High Frequency Spectra of Resonance-Free Integral Equations and Related Algorithms

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Extended Abstract

Integral Equations are widely used in Computational Electromagnetics for simulation and modeling of radiation and scattering scenarios. When solved with the Boundary Element Method (BEM), integral equations give rise to dense linear systems. When the system's dimensionality is large, fast iterative or direct solvers must be used and the BEM matrix has to be well-conditioned to ensure numerical stability. In fact, in solving the matrix associated linear system, the matrix condition number (the ratio of the matrix largest and smallest singular value) is related to iterative solvers' convergence rates and to the error sensitivity of the solution: the highest the condition number, the highest the convergence time and the error sensitivity. Unfortunately, many commonly used formulations suffer from severe ill-conditioning especially for large and complex problems. The problem complexity of current applications is steadily and rapidly increasing. For this reason, the impact of well-conditioned formulations on state-of-the-art computational technology is destined to be more and more predominant and pervasive.

Several preconditioning techniques have been developed to control the condition number of the linear system matrices arising from most the integral formulations for electromagnetics problems. The performance of these techniques is usually assessed in two regimes: (i) the case where the number of unknowns is kept constant and the frequency decreases (low-frequency regime) and (ii) the case where the frequency is kept constant and the number of unknowns increases (dense discretization regime). A third regime, however, can be identified: (iii) the case where both frequency and the number of unknowns grow since the discretization size in wavelengths is kept constant and the frequency increases (high frequency regime).

This contribution will focus on the analysis of the spectra of several relevant integral operators in this third regime. It is well-known that some integral formulations suffer from internal resonances at high frequency. This, however, is not the focus of this talk. Instead, the high-frequency spectral problems under analysis here are those remaining after eliminating internal resonances. Otherwise said, only resonance-free formulations will be analyzed. The analysis and results we will present, will pave the way for effective regularization and fast solution strategies. In fact, our spectral analysis will be complemented with efficient fast algorithms that will impact both solution and operators' numerical stability. Numerical examples will complement the theory and show the practical impact of all our theoretical findings.