A Comparative Study of Singularity Treatment Schemes in Higher-Order Nyström Method for Acoustic Scattering

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1. Extended Abstract

Time harmonic acoustic scattering from arbitrarily shaped objects can be numerically analyzed by solving various types of surface integral equations (SIEs). SIE solvers have several advantages over differential equation solvers [1]: (i) Radiation condition is implicitly enforced and (ii) only surface of the scatterer is discretized. SIEs are oftentimes solved using the method of moments (MoM). The MoM first expands the unknown acoustic potential in terms of basis functions. Inserting this expansion to the SIE and testing the resulting equation with the same functions yield a matrix system that can be solved for the unknown expansion coefficients. Nyström method operates in a similar manner, where the SIE is converted into a summation using quadrature rules over discretization elements. Testing this summation at the same quadrature points yields a matrix system that can be solved for unknown samples of the potential at the quadrature points. Nyström method is easier to implement even when the geometrical discretization is non-conforming and higher-order modeling of the geometry and the acoustic potential is desired. On the other hand, treatment of the Green function singularities is more challenging due to the fact that singularities are not "smoothened" through the use of a testing function [1].

In this work, two methods to treat singularities in higher-order Nyström method for analyzing soft acoustic scattering are compared. The first method is built upon the approach developed in [2] for the (vector) SIEs of electromagnetics. It uses Gaussian quadrature points to sample the potential on every curvilinear discretization element. When the testing point is on the same discretization element, Duffy singularity cancellation technique [3] is used to compute the matrix entries. Duffy techniques calls for division of the discretization element into three and therefore a numerically constructed polynomial interpolation scheme has to be applied on the unknown samples at the initial Gaussian quadrature points. Unlike the first method, the second method uses Lagrange polynomials [4, 5] together with the singularity extraction technique of [6] to smoothen integrals for computation of the matrix entries.

Numerical results, which compare the two methods in terms of higher-order convergence, accuracy, efficiency, and robustness in singularity treatment, will be presented.

2. References

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