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Scattering from Finite Periodic Arrays of Scatterers Using Broadband Green's Function of Infinite Periodic Scatterers

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Abstract

In this paper, we use full wave simulation to study scattering from finite arrays of periodic scatterers. The formulation is based on surface integral equations. Instead of using the free space Green's function, we formulate a dual surface integral equation utilizing the Green's function of an infinite array of periodic scatterers. The Green's function of the periodic scatterer satisfies all the boundary conditions on the scatterers. Thus, the unknowns are only limited to the confining boundary of the finite periodic array. This greatly improves the computing efficiency and is distinct from the effective medium theory where the periodic structure is replaced by a homogeneous material of the effective permittivity and permeability. The effective medium theory is only valid at the long wave limit, while this new approach provides exact solution at all wavenumbers. The Green's function of the periodic scatterer is represented in terms of the band solutions of the infinite periodic array. A low wavenumber component is extracted out from the modal summation series, representing the reactive near field interactions. The remaining part converges fast and is free of singularity. The remaining part can be efficiently evaluated at a broadband of wavenumbers. The band solution of the infinite periodic array is derived from a surface integral equation using the periodic Green's function of the empty lattice, where the technique of broadband Green's function with low wavenumber extraction (BBGFL) is applied, converting the surface integral equation into a linear eigenvalue problem, where the modes are independent of wavenumbers. The Green's function of the periodic scatterer represents field responses in a periodic array due to a point source. The Green's function is an important physical concept and is key to the formulation of integral equations. The application of such Green's function is demonstrated by calculating the reflections from a half-space of periodic scatterers.

1. Introduction

Scattering from finite periodic arrays is an important problem in practical engineering of periodic wave functional materials such as photonic crystals and metamaterials. Many applications depend on structuring the artificial material into specific shapes, such as in the design of cloaks, radomes, prisms, and lens. The supporting of unidirectional surface waves in a photonic topological insulator is only of interest when the periodic structure is finite. Approximation methods using effective medium theory to replace the periodic structure with its effective permittivity and permeability are commonly used.

For full wave simulation of finite periodic arrays, the common methods are the finite difference in time domain (FDTD) and the finite element method (FEM). Both methods involve volumetric discretizations of the entire structure and lead to large number of unknowns. The T-matrix method has also been used to formulate Foldy-Lax multiple scattering equation. In the numerical solutions, the T-matrix coefficients need to be associated with each scatterer. Methods based on surface integral equations have also been developed, using Green's function of the free space, where unknowns are associated with the surface of each scatterer.

In this paper, we developed a new method to solve wave propagation and scattering in finite periodic arrays based on a pair of dual surface integral equation (SIE). We divide the space into two regions. For the region filled with periodic array, we use the broadband Green's function of the scatterer with low wavenumber extraction (BBGFL) to formulate the surface integral equation. The Green's function of the scatterer represents field responses due to a point source at the presence of an infinite array of periodic scatterers. This Green's function satisfies boundary conditions on the scatterer. For the region beyond the periodic array, we choose the free space Green's function of the surrounding media to formulate the surface integral equation. The resulting coupled dual surface integral equations are only limited to the confining boundary of the finite array of the periodic scatterer instead of the surfaces of all the scatterers. The number of surface unknowns in method of moments (MoM) in solving such SIE is greatly reduced, significantly improving the computational efficiency.

The Green's function of the periodic scatterers is represented in terms of multiple band solutions of the infinite periodic structure. A low wavenumber component of the Green's function is subtracted out, representing reactive near field interactions. The remaining part is singularity free and converges fast, requiring few band solutions, making it ready and efficient to be computed over a broadband spectrum [1]. The technique of broadband Green's function with low wavenumber extraction (BBGFL) is again applied to the periodic Green's function of the empty lattice to derive multiple band solutions of the periodic structure using a surface integral equation on the boundary of a single scatterer. The method of moments (MoM) is applied to the surface integral equation, converting it into a linear eigenvalue problem of small size. Because the matrix of the linear eigenvalue problem is independent of wavenumber, multiple band solutions are calculated readily [1-3].

This new method enables the efficient broadband analysis of finite periodic arrays of large electrical size using full wave simulations. The periodic scatterer can be of arbitrary shape with arbitrary volume filling ratio. The truncation of the periodic array can be also of arbitrary boundary. The effectiveness of such method is demonstrated by calculating the reflections from a half-space of periodic scatterers from full wave simulations as compared to the results using the effective medium theory.

2. Methodology and Illustration

The periodic Green's function $g_P^S\left(k,\bar{k}_i;\bar{\rho},\bar{\rho}'\right)$ at a single wave vector $\bar{k}_i$ including the periodic scatterers due to a point source array with progressive phase shift is expressed in terms of the band solution in (1) with a low wavenumber component k_L being extracted out, where k_β^S and ψ_β^S represents the modal wavenumber and the normalized modal field distribution, respectively.

$$g_P^S\left(k,\bar{k}_i;\bar{\rho},\bar{\rho}'\right)=\sum_\beta \frac{\psi_\beta^S\left(\bar{k}_i;\bar{\rho}\right)\psi_\beta^{S*}\left(\bar{k}_i;\bar{\rho}'\right)}{\left[k_\beta^S\left(\bar{k}_i\right)\right]^2-k_L^2}$$
$$+\left(k^2-k_L^2\right)\sum_\beta \frac{\psi_\beta^S\left(\bar{k}_i;\bar{\rho}\right)\psi_\beta^{S*}\left(\bar{k}_i;\bar{\rho}'\right)}{\left(\left[k_\beta^S\left(\bar{k}_i\right)\right]^2-k^2\right)\left(\left[k_\beta^S\left(\bar{k}_i\right)\right]^2-k_L^2\right)} \quad (1)$$

Note the second term, defined as the broadband Green's function, converges as fast as $\left[k_\beta^S\left(\bar{k}_i\right)\right]^{-4}$, and the only wavenumber dependence is in the multiplicative factor k^2, and it is efficient to be evaluated over multiple wavenumbers. There is no singularity in this term at any source and field points. In contrast, the first term denotes the low wavenumber contribution, and converges slowly as $\left[k_\beta^S\left(\bar{k}_i\right)\right]^{-2}$. The slowly converging series contains reactive near field interactions and is only required to be evaluated once at a single wavenumber. This can be calculated readily from a surface integral equation [1]. The solution of the multiple band solutions and their normalization using the technique of broadband Green's function with low wave number extraction (BBGFL) is detailed in [1-3]. The Green's function due to a single point source $g^S\left(k;\bar{\rho},\bar{\rho}'\right)$ is obtained by integrating $g_P^S\left(k,\bar{k}_i;\bar{\rho},\bar{\rho}'\right)$ over $\bar{k}_i$ in the first Brillouin zone [1].

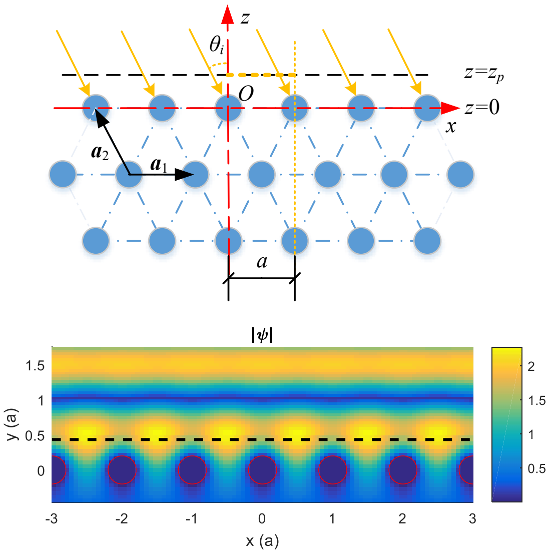

Figure 1. Scattering of a plane wave from a half-space of periodic scatterers. (a, top) illustration of the boundary. (b, bottom) total field distribution near the boundary at normalized frequency of 0.2 (Units of c/a).

Note that $g^S\left(k;\bar{\rho},\bar{\rho}'\right)$ is the field response in an infinite array of periodic scatterers that satisfies all the boundary conditions of the periodic scatterers. Such Green's function can be applied to solve scattering from a truncated periodic array, reducing the unknowns to be only on the interface of truncation. Such concept is demonstrated in Fig. 1 (a), where a half-space of periodic scatterers is illuminated by an incoming plane wave polarized along the cylinder axis. Above the virtual boundary at $z=z_p$, we use free space Green's function g^0 to formulate the surface integral equation (SIE); below the boundary we choose $g^S\left(k;\bar{\rho},\bar{\rho}'\right)$. The coupled SIEs are as follows,

$$\psi_{inc}\left(\bar{r}\right)-\int_0^a dx'\left[g_{Px}^0\frac{\partial\psi}{\partial z'}-\psi\frac{\partial g_{Px}^0}{\partial z'}\right]=0, z\to z_p^-$$
$$\int_0^a dx'\left[g_{Px}^S\frac{\partial\psi}{\partial z'}-\psi\frac{\partial g_{Px}^S}{\partial z'}\right]=0, z\to z_p^+ \quad (2)$$

Note in this specific example, we have further applied periodic boundary condition along x to reduce the integration domain to one period, where g_{Px}^0 and g_{Px}^S are related to g^0 and g^S, by

$$g_{Px}^{0,S}\left(k_{ix},k;x,z;x',z'\right)=\sum_{n=-\infty}^{\infty} g^{0,S}\left(x,z;x'+na,z'\right)\exp\left(ik_{ix}na\right) \quad ,$$

respectively.

After solving the surface field ψ, the field everywhere can be derived from the extinction theorem. The magnitude of the total field distribution is depicted in Fig. 1 (b) near the boundary of the PEC cylinders. All the incident wave is being reflected as the wave frequency falls in the first stop band of the wire media, and the media behaves as a homogeneous material with negative permittivity away from the boundary.

3. Conclusions

The Green's function of periodic scatterers is derived and next used to formulate integral equations for scattering from finite periodic arrays. The Green's function is expressed in terms of band solutions and is broadband. The BBGFL method described is a method of full wave simulation of finite periodic arrays of large electrical size.

4. References

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