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Stored energy in dispersive and piecewise homogenous media

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Abstract

A method for calculation of stored energy and Q-factors based on a state-space model is presented. It is shown that the method can be extended to temporally dispersive and piecewise homogeneous media. Numerical examples indicate that this method of calculating the stored energy produce accurate Q factors for such environments.

1 Introduction

Stored energy is used to calculate the Q-factor which is related to bandwidth of small antennas [1, 2, 3, 4]. Bandwidth is of paramount performance when designing small antennas, as its availability is restricted by size. Thus, it is essential that the stored energy is calculated accurately and efficiently for the design case.

Previous methods have calculated stored energy for antennas embedded in free-space [3, 5, 6]. Here, the background media is generalized to dispersive permittivity and permeability. We calculate the stored energy from quadratic forms based on a state-space representation derived from the electric field integral equation (EFIE) and magnetic field integral equation (MFIE) [7]. This method has the ability to calculate the stored energy in dispersive media without predicting unrealistic Q values.

In many applications antennas are placed inside or close to lossy or inhomogenous media, *e.g.*, on- and in-body systems [8]. These configurations are naturally inhomogeneous. However, simulating the performance of antennas in such environments can be both inaccurate and time consuming. In this paper we illustrate how the state-space method can calculate the stored energy in piecewise homogeneous media resembling common in-body situations.

2 Stored Energy and Q-factor

Stored energy for antennas is a concept which arose by necessity to calculate the Q-factor [1, 2, 3, 9]. The Q-factor measures how well an oscillating system stores energy as apposed to dissipating it, and is defined for an antenna tuned to resonance as [1, 2, 3]

$$Q = \frac{2\omega \max\{W_{\mathrm{e}}, W_{\mathrm{m}}\}}{P_{\mathrm{d}}} = \frac{\omega(W + |W_{\mathrm{m}} - W_{\mathrm{e}}|)}{P_{\mathrm{d}}}, \qquad (2.1)$$

where $W = W_{\mathrm{e}} + W_{\mathrm{m}}$, W_{e}, W_{m} denote the stored electromagnetic, electric, and magnetic energies, respectively, and P_{d} is the dissipated power. Because of its relation to the bandwidth, the Q-factor is an important parameter for antenna design. Thus, it has become imperative to define stored energy for antennas [1, 2, 3, 4, 5, 6].

3 State-space model

The state-space model for stored energy is based on the EFIE impedance matrix $\mathbf{Z}$. A standard Method of Moments (MoM) implementation of the EFIE determines the impedance matrix $\mathbf{Z} = \mathbf{R} + \mathrm{j}\mathbf{X}$ [10, 11, 12] that can be written as

$$\mathbf{Z} = s\mu\mathbf{L} + \frac{1}{s\varepsilon}\mathbf{C}_{\mathrm{i}} \quad \text{with } \mathbf{Z}\mathbf{I} = (s\mu\mathbf{L} + \frac{1}{s\varepsilon}\mathbf{C}_{\mathrm{i}})\mathbf{I} = \mathbf{B}V_{\mathrm{in}} \quad (3.1)$$

where s is the Laplace parameter, ε and μ ares the background permittivity and permeability, respectively, $\mathbf{I}$ is the current matrix, $\mathbf{B}V_{\mathrm{in}}$ is the feeding voltage, and the matrices $\mathbf{L}$ and $\mathbf{C}_{\mathrm{i}}$ depend on the wavenumber [10, 13]. A voltage state $\mathbf{U} = \frac{1}{s\varepsilon}\mathbf{C}_{\mathrm{i}}\mathbf{I}$ can be introduced to rewrite the second order system (3.1) as a first order system,

$$s\begin{pmatrix} \mu\mathbf{L} & \mathbf{0} \\ \mathbf{0} & \varepsilon\mathbf{C} \end{pmatrix}\begin{pmatrix} \mathbf{I} \\ \mathbf{U} \end{pmatrix} = \begin{pmatrix} \mathbf{0} & -\mathbf{1} \\ \mathbf{1} & \mathbf{0} \end{pmatrix}\begin{pmatrix} \mathbf{I} \\ \mathbf{U} \end{pmatrix} + \begin{pmatrix} \mathbf{B} \\ \mathbf{0} \end{pmatrix} V_{\mathrm{in}}. \quad (3.2)$$

This system is a classical state-space model for the free-space case $\varepsilon = \varepsilon_0$ and $\mu = \mu_0$ in the limit of small antennas $ka \ll 1$, where k is the wavenumber, and a is the radius of the smallest sphere circumscribing the antenna. The symmetry in the system implies that the stored energy is defined for minimal representations [14]. The stored energy is given by the quadratic form generated by the matrix that multiples s (temporal derivative). However, the frequency dependence of $\mathbf{L}$ and $\mathbf{C}_{\mathrm{i}}$ cannot be neglected for finite sized antennas. To resolve this issue, we use differentiation with respect to s of the state-space model to estimate the term that is proportional to s and hence the time average stored

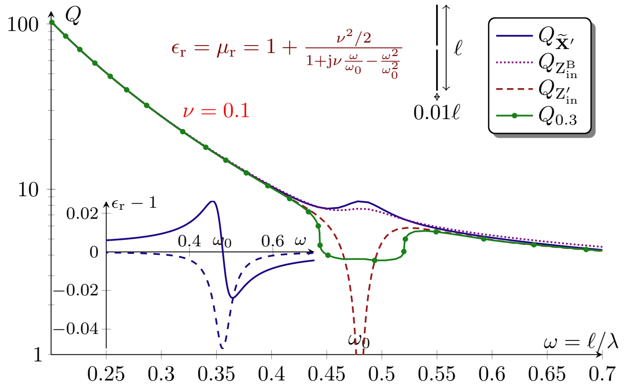

Figure 1. Q-factors for a strip dipole with length ℓ, width $\ell/100$, fed at the center, and placed in a homogeneous electric and magnetic Lorentz medium with relative permittivity and permeability as depicted in the top of the figure and plotted in the bottom left inset, where $\omega = 2\pi\ell/\lambda$ and $\nu = 10^{-1}$. The Q-factors are computed using the state space model $Q_{\widetilde{\mathbf{X}}'}$, Brune synthesis $Q_{Z_{\mathrm{in}}^{\mathrm{B}}}$, differentiation of the input impedance $Q_{Z'_{\mathrm{in}}}$, and fractional bandwidth $Q_{0.3}$ with $\Gamma_0 = 0.3$. The frequency axis is represented in the dimensionless parameter ℓ/λ where λ is the free-space wavelength.

energy, *i.e.*,

$$W_{\widetilde{\mathbf{X}}'} = \frac{\mathrm{Re}}{4}\begin{pmatrix}\mathbf{I}\\ \mathbf{U}\end{pmatrix}^{\mathrm{H}}\begin{pmatrix}\mu_0(\mathbf{L}+\mathrm{j}\omega\mathbf{L}') & \mathbf{0}\\ \mathbf{0} & \varepsilon_0(\mathbf{C}+\mathrm{j}\omega\mathbf{C}')\end{pmatrix}\begin{pmatrix}\mathbf{I}\\ \mathbf{U}\end{pmatrix}$$
$$= \frac{\mathrm{Re}}{4}\left(\mu_0\mathbf{I}^{\mathrm{H}}(\mathbf{L}+\mathrm{j}\omega\mathbf{L}')\mathbf{I}+\varepsilon_0\mathbf{U}^{\mathrm{H}}(\mathbf{C}+\mathrm{j}\omega\mathbf{C}')\mathbf{U}\right)$$
$$\simeq \frac{\mathrm{Re}}{4}\mathbf{I}^{\mathrm{H}}\left(\mu_0(\mathbf{L}+\mathrm{j}\omega\mathbf{L}')+\frac{1}{\omega^2\varepsilon_0}(\mathbf{C}_{\mathrm{i}}-\mathrm{j}\omega\mathbf{C}_{\mathrm{i}}')\right)\mathbf{I} = \frac{1}{4}\mathbf{I}^{\mathrm{H}}\frac{\partial\mathbf{X}}{\partial\omega}\mathbf{I}, \tag{3.3}$$

where $\mathbf{C}' = -\mathbf{C}\mathbf{C}_{\mathrm{i}}'\mathbf{C}$ is used. This expression for stored energy is identical to the expressions proposed by Harrington [15] and Vandenbosch [5] for antennas operating in free space. However, the strength of this method is that it can be generalized to dispersive media. Using the same methodology while introducing complex, frequency dependent ε and μ results in energy expressions for temporally dispersive background media, see [7].

Fig. 1 shows Q-factors calculated for a strip dipole in a homogeneous electric and magnetic Lorentz medium. The Q-factors agree well except around the resonance frequency ω_0, which corresponds with the antennas resonance, where $Q_{Z'_{\mathrm{in}}}$ has a dip down to very low Q values. $Q_{0.3}$ also decreases down to a lower value in this region where as $Q_{\widetilde{\mathbf{X}}'}$ and $Q_{Z_{\mathrm{in}}^{\mathrm{B}}}$ exhibit an increase in values instead, with $Q_{Z_{\mathrm{in}}^{\mathrm{B}}}$ having slightly lower values than $Q_{\widetilde{\mathbf{X}}'}$. This indicates that there is disagreement between methods in this type of media.

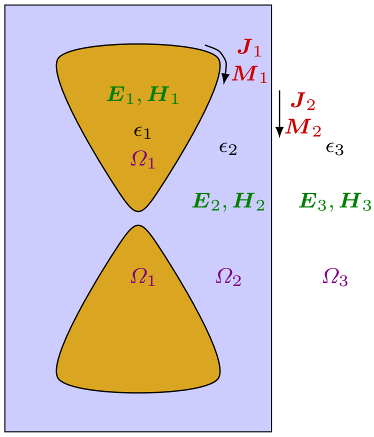

Figure 2. Illustration of an arbitrary antenna embedded in a dielectric material. Currents J, M, fields E, H, and permittivity ε is displayed for the three regions $\Omega_{1,2,3}$.

4 Piecewise homogeneous background media

The surface equivalence principle is used to express the electromagnetic fields in piecewise homogeneous media [10, 11, 12, 16, 17]. Consider for simplicity a perfect electric conductor (PEC) antenna structure embedded in a media with permittivity and permeability as depicted in Fig. 2. The geometry is divided into the three regions Ω_p with $p = 1,2,3$ and corresponding material parameters ε_p and μ_p. The field in region Ω_p is expressed by the equivalent currents J_{p-1}, J_p, M_{p-1}, and M_p, where $J_0 = M_0 = 0$. We follow the state-space approach and construct a system for the input impedance $Z_{\mathrm{in}} = V_{\mathrm{in}}/I_{\mathrm{in}}$.

The EFIE and MFIE for the inner region Ω_2 is written

$$\begin{pmatrix}\mathbf{Z}_{2,11} & \mathbf{Z}_{2,12} & \mathbf{K}_{2,12}\\ \mathbf{Z}_{2,21} & \mathbf{Z}_{2,22} & \mathbf{K}_0+\mathbf{K}_{2,22}\\ -\mathbf{K}_{2,21} & \mathbf{K}_0-\mathbf{K}_{2,22} & \frac{1}{\eta_2^2}\mathbf{Z}_{2,22}\end{pmatrix}\begin{pmatrix}\mathbf{I}_1\\ \mathbf{I}_2\\ \mathbf{M}_2\end{pmatrix} = \begin{pmatrix}\mathbf{V}_1\\ \mathbf{0}\\ \mathbf{0}\end{pmatrix} \tag{4.1}$$

where $\mathbf{Z}_{p,oq}$ denotes the EFIE impedance matrix (3.1) connecting the surfaces o and q and evaluated using materials ε_p and μ_p. The EFIE and MFIE for the exterior region Ω_3 is similarly

$$\begin{pmatrix}\mathbf{Z}_{3,22} & -\mathbf{K}_0+\mathbf{K}_{3,22}\\ -\mathbf{K}_0-\mathbf{K}_{3,22} & \frac{1}{\eta_3^2}\mathbf{Z}_{3,22}\end{pmatrix}\begin{pmatrix}\mathbf{I}_2\\ \mathbf{M}_2\end{pmatrix} = \begin{pmatrix}\mathbf{0}\\ \mathbf{0}\end{pmatrix}. \tag{4.2}$$

In order to suppress internal resonance problems the EFIE and MFIE in (4.1) and (4.2) can be combined in different ways *e.g.*, as the PMCHWT and Müller integral equations [17, 11]. These equations can be decomposed to second order state-space equations as in (3.1). Here we consider the two regions as two formally identical parts,

$$\begin{pmatrix}s\mu\mathbf{L}+\frac{\mathbf{C}_{\mathrm{i}}}{s\varepsilon} & \mathbf{K}\\ -\mathbf{K} & s\varepsilon\mathbf{L}+\frac{\mathbf{C}_{\mathrm{i}}}{s\mu}\end{pmatrix}\begin{pmatrix}\mathbf{I}\\ \mathbf{M}\end{pmatrix}, \tag{4.3}$$

where the material parameters ε_p and μ_p are used in region Ω_p. This second order system can be rewritten similarly

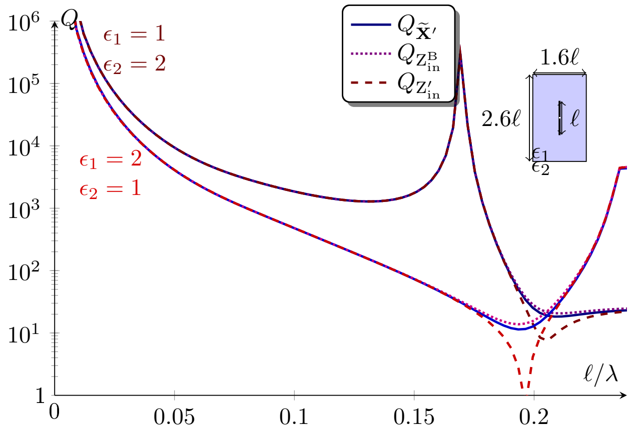

Figure 3. Q-factors for a cylindrical dipole with length ℓ, width $\ell/100$, fed at the center, and placed in a dielectric cylinder of height 2.6ℓ, diameter 1.6ℓ and permittivity ε_1. The background has permittivity ε_2. The Q-factors are computed using the state space model $Q_{\widetilde{\mathbf{X}}'}$, Brune synthesis $Q_{Z_{\rm in}^{\rm B}}$, and differentiation of the input impedance $Q_{Z'_{\rm in}}$ for the two cases $\{\varepsilon_1,\varepsilon_2\} = \{1,2\}$ and $\{\varepsilon_1,\varepsilon_2\} = \{2,1\}$.

as (3.2) with the voltage states $\mathbf{U}$ and $\mathbf{U}_{\rm m}$,

$$\begin{pmatrix} s\mu\mathbf{L} & \mathbf{1} & \mathbf{K} & \mathbf{0} \\ -\mathbf{1} & s\varepsilon\mathbf{C} & \mathbf{0} & \mathbf{0} \\ -\mathbf{K} & \mathbf{0} & s\varepsilon\mathbf{L} & \mathbf{1} \\ \mathbf{0} & \mathbf{0} & -\mathbf{1} & s\mu\mathbf{C} \end{pmatrix} \begin{pmatrix} \mathbf{I} \\ \mathbf{U} \\ \mathbf{M} \\ \mathbf{U}_{\rm m} \end{pmatrix}. \tag{4.4}$$

By taking the quadratic form of the frequency differentiation of this matrix, as in (3.3), the stored energy is calculated, and total stored energy is gained by summation over all regions [7]. This method can be extend for an arbitrary number of regions, however the computational complexity is the limiting factor when meshing subsequent layers.

In Fig. 3 Q-factors calculated by the state space model $Q_{\widetilde{\mathbf{X}}'}$, Brune synthesis $Q_{Z_{\rm in}^{\rm B}}$ [18, 19], and differentiation of the input impedance $Q_{Z'_{\rm in}}$ [3] are shown for a cylindrical dipole embedded in a dielectric cylinder. The Q-factors agree well for most of the simulated frequencies. However, $Q_{Z'_{\rm in}}$ seems to have a significant dip in both configurations around $\ell/\lambda = 0.2$, $Q_{\widetilde{\mathbf{X}}'}$ and $Q_{Z_{\rm in}^{\rm B}}$ on the other hand retain Q values above 10 in this region. This divergence is most likely caused by multiple resonance phenomena.

5 Conclusions

The state space method of calculating stored energy has been illustrated for homogeneous and piecewise homogeneous media. It has been shown that this method has the capability to calculate accurate Q values for these cases and seems to avoid inconsistencies displayed by previous methods designed for free-space. These results contribute to the accurate calculation of Q-values for in-body antennas inside of their operating environment. This can enable faster antenna optimization in such environments as well as the establishment of physical bounds for those environments.

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