

## Going Beyond Chu Harrington Limit: ULF Radiation with a Spinning Magnet Array

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### Abstract

ULF antennas, if made portable within a diameter of 1m, will operate under an electrical length of the order of  $10^{-4}$  to  $10^{-6}$  wavelengths in free space. Like for any other electrically small antenna, Chu's limit kicks in and defines the lower bound of the radiation quality factor. In this paper we present our study on an electromechanical radiating system based on spinning permanent magnets which is not subject to Chu's limit. Based on our simulation study, a spinning magnet antenna array is proposed.

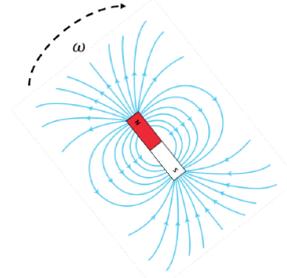
### 1 Introduction

The frequencies lying between 300 Hz to 3 KHz have been designated as Ultra Low Frequency (ULF) with corresponding wavelengths from 1000 Km to 100 Km. ULF has been widely used for military applications involving long range and underground communications. Any practical antenna at ULF would be electrically small making it very inefficient as it is subjected to the Chu Harrington limit. It has been demonstrated in [1–5] that the concept of Direct Antenna Modulation (DAM) can help overcome the efficiency and bandwidth product with high frequency switching schemes. Hence the technique of Direct Antenna Modulation when used in conjunction with Spinning Magnetic Arrays can directly modulate the spinning magnets with On-Off Keying (OOK) or Frequency Shift Keying (FSK) without being constrained by Chu's limit. The elegance of a spinning magnet system is that, DAM can be performed by changing the frequency of spinning of magnets without the use of high speed switches. In this paper, we propose an electro-mechanical system consisting of spinning magnets as a potential approach to overcome the Chu-Harrington limit to the efficiency bandwidth product of electrically small antennas at ULF.

### 2 Theory

Chu's limit places a fundamental limit on the electrical size of an antenna [6–8] and defines a lower bound for the radiation quality factor  $Q_{\text{rad}}$  as represented by:

$$Q_{\text{rad}} > \frac{1}{(ka)^3} + \frac{1}{ka} \quad (1)$$



**Figure 1.** Physically spinning a magnet creates radiation of electromagnetic waves.

where it is assumed that the antenna is enclosed within a sphere of radius  $a$  and  $k = \frac{2\pi}{\lambda}$  with  $\lambda$  being the wavelength. The complete field expression of a rotating magnet can be derived by superimposing the two orthogonal magnetic dipole solutions in space with a 90-degree phase [9]. Assuming the rotation is along the elevation axis (Z-axis) with an angular frequency  $\omega$ , the following electromagnetic fields  $\vec{E}_{\text{rad}}$  and  $\vec{B}_{\text{rad}}$  are obtained in the far field at a distance  $r$ :

$$\vec{E}_{\text{rad}} = \frac{\mu_0 m_0 c}{4\pi} k^2 \frac{e^{i(kr - \omega t)}}{r} (\cos\theta \hat{y} - i\hat{z}) \quad (2)$$

$$\vec{B}_{\text{rad}} = \frac{\mu_0 m_0}{4\pi} k^2 \frac{e^{i(kr - \omega t)}}{r} (\cos\theta \hat{z} + i\hat{y}) \quad (3)$$

where

$\mu_0$  is the permeability of free space

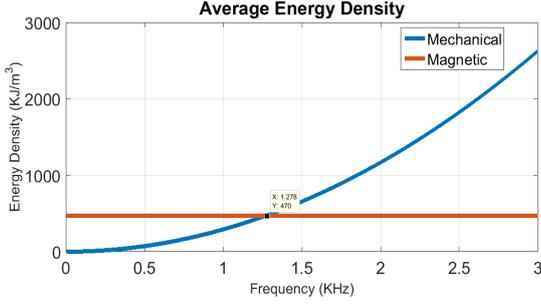
$m_0$  is the magnetic dipole moment

$c$  is the speed of light in vacuum

$k = \frac{\omega}{c}$  is the wave number

$\hat{I} = \hat{y} \times \hat{n}$  and  $\hat{n}$  is the direction of observation

In a conventional electrically small antenna that operates in a linear, time-invariant fashion, the radiated power is always proportional to the stored near field energy by the radiation quality factor, with its lower bound defined in (1). However, in a rotating magnet system, it is important to note that the energy stored in the near field of a permanent magnet is



**Figure 2.** Energy Density as a function of frequency (Radius of magnet=0.2 cm).

constant regardless of the frequency of spinning or orientation of the magnet, while the radiated field is proportional to the square of the spinning frequency as per (2) and (3). It is thus concluded the radiation of a spinning magnet system is not subject to Chu's limit.

### 3 Mechanical and Magnetic Energy Constraints

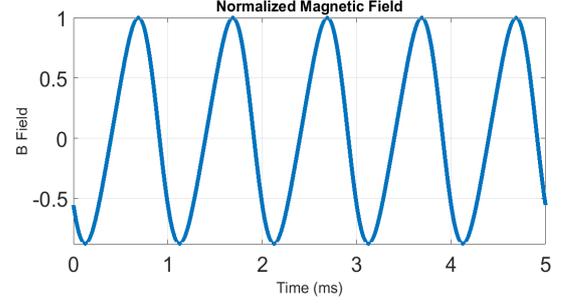
A fundamental issue with this approach is the amount of mechanical energy required to spin the magnet to the designated speed may overwhelm the dynamic magnetic energy it creates. When modulation of the spinning speed is introduced, the energy difference between the two modulation states may be dissipated and require replenishment, which would defeat the original purpose of achieving high efficiency. Therefore, one must take actions to minimize the required mechanical energy density in reference to its magnetic energy density in such a system. The average energy density associated with a circular disc of radius  $r$  and volume  $V$ , rotating at an angular velocity  $\omega$  can be written as

$$\frac{W_{ME}}{V} = \frac{1}{4} \omega^2 r^2 \rho \quad (4)$$

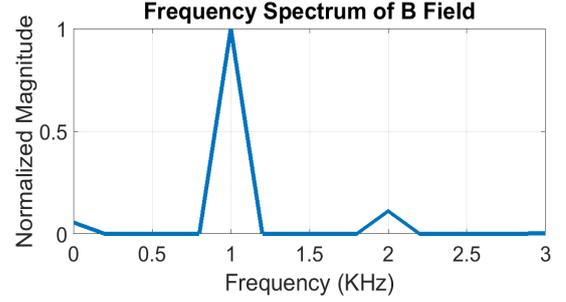
where  $\rho$  is the density. The typical energy density of an Nd-FeB magnet obtained from its  $BH_{max}$  value is around 470  $\text{KJ/m}^3$  [10]. The mechanical energy density turns out to be an order of magnitude greater than magnetic energy density at higher ULF for a magnet of radius 0.2 cm as seen in figure 2. Hence it is feasible to pursue this approach at lower ULF.

### 4 Simulation Study

To validate our concept a simulation study was performed using Ansys Maxwell. A two pole NdFeB magnet with relative permeability of 1.099 and magnetic coercivity of 890  $\text{kA/m}$  was chosen for the study. The frequency of spinning was 1 KHz. The B field at a point 5 cm away from the magnet was plotted as a function of time in figure 3a. The Fourier transform of the signal shows a peak at 1 KHz as expected in figure 3b. A harmonic at 2 KHz can also be seen. It can be explained as the permanent magnet is not a perfect dipole which leads to the existence of higher order



(a) Time varying magnetic field



(b) Frequency Spectrum

**Figure 3.** Simulation Results

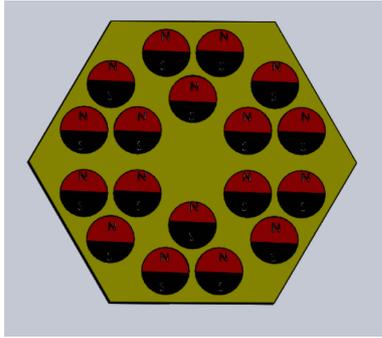
modes giving rise to harmonics. In practice, transmitting ULF signals at 1 KHz requires physically spinning magnets at speeds of 60,000 rpm, which is not a trivial task in the mechanical design. One must design the system to meet the tensile stress limit and reduce the energy loss due to the friction.

### 5 Spinning Magnet Array

As shown in figure 4, we propose an array of smaller magnets that are rotating in a synchronized fashion to emulate a larger spinning disc in its magnetic field behavior while lowering its mechanical energy density significantly. The 18 magnets are divided into 6 groups that are spaced out in a hexagonal fashion. The quasi-axially symmetric distribution of the magnets is to keep the uniformity of the magnetic energy during the rotation, which also helps to improve the efficiency of the overall system. The 18 unit magnet array can be used as a sub-array for a larger scale ULF antenna system. A ferrite plate can be placed that fills the gap in between the magnets to conduct the magnetic flux uniformly and to effectively increase the size of the magnetic dipole for efficient radiation of ULF fields to space.

### 6 Conclusion

An electro-mechanical system consisting an array of spinning magnets that is not constrained by Chu Harrington limit for ULF communications was proposed. DAM technique can be used to modulate the antenna with OOK or FSK by varying the angular frequency of spinning. It was shown that the system operates efficiently for the lower frequency range of ULF band. Future work involves explor-



**Figure 4.** Proposed Spinning Magnet Array

ing different electromechanical systems consisting of magnets and magnetic meta materials for ULF and VLF applications.

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