

Analysis of Multilayer Interconnects Distributed Energy-Per-Bit and Power Integrity with Kron-Branin Formalism

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Abstract

With the increase of the operation speed and the miniaturization, the power consumption assessment of multilayer printed circuit board (PCB) becomes a crucial task during the design phase. More accurate power integrity (PI) analysis is necessary for the complex multilayer PCB. An unfamiliar modelling of static and dynamic power distributed through a four-layer interconnect structure is introduced in this paper. The modelling concept is based on the Kron-Branin (KB) formalism. The different steps of the proposed KB modelling are described with the consideration of the RLC equivalent model and its equivalent graph topology. The KB model efficiency is validated with the static and dynamic powers distributed through the multilayer structure by considering 20 ns duration and 0.5 ns rise/fall time a pulse signal. The computed energy-per-bit and average powers are compared with simulations from a commercial 3D EM computational tool.

1. Introduction

With the design complexity and the operation speed, the multilayer technology is assumed as the best solution in term of miniaturization [1]. The analysis of multilayer printed circuit boards (PCBs) remains a challenging task for the design and manufacturing engineers [2]. Nowadays, the power integrity (PI) assessment becomes challenging issue encountered during the design process [3]. The power management constitutes a crucial aspect in addition to the electromagnetic compatibility/interference (EMC/EMI) and signal integrity (SI) analyses [4-5]. Various problems of PCB functionality were reported due to the PI issue. Despite the electromagnetic (EM) computational tools available in the literature, more efficient PI analysis is still required [6]. Full wave computational methods as FDTD, FEM and FIT are generally used for the multilayer high density PCB PI analysis. However, these solvers are practically memory and time consuming. Therefore, the EMC/SI engineers are constantly looking for higher performant computational methods for the PCB electrical interconnects modelling.

One of solutions can be the exploitation of an unconventional modelling method as the Kron-Branin (KB) formalism. It is based on the tensorial analysis of electrical network initiated by Kron in 1930s [7]. Associated with the Branin's model [8], the method is outstandingly useful for the complex structures analyses [9-10]. The KB formalism was recently applied to the SI

analysis through the microstrip Y-tree [11]. However, the KB formalism effectiveness for the PI analysis notably for the multilayer PCB remains an open question. For this reason, the present paper purpose is the PI analysis of multilayer interconnect structure by using the KB formalism.

2. Multilayer Kron-Branin (KB) modelling methodology

The present section describes the methodology of the KB formalism application for PI analysis. The formalism is applied to the multilayer interconnects PCB.

2.1 Description of the multilayer structure under study

Fig. 1(a) and Fig. 1(b) represent respectively the cross and top views of the four-layer PCB structure under investigation. The overall structure is constituted by two different dielectric substrates with different relative permittivity and same thickness $h_{1,2,3,4,5} = 2$ mm. The KB modelling methodology suggests to combine all the elementary cells' equivalent circuit topology. In this case, the main elements constituting the posed problem assumed as the network under study are interconnects, vias and pads.

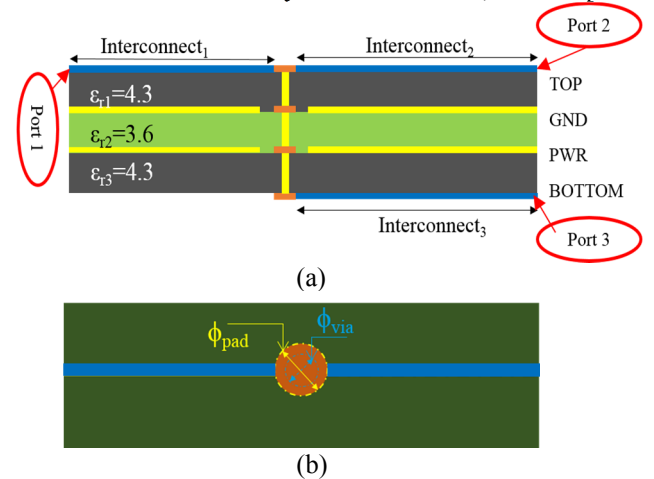


Figure 1. (a) Crossection of the considered multilayer structure and (b) top view.

2.2 Implementation of the multilayer structure KB graph topology

The KB modelling can be implemented in four basic synthetic steps which can be described as follows:

- Step 1 consists in splitting the structure into several parts constituted by via cells and interconnect TLs presenting characteristic impedances $Z_{1,2}$ and electrical lengths $\theta_{1,2}$.

- Step 2 is the extraction of equivalent electrical model of each elementary object constituting the initial structure. As proposed in [12-13], the via model parameters are formulated as:

$$L_{via} = \frac{\mu_0}{2\pi} \left[h \cdot \ln \left(\frac{2h + \sqrt{\phi_{via}^2 + 4h^2}}{\phi_{via}} \right) + 3 \left(\phi_{via} - \sqrt{\phi_{via}^2 + 4h^2} \right) \right], \quad (1)$$

$$C_{anti-pad} = \frac{1.41 \epsilon_r t \cdot \phi_{pad}}{\phi_{anti-pad} - \phi_{pad}}, \quad (2)$$

with t is the interconnect metallization thickness and μ_0 is the vacuum permeability.

- Step 3 consists in elaborating the structure as an equivalent graph topology. The graph diagram is essentially constituted by the combination of the network of the elementary cells constituting the initial structure. Fig. 2 sketches the established equivalent KB graph topology.

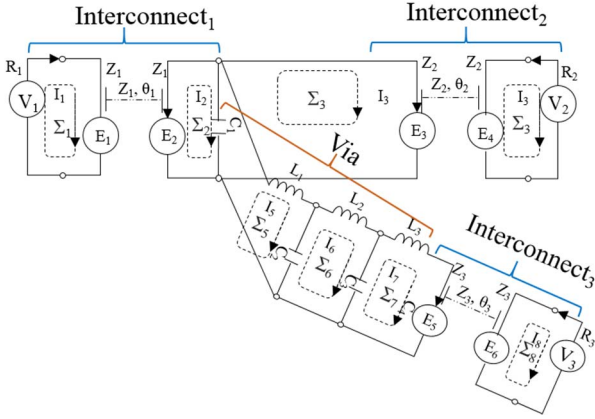


Figure 2. KB equivalent graph topology of the structure shown in Fig. 1.

- Step 4 is the abstraction of the posed problem. It consists in traducing the graph topology into mathematical expressions. It is noteworthy that the equivalent KB graph topology is constituted by Branin's model in mesh ($\Sigma_{1,...,4}$) with the input $R_{1,2,3}$ resistive source $V_{1,2,3}$ in mesh (Σ_1), via LC-ladders with meshes ($\Sigma_{5,...,7}$). The initially posed-problem solutions can be obtained from the abstracted matrix equations. This last step will be described in more details in the next paragraph.

2.3 Analytical abstraction of the equivalent KB topology

From the proposed graph topology, the tensorial network equations can be easily developed with mesh laws. By applying Kirchhoff's laws to those equations, the associated characteristic matrix is depicted in matrix relation (3) by taking (for $m=\{1,2,3\}$). In this expression, $x_m = \exp[-(\alpha_m + j\beta_m)d_m]$ is the parameter associated to the electrical length of elementary TL TL_m , with α_m is the attenuation constant, β_m is the phase constant, d_m is the physical length and $j=\sqrt{-1}$. The mathematical solution of matrix equation (3) enables to calculate the dynamic and static power distribution through the multilayer interconnect structure. The present study is dealing with the analysis of the input, and output powers. The instantaneous power expressions through the branch m ($m=\{1,2,3\}$) can be written as:

$$p_m(t) = v_m(t) \cdot i_m(t), \quad (4)$$

In function of instantaneous voltage v_m and current i_m . The output equivalent average power when considering an input excitation presenting time duration T one-bit data signal can be calculated from frequency or time-domain as defined by the Parseval's theorem:

$$P_m = \frac{R_m}{T} \int_0^T |i_m(t)|^2 dt = \frac{R_m}{T} \int_0^{f_{max}} |I_m(f)|^2 df, \quad (5)$$

with $I_m(f)$ for $m=\{2,3\}$ is the current power spectrum. By assuming the discrete samples $i_m(k \cdot \Delta t)$ in time domain and $I_m(k \cdot \Delta f)$ in frequency domain, the previous equality can be written as:

$$P_m = \frac{R_m \Delta t}{T} \sum_{k_t=0}^{k_{t_{max}}} |i_m(k \cdot \Delta t)|^2 = \frac{R_m \Delta f}{T} \sum_{k_f=0}^{k_{f_{max}}} |I_m(k \cdot \Delta f)|^2. \quad (6)$$

where Δt is the time step, Δf is the frequency step, $k_{t_{max}} = \text{int}(T / \Delta t)$ and $k_{f_{max}} = \text{int}(f_{max} / \Delta f)$ by taking the signal spectrum is limited by f_{max} . $\text{int}(x)$ expresses the superior entire part of real x . The energy-per-bit is:

$$E_m = \int_0^T p_m(t) dt = \int_0^T v_m(t) \cdot i_m(t) dt, \quad (7)$$

3. Proposed KB model based PI analysis numerical validations

It is noteworthy that the proposed unfamiliar PI analysis is validated with simulations of the structure introduced in Fig. 1 with the commercial tool ADS®.

$$\begin{bmatrix} V_1 \\ V_1 \\ V_2 \\ V_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_3 \\ V_3 \end{bmatrix} = \begin{bmatrix} Z_1 + R_1 & (Z_{C1} - Z_1)x_1 & -Z_{C1}x_1 & 0 & -Z_{C1}x_1 & 0 & 0 & 0 & 0 & 0 \\ R_1 - Z_1 & (Z_{C1} + Z_1)/x_1 & -Z_{C1}/x_1 & 0 & -Z_{C1}/x_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{C1}x_2 & -(Z_{C1} + Z_2)/x_2 & R_2 - Z_2 & -Z_{C1}/x_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & Z_{C1}x_2 & (Z_2 - Z_{C1})x_2 & R_2 + Z_2 & -Z_{C1}x_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & -Z_{C1} & Z_{C1} & 0 & Z_{L1} + Z_{C2} + Z_{C1} & -Z_{C2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -Z_{C2} & Z_{L2} + Z_{C3} + Z_{C2} & -Z_{C3} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -Z_{C3} & Z_{L3} + Z_{C4} + Z_{C3} & -Z_{C4} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{C4}/x_3 & -(Z_{C4} + Z_3)/x_3 & R_3 - Z_3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_{C4}x_3 & (Z_3 - Z_{C4})x_3 & R_3 + Z_3 & 0 \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{bmatrix} \quad (3)$$

The present section introduces the comparison of the KB model computed results and full wave simulations.

3.1 Description of the computation parameters

To validate the proposed KB model PI analysis, comparison between the computed model and ADS® simulation was performed. Table 1 summarizes the physical parameters of the multilayer interconnects structure under consideration. After power spectrum analyses, the frequency- and time-domain analyses were performed. To do this, the one-bit test signal is assumed as a pulse wave with rise- and fall-times denoted t_r and t_f . It presents a limited bandwidth:

$$f_{max}=0.35/\min(t_r, t_f). \quad (8)$$

Table 1. Parameters of the structure under study.

Element	Parameter
TLs	$t = 18 \mu\text{m}$, $w = 0.2 \text{ mm}$, $d = 20 \text{ mm}$
Via	$\phi_{via} = 0.4 \text{ mm}$
Pad	$\phi_{pad} = 0.6 \text{ mm}$
Anti-pad	$\phi_{anti-pad} = 1 \text{ mm}$

The following paragraphs described the obtained results after the implementation of the previous abstracted mathematical expressions as a Python program.

3.2 Frequency domain analysis

Fig. 3 presents the plot of the input excitation signal power spectrums. The compared KB model computations and ADS simulations are respectively plotted in black and red dotted lines. A good correlation is realized from 50 MHz up to 1 GHz.

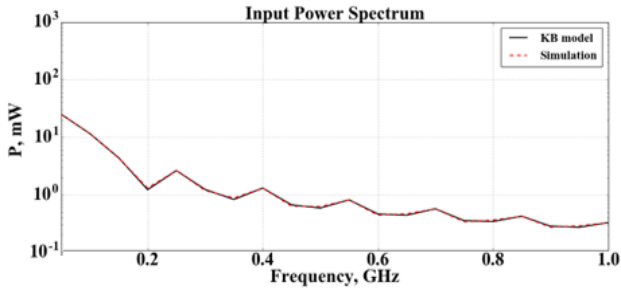


Figure 3. Frequency dependent plots of the input power and the simulated/computed distributed powers through the multilayer structure introduced in Fig. 1.

3.3 Time-domain analysis

A transient analysis was also carried out by considering a time-domain input excitation signal presenting characteristics addressed in Table 2.

Table 2. Time domain characteristics of the input excitation signal.

Parameter	Value
Pulse duration	$T=20 \text{ ns}$
Rise-time	$t_r = 0.5 \text{ ns}$
Fall-time	$t_f = 0.5 \text{ ns}$
Steady state level	$V_{in} = 1 \text{ V}$

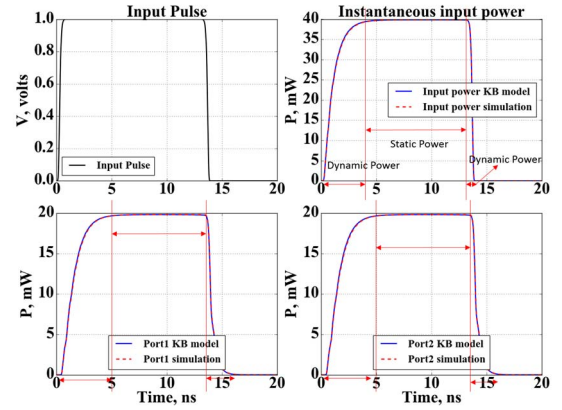


Figure 4. Comparisons of the distributed powers through loads in the time-domain.

By using the previously described input, the transient output powers plotted in Fig. 4 are obtained. The compared KB model computations and ADS simulations are respectively plotted in blue and red dotted lines. Once again, the KB model computation results and ADS simulations are particularly in good agreement.

3.4 Average dynamic and static power assessments

In addition to the previous frequency- and time-domain analyses, the average output powers were also assessed in function of the conductivity σ and thickness t . Fig. 5 displays the plot of the associated average powers versus (σ, t) which are respectively varied from 7.25-to-116 MS/m and 18-to-90 μm . The associated average static and dynamic powers and energy per bit delivered from the multilayer structure input onto the output loads are shown in Table 3 and Table 4 for $t = 18 \mu\text{m}$ and $\sigma = 58 \text{ MS/m}$. In this case, the errors are mainly caused by the calculations of the interconnect attenuations and via parameters.

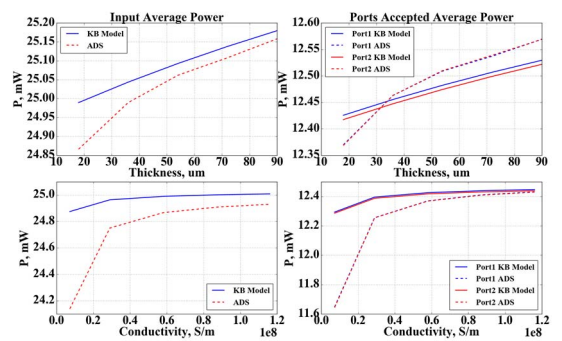


Figure 5. Computed and simulated average powers distributed through the structures introduced in Fig. 1 in function of (σ, t) .

Table 3. Average powers with $t = 18 \mu\text{m}$ and $\sigma = 58 \text{ MS/m}$.

Approach	Average Power (mW)	Input	Port 1	Port 2
KB model	P_{static}	39.76	19.77	19.77

	$P_{dynamic}$	34.65	16.73	16.68
ADS®	P_{static}	39.75	19.78	19.78
	$P_{dynamic}$	34.65	16.72	16.67

Table 4. Energy-per-bit with $t = 18 \mu\text{m}$ and $\sigma = 58 \text{ MS/m}$.

Approach	Energy (pJ)	Input	Port 1	Port 2
KB model	E_{static}	317.5	157.9	157.9
	$E_{dynamic}$	180.7	89.9	89.7
ADS®	E_{static}	318.0	158.2	158.2
	$E_{dynamic}$	179.8	89.7	89.4

3.5 Discussion on the benefits and drawbacks of the KB model for the PI analysis

The unfamiliar KB model presents certain considerable benefits for the EMC/EMI/SI/PI analyses thanks to its implementability in tensorial approach. It can be emphasized that compared to the familiar solvers (FEM, FDTD, FIT, ...), the KB model is mainly advantageous in terms of adaptability to complex structures, simplicity in data processing, less time consuming, less memory consuming and possibility to be implemented with low cost process. However, the KB model is not a fully independent computational method because it requires input parameters as the physical and electric characteristics (as RLC networks equivalent to the interconnect TLs, via and pads) of the structure under study. It means that the KB model is systematically sensitive to the accuracy of the RLC networks constituting the elementary network constituting the overall multi-layer structure. Such inaccuracy can be considerably significant when the case of the structure operating in ultra wideband (UWB) frequency which includes the network cell resonance frequencies.

4. Conclusion

An unfamiliar PI analysis of four-layer structure is proposed. The PI analysis is based on the unfamiliar modelling concept using the KB formalism. The principle of the modelling methodology is described.

The PI analysis model efficiency is validated ADS® simulations. By considering a limited bandwidth pulse excitation power, comparison between static and dynamic powers in both frequency and time-domains are performed. Moreover, the associated dynamic average powers were also assessed. As expected, a good correlation between the KB model computations and simulations were realized.

As future work, the KB modelling approach will be employed to optimize more complex structures of power delivery network (PDN) for the power and energy management.

5. Acknowledgements

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6. References

- [1] International Technology Roadmap for Semiconductors (ITRS) Reports, 2015, Available Online [2016], <http://www.itrs2.net/itrs-reports.html>
- [2] B. Archambeault and A. E. Ruehli, "Analysis of Power/Ground-Plane EMI Decoupling Performance Using the Partial-Element Equivalent Circuit Technique," IEEE Trans. Electromagn. Compat., Vol. 43, No. 4, Nov. 2001, pp. 437-445.
- [3] A. E. Ruehli and A. C. Cangellaris, "Progress in the methodologies for the electrical modeling of interconnects and electronic packages," Proceedings of the IEEE, Vol. 89, No. 5, pp. 740-771, 2001.
- [4] F. Jun, Y. Xiaoning, J. Kim, B. Archambeault, and A. Orlandi, "Signal integrity design for high-speed digital circuits: Progress and directions," IEEE Trans. Electromagn. Compat., Vol. 52, No. 2, pp. 392-400, May 2010.
- [5] J. Kim and E. Li, "Special issue on PCB level signal integrity, power integrity, and EMC," IEEE Trans. Electromagn. Compat., Vol. 52, No. 2, pp. 246-247, May 2010.
- [6] C. Wang, J. Mao, G. Selli, S. Luan, L. Zhang, J. Fan, D. J. Pommerenke, R. E. DuBroff and J. L. Drewniak, "An efficient approach for power delivery network design with closed-form expressions for parasitic interconnect inductances," IEEE Trans. Advanced Packaging, Vol. 29 No. 2, pp. 320-334, 2006.
- [7] G. Kron, Tensor analysis of networks, Wiley, New York; Chapman & Hall, London, 1939.
- [8] F. H. Branin, "Transient analysis of lossless transmission lines," Int. J. Proceedings of the IEEE, Vol. 55, No.11, pp. 2012-2013, Nov. 1967.
- [9] O. Maurice, A. Reineix, P. Durand and F. Dubois, "Kron's method and complexes for magnetomotive and electromotive forces," Int. J. Applied Mathematics, Vol. 44, No. 4, pp. 183-191, 2014.
- [10] O. Maurice, A. Reineix, P. Hoffmann, B. Pecqueux and P. Pouliguen, "A formalism to compute the electromagnetic compatibility of complex networks," Advances in Applied Science Research, Vol. 2, No. 5, 2011, pp. 439-448.
- [11] B. Ravelo, and O. Maurice, "Kron-Branin Modeling of Y-Y-Tree Interconnects for the PCB Signal Integrity Analysis," IEEE Trans. Electromagn. Compat., Vol. 59, No. 2, Apr. 2017, pp. 411-419.
- [12] Y. Shang, C. Li and H. Xiong, "One method for via equivalent circuit extraction based on structural segmentation," Int. J. Computer Science Issues, Vol. 10, No. 3, pp. 18-22, May 2013.
- [13] M. E. Goldfarb and R. A. Pucel, "Modeling via hole grounds in microstrip," Int. J. IEEE Microwave and Guided Wave Letters, Vol. 1, No. 6, pp. 135-137, 1991.