVX package [15].

To enhance the sparsity of the solution and hence reduce the number of perturbed array elements, the iterative reweighted ℓ_1 norm algorithm is adopted here. Then, a reweighted ℓ_1 norm of the vector $\mathbf{X}$ is minimized at iteration i as

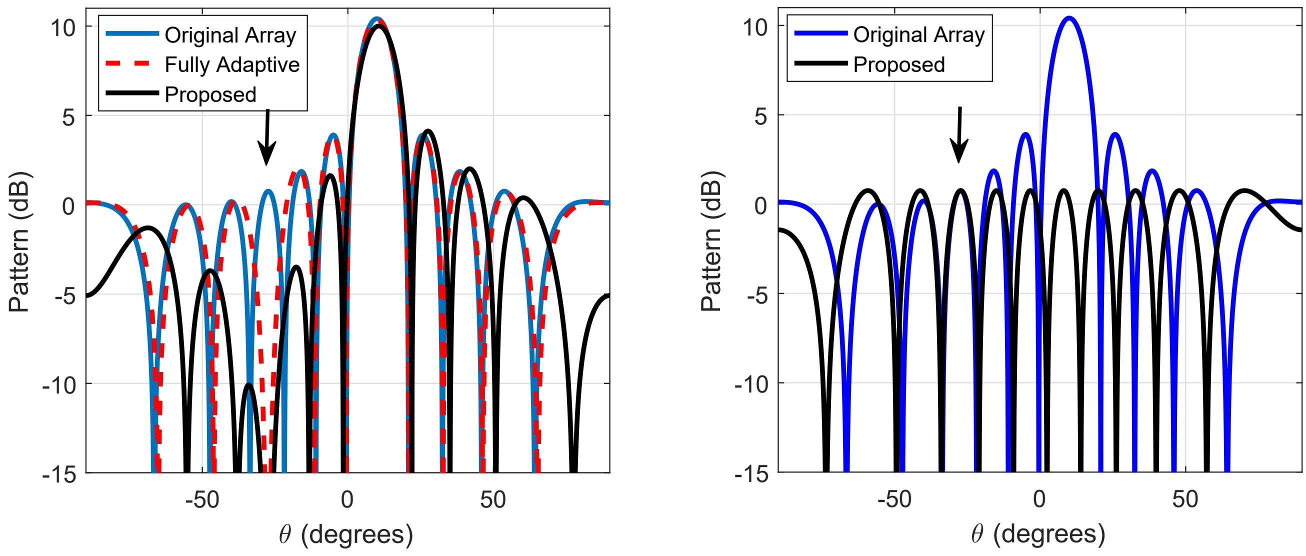
$$\begin{gathered} \min \|\xi\left(\mathbf{X}^{i-1}\right)\mathbf{X}^i\|_1 \\ \text{subject to } \mathbf{AX} - \mathbf{y} \leq \varepsilon \end{gathered} \tag{15}$$

where the weightings $\xi\left(\mathbf{X}^i\right) = 1/\left(\mathbf{X}^{i-1} + \delta\right)$ are assigned to each of the elements of $\mathbf{X}^i$ ($\mathbf{X}$ at ith iteration). δ is a small positive number that provides numerical stability and ensures that a zero-valued component in $\mathbf{X}^i$ does not prevent a nonzero estimate at the next iteration. In the first iteration $(i = 1)$, the weightings $\xi\left(\mathbf{X}^0\right)$ initialized to all ones which results in the objective function in (14). With the obtained $\xi\left(\mathbf{X}^i\right)$, those of small magnitudes are given larger weightings in the next iteration and vice versa. The process suppresses the small entries in $\mathbf{X}$ to zero.

3 Simulation Results

For the first example, a uniform linear array with 11 elements and 0.5λ element spacing is assumed. α_n is set to one for all antenna elements, and the main beam is steered to $\theta_s = 10°$. The direction of arrival of the interfering signal is assumed at $\theta_i = -7.5°$. To achieve the required wide null, two nulls around θ_i with small spacing of 0.05° are introduced, i.e., nulls at $-7.45°$ and $-7.55°$. Only the edge elements (elements number 1 and 11) were required by the algorithm at the first iteration to impose the required wide null. The elements perturbations acquired by the algorithm are $x_1 = 0.5955\angle 52.9344°$ and $x_{11} = 0.5955\angle -52.9344°$.

Figure 1(a) shows the patterns of the original linear array, the fully adaptive array, and the proposed method. Clearly, the proposed method is able to create the required wide null by canceling the whole sidelobe which contains the angle θ_i. Also, the proposed method resulted in a wider null than the fully adaptive array. Figure 1(b) shows the patterns of the original array, and the selected elements with their corresponding perturbations. It can be seen that the sidelobe of the full array which contains the angle θ_i is matched in width and magnitude with the corresponding grating lobe of the selected elements. And since both patterns are antiphase with each other at null angles, the whole sidelobe is canceled in the total pattern of the proposed method.



(a) **(b)**

Figure 1. Null steering in 11 elements array: (a) Patterns of the different methods, (b) Pattern of the selected elements.

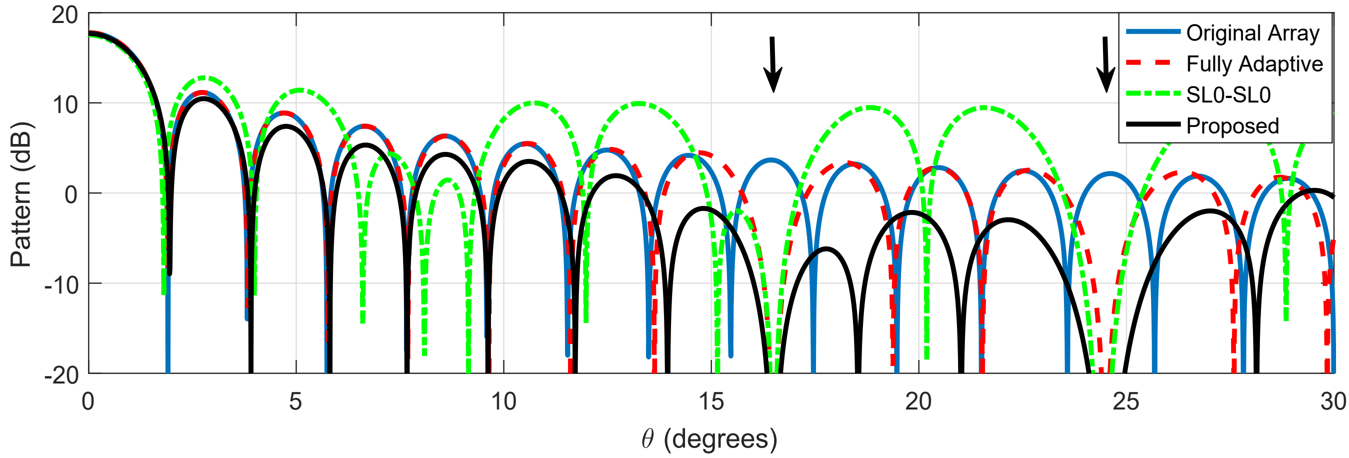


Figure 2. Patterns of the different methods for the 60 elements array.

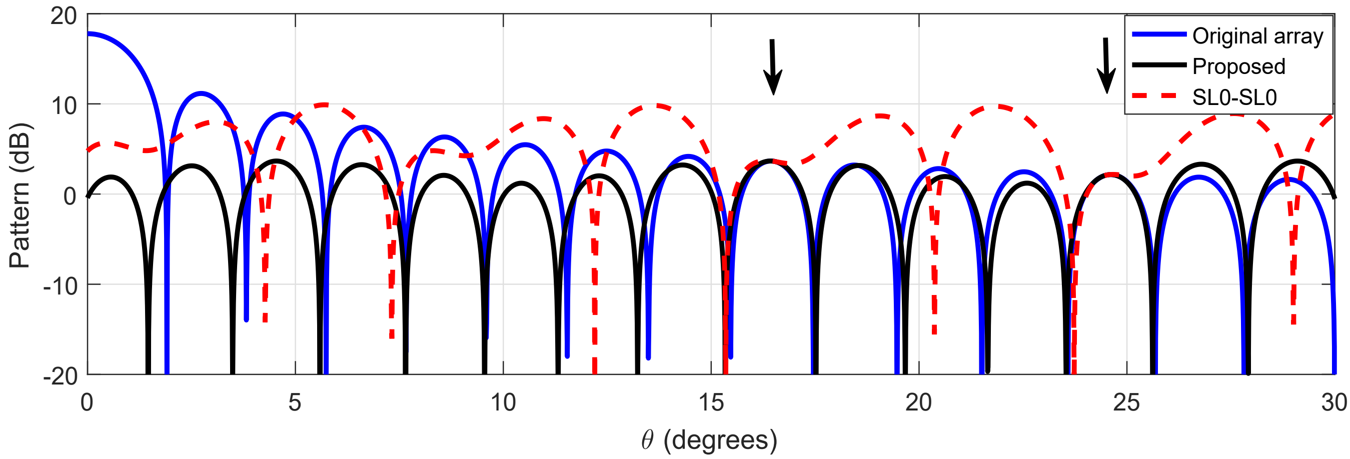


Figure 3. Patterns of the selected elements for the 60 elements array.

For the second example, a uniform linear array with 60 elements and 0.5λ element spacing is assumed. The directions of arrival of the interfering signals are assumed at $\theta_i = 16.5°$ and $24.5°$. Four nulls were imposed at an-

Table 1. Perturbations of selected array elements

Proposed Method			SL0-SL0		
n	$\lvert\mathbf{x_n}\rvert$	$\angle\mathbf{x_n}$ (deg)	n	$\lvert\mathbf{x_n}\rvert$	$\angle\mathbf{x_n}$ (deg)
1	0.9110	118.7340	1	1.6429	-156.2271
11	0.2517	-94.1930	16	3.3281	90.0003
50	0.2517	94.1930	46	3.3281	-90.0003
60	0.9110	-118.7340	60	1.6429	156.2271

gles 16.45°, 16.55°, 24.45°, and 24.55°. For comparison purposes, the two-step SL0 (SL0-SL0) algorithm from [10] was used to obtain the same wide nulls. Only four antennas were chosen by the proposed algorithm to steer the nulls. For a fair comparison, the number of perturbed antennas was set to four in the SL0-SL0 algorithm. The selected antennas and their perturbations are listed in Table 1. The patterns of the two techniques, the fully adaptive array, and the original array are shown in Figure 2. Obviously, the pattern of the proposed algorithm has wider nulls and lower sidelobe level than the pattern of the SL0-SL0 algorithm. Figure 3 shows the patterns of the selected elements by the two algorithms. The grating lobes of the selected elements by the proposed algorithm have better matching with the sidelobes at which the interfering signals arrive and lower grating lobes levels than those of the SL0-SL0 algorithm.

4 Conclusions

An algorithm for wide null steering in linear antenna arrays was presented. The algorithm can cancel the sidelobes at the interference directions by changing the weights of a minimum number of antenna elements. Results showed that changing the weights of the edge elements only were sufficient to impose a single wide null, while four elements were required for two separated wide nulls.

References

[1] Y. Gao, J. Xu, Y. He, and X. Wang, "A novel adaptive nulling of interference method for array radar," in *IET International Radar Conference 2015*, Oct 2015, pp. 1–4.

[2] H. O. Kpojime and G. A. Safdar, "Interference mitigation in cognitive-radio-based femtocells," *IEEE Communications Surveys Tutorials*, vol. 17, no. 3, pp. 1511–1534, thirdquarter 2015.

[3] S. K. Mahto and A. Choubey, "A novel hybrid iwo/wdo algorithm for interference minimization of uniformly excited linear sparse array by position-only control," *IEEE Antennas and Wireless Propagation Letters*, vol. 15, pp. 250–254, 2016.

[4] M. H. Er, "Linear antenna array pattern synthesis with prescribed broad nulls," *IEEE Transactions on Antennas and Propagation*, vol. 38, no. 9, pp. 1496–1498, Sep 1990.

[5] K. W. Lo, "Null steering in monopulse planar array using amplitude only control," *Electronics Letters*, vol. 26, no. 20, pp. 1700–1701, Sept 1990.

[6] T. Lam, "Null steering antenna array using phase-only weights," in *2014 IEEE Antennas and Propagation Society International Symposium (APSURSI)*, July 2014, pp. 1660–1661.

[7] I. El-Azhary, M. S. Afifi, and P. S. Excell, "A simple algorithm for sidelobe cancellation in a partially adaptive linear array," *IEEE Transactions on Antennas and Propagation*, vol. 36, no. 10, pp. 1482–1486, Oct 1988.

[8] J. R. Mohammed and K. H. Sayidmarie, "Null steering method by controlling two elements," *IET Microwaves, Antennas Propagation*, vol. 8, no. 15, pp. 1348–1355, 2014.

[9] H. Mohimani, M. Babaie-Zadeh, and C. Jutten, "A fast approach for overcomplete sparse decomposition based on smoothed ℓ^0 norm," *IEEE Transactions on Signal Processing*, vol. 57, no. 1, pp. 289–301, Jan 2009.

[10] M. Khosravi, M. Fakharzadeh, and M. H. Bastani, "Large array null steering using compressed sensing," *IEEE Signal Processing Letters*, vol. 23, no. 8, pp. 1032–1036, Aug 2016.

[11] A. Massa, P. Rocca, and G. Oliveri, "Compressive sensing in electromagnetics - a review," *IEEE Antennas and Propagation Magazine*, vol. 57, no. 1, pp. 224–238, Feb 2015.

[12] Z.-Q. Luo and W. Yu, "An introduction to convex optimization for communications and signal processing," *IEEE Journal on Selected Areas in Communications*, vol. 24, no. 8, pp. 1426–1438, Aug 2006.

[13] E. J. Candès, M. B. Wakin, and S. P. Boyd, "Enhancing sparsity by reweighted $\ell 1$ minimization," *Journal of Fourier Analysis and Applications*, vol. 14, no. 5, pp. 877–905, 2008.

[14] R. Shore, "A unified treatment of nulling in linear array patterns with minimized weight perturbations," in *1982 Antennas and Propagation Society International Symposium*, vol. 20, May 1982, pp. 703–706.

[15] M. Grant and S. Boyd, "CVX: Matlab software for disciplined convex programming, version 2.1," http://cvxr.com/cvx, Mar. 2014.