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Unstructured Block Meshing in Time-Domain TLM Method with Local Time-Step

Abdelrahman A. Ijjeh (1) and Michel M. Ney (2)
(1) LEAT, UMR-CNRS 7248 University of Nice Sophia-Antipolis 930 route des Colles, 06903 Sophia Antipolis, France
(2) Lab-STICC/Telecom-Bretagne Institute, CS 83818, 29238 Brest Cedex 3, France, michel.ney@telecom-bretagne.eu

Abstract

Many problems in nature and engineering are multiscale. Thus, for adequately representing structure geometrical details, one has to use small cell sizes which impose small time-step values. In addition, if the mesh is irregular but structured it might lead to exhaustive computer expenditure and higher dispersion. The alternate approach is to use block-meshing techniques which allow fine discretization in regions with fine details and coarse discretization in regions with smoothly-varying details. However, the mesh is said "unstructured" in this case and beside spatial interpolation that provokes some additional error, the choice of time-steps is of crucial importance when time-domain methods are considered. The main consequence is the long-term stability of the process. In this article, we present an algorithm that allows the use of local time-steps unlike the global time-step approach usually considered. Some experiments are presented to show the behavior and performances of a block-meshing algorithm used in time-domain TLM computation with local time-step.

1. Introduction

Transmission-line matrix (TLM) method is a computational scheme that is based on the analogy between EM waves propagation in media and the corresponding currents and voltages propagating across a mesh of transmission lines [1]. Unlike FDTD [2] or FIT [3] methods, the analogy with circuit theory in case of the TLM method allows one to utilize the concept of ideal transformer to govern the fields exchange between domains with different mesh sizes. Since an ideal transformer is used, one can expect a lossless exact procedure for going from fine mesh toward coarse one and vice versa. On the other hand, FDTD and FIT require spatial interpolation techniques that always produce some error [2-3].

In the previous publications, global time-steps were used in the entire computational domain. This global time-step is corresponding to the CFL limit in the finest mesh [2,4]. Many attempts to use local time-step failed because of middle to long-term instabilities, which are caused by temporal interpolation. Later, in [3], authors proposed a new scheme for FIT method. This scheme can use the CFL time-step limit in the small cells, and its double in the regions of course mesh. This factor of two was independent of the subgridding ratio utilized. This technique developed with FIT was transposed to TLM and validated.

The remaining question is whether or not one can use the optimal local time-step, i.e., in every subdomain with different mesh size we use the maximum allowed time-step. In this article, we present an approach that was initially used for synchronizing multi-rate linear feedback systems [5]. It will be shown that this approach yields a stable block-meshing TLM scheme that uses local time-steps.

After this introduction, we present the mathematical model for the proposed approach in section two, and in section 3, two experiments are presented to validate the proposed temporal up/down sampling scheme.

2. Mathematical Model

A. *Spatial voltages-currents upsampling/downsampling*

For spatial fields exchange between domains with different discretization levels the approach of Wlodarczyk is adopted [4]. This approach is based on the concept of ideal transformer as shown in fig. 1. Note, that in this approach we assume that every big cell in the first domain is adjacent to an integer number of small cells in the second domain. However, once the previous condition is respected, irregular structured meshing can be used inside every domain. Moreover, this approach is valid for multi levels of subgridding.

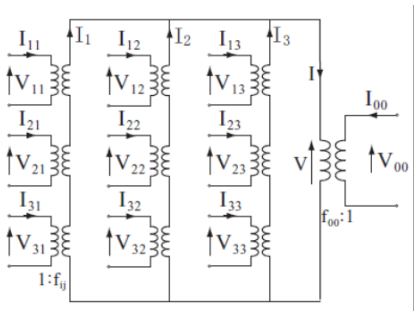

Figure 1, schematic diagram of the equivalent circuit of the interface between two domains

Inside every domain, the update equations are the standard TLM update equations depending on the type of TLM node. However, at the interface between domains with different mesh size an extra step is necessary to correctly exchange fields between both domains. The fields exchange is governed by:

$$v_{ij} = u_{ij} + (V - V_j) Y_j f_{ij} Z_{ij} \qquad (1.a)$$

where

$$f_{ij} = \frac{\Delta x_{ij}}{\Delta y_{ij}} \quad (1.b)$$

$$Y_j = \frac{1}{\sum_i {f_{ij}}^2 Z_{ij}} \quad (1.c)$$

$$V_j = 2\sum_i f_{ij} u_{ij} \quad (1.d)$$

$$V = \frac{\sum_j Y_j V_j}{\sum_j Y_j} \quad (1.e)$$

in which u_{ij} are the incident voltages, and v_{ij} are the reflected voltages from the interface, Z_{ij} is the arm impedance that is dependent on the type of TLM node. For instance, Z_{ij} equals the free-space impedance in the case of SCN node [6].

B. *Temporal upsapling/downsampling*

In [4], the algorithm was shown to be stable when global time-step is used. However, there are two drawbacks of adopting a global time-step. First, this global time-step is determined by the CFL limit in the small cells. Thus, the large cells are not operating at their maximum time-step. As a result, numerical dispersion increases in those cells (usually the majority of cells belong to this group in the computational domain). Secondly, we are increasing the CPU time many-folds as compared to the case for which local time-steps are used. However, one can observe that every subdomain is stable by its own when using its local time-step. Instability arises only when different time-steps are used in the computational domain. Thus, one can notice that the problem appears at the interface between different regions with different time-steps. From system theory point of view, one can notice that the temporal aspect at the interface is reduced to two systems operating at different sampling frequencies as shown in fig. 2.

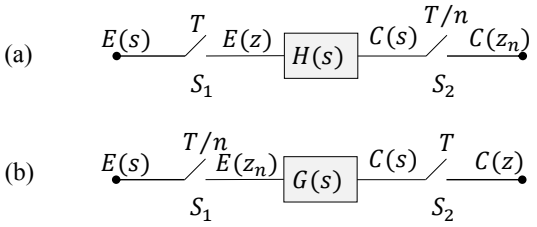

Figure 2, Multi-rate open loop system equivalent to the temporal interface between different domains using different Δt values

When voltages come from large cells to the small ones we keep a constant value through the period of the large time-step as shown in fig.2. a. However, when the voltages go from fine mesh to the coarse one, a filtering process is required [5]. Hence, the equivalent transfer function of fig.2.b writes as:

$$C(z) = \frac{1}{N_s}\sum_{k=1}^{N_s} E\left(z_n e^{\frac{j2\pi}{k}}\right) H\left(z_n e^{\frac{j2\pi}{k}}\right) \quad (2)$$

Implementing (2) at the interface between both regions with mesh ratio of factor N_s has shown no signs of instability, although some small dissipation was observed in the long term. However, by proper choice of the function $H(s)$ one can obtain stable simulations with negligible losses. Moreover, many applications don't need a huge number of iterations. Therefore, the effect of dissipation might be neglected.

2. Results and Discussions

In this section, two experiments that use block meshing with local time-step are presented to show the validity of the proposed approach.

A. *TE mode propagation in a rectangular waveguide*

In this experiment, we show the E-field distribution for a TE_{20} mode in a rectangular waveguide of cross-section dimensions 10 cm x 20 cm (figure 3). The waveguide is air filled and matched by an impedance boundary condition Z_{TE} for the TE_{20} mode of propagation. The structure was excited by TE_{20} mode template of frequency 7 GHz. The mesh size is $\lambda_o/17$ in coarse grid and $\lambda_o/51$ in fine grid. CLF time-step limits were used in both fine and coarse meshes 4.2 ps and 1.4 ps, respectively.

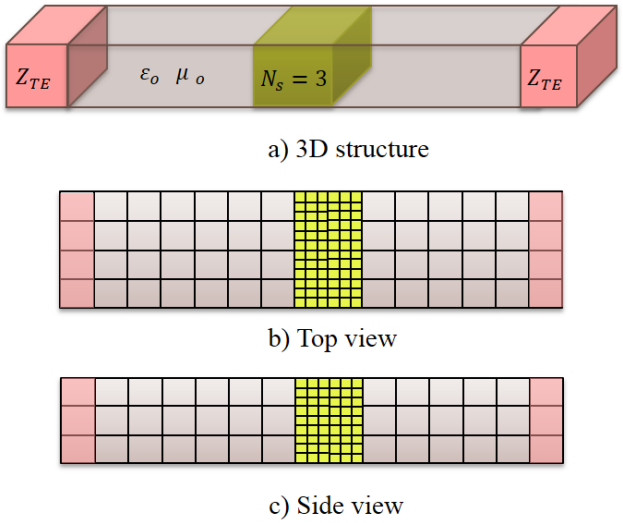

Figure 3, rectangular waveguide meshed with a non-uniform non-structured mesh

Figure 4 shows the E-field distribution of the TE_{20} mode. In the regions in between vertical dashed lines, a 3-times refined mesh was used ($N_s = 3$).

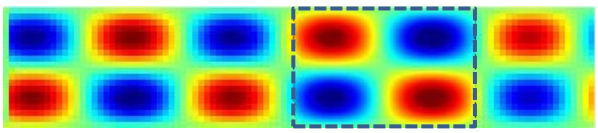

Figure 4, E field in a TE_{20} mode in a rectangular waveguide (time frame after 150000 iterations)

We can observe the better special distribution in the small-mesh region within the dashed rectangle. The simulation didn't show any sign of instability even after 1500000 iterations.

B. Cut-off frequencies of a ridged waveguide

In this experiment, we exploit the concept of unstructured bloc-meshing to compute cutoff frequencies in a rectangular ridged waveguide shown in figure 5 below. The waveguide is air-filled and matched by a PML layer from both sides as an absorbing boundary condition, the PML layer is then terminated by a matched impedance to enhance the absorbing characteristics of the PML. The structure was excited by Dirac delta function impulse. The mesh size is 5 mm in coarse grid and 1.667 mm in fine grid. CLF time-step limits were used in both fine and coarse meshes, namely, 8.33 ps and 2.77 ps.

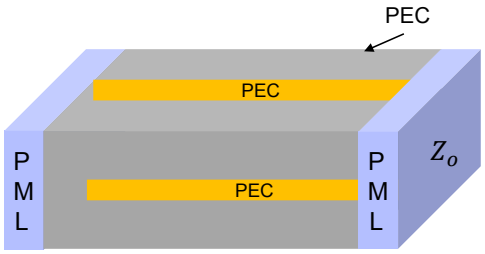

Figure 5, ridged rectangular waveguide terminated by a PML from both sides

The cross-section of the waveguide is a square of 10 cm side length and, two ridged of 3 cm length. The subgridding is manly used to enhance the spatial resolution around the fins in the waveguide. A subgridding ratio of $N_s = 3$ was used as shown in figure 6.

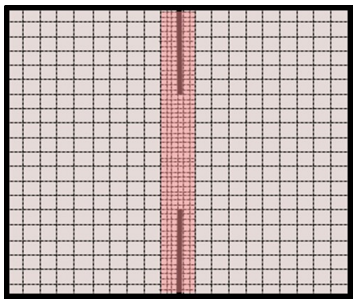

Figure 6, transversal cross-section of waveguide in figure 5, meshed using meshing per block to enhance the spatial resolution around the discontinuities

For the first five cutoff frequencies, the relative errors in the obtained results were within 0.23% as compared to a computational domain that uses fine mesh everywhere. However, the CPU time was six times smaller and the memory requirement was around 40% lower compared to the case when a uniform fine mesh was used everywhere in the computational domain. Furthermore, the simulation did not show any signs of instability even after 2 million iterations.

4. Conclusion

In this paper a new approach of block-meshing that uses local time-step was presented. The idea is to treat the computational domain as a multi-rate network. Thus, using appropriate filters at the interface between differently meshed regions remove the instability usually observed. Simulation experiments were presented to show the long-term stability and accuracy with, in addition, some computer cost reduction. On-going work concerns theoretical and numerical investigation to assess the stability and the gain in computer expenditure.

4. References

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