

Analysis of the Uniform Theory of Diffraction (UTD) Formulation in the High Frequency Planar Limit

Whitney B. Larsen^{*(1)} and Deb Chatterjee⁽²⁾

(1) Honeywell Federal Manufacturing & Technologies (FM&T), KC, MO 64147, USA

(2) CSEE Department, University of Missouri at Kansas City (UMKC), KC, MO 64110, USA

Extended Abstract

Planar limit of high frequency asymptotic formulations often serves as a validation check in terms of providing insight into the consistency of the physical of the solutions to the problem. For electromagnetic scattering by perfectly conducting (PEC) convex bodies [1]–[3], the various asymptotic formulations commonly utilize the Watson transform for conversion of the exact eigenfunction series solution to a contour integral, to be followed by its subsequent evaluation along the steepest descent contours passing through appropriate saddle points. In the deep shadow region, the result of such an evaluation is the *creeping wave* representation whose convergence deteriorates near the paraxial regions [4],[5].

The z -component of the scattered field, E_z^s , due to an electric line source in presence of a PEC circular cylinder of electrical radius ka can be approximated for $ka \rightarrow \infty$ from the exact, infinite eigenfunction solution as:

$$E_z^s \approx \frac{j\eta I}{4\pi} \left(\frac{e^{-jk(\rho+\rho')}}{\sqrt{\rho\rho'}} \right) \sum_{n=-\infty}^{+\infty} \exp \left\{ j[n(\pi + \phi - \phi')] + 2j \tan^{-1} \left(\frac{8ka}{4n^2 - 1} \right) \right\}. \quad (1)$$

In (1) (ρ, ϕ) and (ρ', ϕ') are the field and source points, respectively, and $(\rho, \rho') \gg a$. (The magnetic field, H_z^s , due to a magnetic line source can be derived from (1) via the duality principle.) To (1), the direct field from the line source in absence of the cylinder needs to be added to obtain the total field.

Equation (1) was obtained by algebraic manipulation of the standard eigenfunction solution [1, p. 197], and using the large argument asymptotic forms for the Hankel functions. The Watson transform [1, Eq. (2), p. 198], [3, Eq. (A2b)] can be used to convert the infinite series in (1) to a contour integral, followed by its steepest descent evaluation for $ka \rightarrow \infty$.

Using image theory, the exact 2-D Green's function for an electric line source radiating in presence of a perfectly conducting infinite ground plane can be derived, for numerical comparisons against (1) above and the UTD formulation [3] when $ka \rightarrow \infty$ in the planar limit for the latter case. These 2-D results, and their extensions to the 3-D case of an axial Hertzian electric dipole radiating in presence of an infinite PEC cylinder, are currently in progress.

References

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