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Fundamental limits on the MIMO capacity using semidefinite programming

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Extended Abstract

Optimization of the antenna current is an efficient technique to determine physical bounds on arbitrary shaped antennas [1]. The antenna performance can be expressed in classical quantities involving the Q-factor, gain, directivity, and efficiency. MIMO (multiple input multiple output) antennas are more intricate and it is not sufficient to express their performance using these quantities. Here, the maximum capacity with fixed Q-factor and signal-to-noise ratio is formulated as a convex optimization problem in the covariance matrix of the current distribution. The computational complexity of the problem is reduced by a model order reduction using current modes.

A classical MIMO system is modeled using the input output model $\mathbf{y} = \mathbf{H}\mathbf{x} + \mathbf{n}$, where $\mathbf{x}$ denotes the $N \times 1$ matrix with input signals, $\mathbf{y}$ the $M \times 1$ matrix with output signals, $\mathbf{n}$ the $M \times 1$ matrix with additive noise, and $\mathbf{H}$ the $M \times N$ channel matrix [2]. The channel matrix models the antennas and the wave propagation between the antennas. The fundamental limitations are determined using a channel matrix $\mathbf{H}$ modeling an arbitrary antenna and an idealized receiver. The transmitting antenna is described by a MoM (method-of-moments) approximation of the current distribution, where each basis function corresponds to an element in $\mathbf{x}$. The idealized receiver is modeled using the radiated spherical modes such that each mode is an element in $\mathbf{y}$ [3]. The capacity of the MIMO system is hence in practice unbounded as the dimension N increases with the mesh refinement and M with the number of included spherical modes. This unrealistic unbounded capacity is similar to the case with superdirectivity and is removed by inclusion of constraints on the efficiency or Q-factor.

The efficiency and Q-factor are expressed as quadratic forms in the current density. Here, we use the stored energy in [4] simply determined from the MoM impedance matrix. The Q-factor is determined from the quotient between the average stored energy and average dissipated power. Maximization of the ergodic capacity for a fixed signal-to-noise ratio and Q-factor is formulated as a convex optimization problem. The convex optimization problem is a semi-definite program expressed in the covariance matrix of the current distribution which has $\approx N^2/2$ parameters. It is hence difficult to solve the problem for a fine mesh (large N) unless model order reduction is applied to reduce the dimension of the optimization problem. Here, approaches using characteristic and energy modes are considered. Numerical results for the maximal capacity as a function of the Q-factor and electrical size are used to illustrate the results. The solution of the optimization problem produce the maximum capacity and the covariance matrix of the current distribution. Moreover, the capacity and covariance matrix are compared with the minimum Q current modes on the antenna structure [5].

References

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