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Space-Time (ST) Reflection Focusing in Dispersion-Engineered Medium

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Abstract

The space-time focusing of a pulse after reflection can be realized in a nonreciprocal dispersive medium. We show here how this task can be accomplished using a space-time (ST) periodic dispersive nonreciprocal medium.

1 Introduction

The spatial focusing of waves is typically achieved using lenses whose curvature deflects rays to produce a focus at a specific position in space. In [1–3], it was shown that focusing in space and time, or space-time (ST) focusing, may be accomplished by a temporal discontinuity using the principle of time reversal. Here, we show that ST focusing may be accomplished in a medium with properly engineered dispersion curvature. In contrast to the ST focusing reported in [1–3], dispersion-engineered ST focusing is fundamental based on medium nonreciprocity, rather than on time reversal.

2 Principle of Dispersive-Medium Space-Time Focusing

The principle of ST focusing is illustrated in Fig. 1. Figure 1(a) traces the ST trajectory of the pulse, first travelling to the right, then reflected at the wall and finally refocused at the point $z = 0$. Figure 1(b) shows the dispersion diagram of a medium that would accomplish such a focusing. Note that a medium with such an asymmetric dispersion with respect to the wavenumber is fundamentally a nonreciprocal one. The two rays correspond to the minimal and maximal frequencies of the pulse, colored red and blue, respectively. The pulse is launched at the ST point $(z,t) = (0,0)$, as illustrated in Fig. 1(a). It then spreads out in space and time, with higher frequencies (blue ray) propagating *faster* than lower frequencies (red ray), given the dispersion (slope $\propto \partial\omega/\partial k_z = v_g$, group velocity) of the medium [Fig. 1(b)]. After reflection, the velocities of the frequencies are inverted, due to the inversion of the dispersion curvatures, so that the higher frequencies are now propagating *slower* than the lower frequencies. Since the velocities are exactly opposite, focusing occurs at $z = 0$.

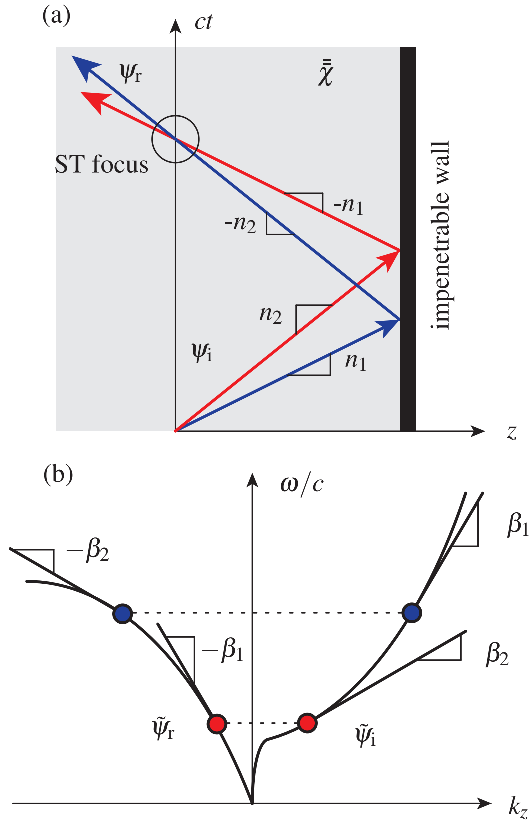

Figure 1. Illustration of space-time (ST) focusing in a dispersive nonreciprocal medium with generic susceptibility tensor $\bar{\bar{\chi}}$. (a) ST rays of a pulse in the Minkowski diagram. The incident pulse ψ_i is launched at the ST point $(z,t) = (0,0)$. It propagates toward the right, reflects on the wall, and the reflected pulse ψ_r focuses later in time at the point $z = 0$. The blue and red rays correspond to the trajectories of the highest and lowest frequencies, respectively. The slopes of the trajectories $n_{1,2}$ correspond to the refractive indices, with $n_2 > n_1$. (b) Qualitative dispersion diagram of the corresponding ST focusing medium, with incident and reflected spectra $\tilde{\psi}_\mathrm{i,r}$. The slopes correspond to the normalized group velocities $\beta_{1,2} = v_{\mathrm{g}1,2}/c = (\partial\omega/\partial k_z)/c$.

3 Periodic Space-Time Focusing System

One method for introducing nonreciprocity is space-time modulation. This is the method we choose here. In particular, we consider a ST-modulated collision-less plasma without magnetic bias, whose wave equation reads

$$\left(\frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} - \frac{\omega_{\mathrm{p}}^2(z,t)}{c^2}\right)\mathbf{E} = 0, \tag{1}$$

with spatially periodic plasma frequency $\omega_{\mathrm{p}} = \omega_{\mathrm{p}0} + \Delta\omega_{\mathrm{p}}\cos(Kz - \Omega t)$ where K and Ω are the modulation wavenumber and frequency. Figure 2(a) represents such a medium in the Minkowski diagram. The wavelength and period of the medium are Λ and T, and the modulation velocity is therefore $v_{\mathrm{m}} = \Lambda/T$. A frame of reference moving along with the perturbation at velocity v_{m} sees only spatial periodicity. Therefore, the time axis ct' is drawn parallel to the interface of the ST modulation.

To mathematically derive the dispersion diagram corresponding to this medium, we first note that the solution of the wave equation (1) takes on the Bloch-Floquet form [4]:

$$\mathbf{E} = e^{i(kz-\omega t)} \sum_{m=-\infty}^{\infty} \mathbf{E}_m e^{im(Kz-\Omega t)}. \tag{2}$$

Inserting the expansion (2) into (1) yields a matrix equation whose determinant roots correspond to the dispersion diagram, schematically drawn in Fig. 2(b) for an infinitesimally small modulation $\Delta\omega_{\mathrm{p}} \to 0$. The temporal and spatial modulation frequencies are $\Omega = 2\pi/T$ and $K = 2\pi/\Lambda$. Notice the nonmodulated dispersion diagram corresponding to a single harmonic $m = 0$ is repeated along the spatial frequency axis k'_z, since in that frame of reference the perturbation is spatial. Note also that the slopes of the dispersion curves are unaffected by the modulation. This is because the modulation does not alter the group velocity of the waves. The picture would be different if the medium was actually in motion, as in that case the velocities in the forward and backward directions would be different.

The highlighted zones on the dispersion curves of Fig. 2 (b) have opposite curvatures, as desired. Figure 2(c) shows the dispersion diagram for non-negligible modulation. In this case, band-gaps open up at the intersections of the dispersion curves. The maximal and minimal frequencies of the incident wave are plotted on the $m = 0$ curve. They are selected such that they fall into the bandgap created at the intersection of the $m = 0$ and $m = 1$ harmonic. In this configuration, all the energy of the reflected wave is found on the $m = -2$ harmonic, where the curvature is opposite to that of the $m = 0$ harmonic.

4 Conclusion

We have shown a how a dispersive nonreciprocal medium may lead to ST focusing. We suggested a periodic ST dispersive medium that has the required characteristics. This could be realized using the apparatus presented in [5]. Other types of nonreciprocal media may have the required characteristics.

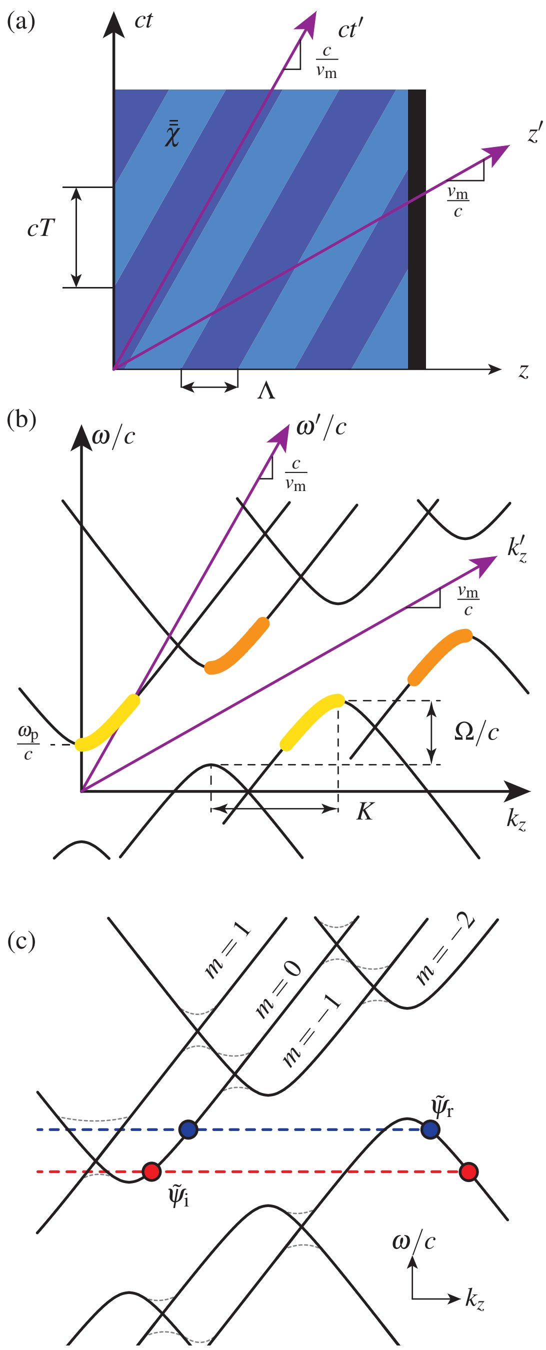

Figure 2. Space-Time periodic dispersive medium. (a) Illustration in Minkowski diagram of ST-periodic medium with spatial and temporal periods Λ and T, with modulation velocity $v_{\mathrm{m}} = \Lambda/T$. The trajectory of the medium is parallel to the ct' axis. (b) Corresponding dispersion diagram for plasma frequency ω_{p}, modulation depth $\Delta\omega_{\mathrm{p}} \to 0$ and spatial and temporal modulation frequencies K, Ω. The periodicity of the dispersion curves is parallel to the k'_z axis. The highlighted zones have opposite curvatures, as required for focusing. (c) Incident spectra and reflected spectra on dispersion diagram with non-zero modulation depth $\Delta\omega_{\mathrm{p}} \neq 0$ with bandgaps in dotted gray curves.

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