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Scattering of an Evanescent Wave from the End-Face of an Ordered Waveguide System

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Abstract

The scattering of an evanescent wave from the end-face of an ordered waveguide system composed of identical cores of equal space is treated by the perturbation method and the scattered field is analytically derived. The complex power of the total field is calculated and it is shown that the effective power incident on the end-face is generated by the interference between the incident and scattered waves.

1 Introduction

The diffraction efficiency is defined by the ratio of the diffracted power to the incident power in an intuitive manner. An evanescent wave incident on a periodic grating is scattered and yields the diffracted plane and evanescent waves. Since the incident power is reactive the diffraction efficiency cannot be defined. Nakayama has derived the effective power of an evanescent wave by using the identity $Im[div(\Psi^* grad\Psi)] = 0$ where Ψ is the electric or magnetic field and defined newly the diffraction efficiency [1]. However, the physical interpretation is not given so far.

The scattering of a plane wave from the end-face of a waveguide system composed of a large number of cores and a cladding has been treated by the perturbation method and the scattered field has been analytically derived [2,3]. The method is applicable to the plane wave scattering from a periodic grating and the diffraction amplitude can be analytically derived.

In this paper the scattering of an evanescent wave from the end-face of an ordered waveguide system composed of identical cores of equal space is treated by the perturbation method and the scattered field is analytically derived. The complex power of the total field is calculated and it is shown that the effective power incident on the end-face is generated by the interference between the incident and scattered waves.

2 Evanescent wave scattering

We consider the scattering of a TE evanescent wave from the end-face of an ordered waveguide system composed of identical cores of equal space (see Figure 1). The total electric field E_y is divided as follows:

$$E_y = \begin{cases} E_y^i + E_y^r + E_y^s, & z > 0 \\ E_y^t + E_y^s, & z < 0 \end{cases} \tag{1}$$

where E_y^i is the incident evanescent wave and E_y^r and E_y^t are the reflected and transmitted waves from the interface between air and cladding, respectively. The scattered wave E_y^s is expressed as the radiated wave from the electric polarization induced in the cores,

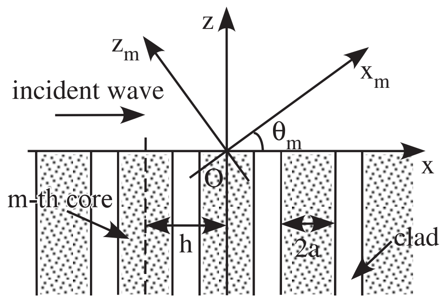

Figure 1. Geometry of the problem.

$$\begin{aligned} E_y^s = \; & k_0^2 \delta n^2 \textstyle\sum_m \int_{S_m} G_y(r,r') E_y^t(r') dr' \\ & + k_0^2 \delta n^2 \textstyle\sum_m \int_{S_m} G_y(r,r') E_y^s(r') dr' \end{aligned} \tag{2}$$

where $\delta n^2 = n_1^2 - n_2^2$ and n_1 and n_2 are the refractive indices of core and cladding, respectively. S_m is the space region occupied by the m-th core. G_y is the Green's function for the two-layered medium composed of air and cladding, which is given by

$$\begin{aligned} G_y(r,r') = \; & \frac{1}{j4\pi} \int_{-\infty}^{\infty} \frac{2}{q_0(\xi)+q_2(\xi)} \\ & \times e^{-j\xi(x-x') - jq_0(\xi)z + jq_2(\xi)z'} d\xi \end{aligned} \tag{3}$$

for $z > 0$ and $z' < 0$ where

$$q_0(\xi) = \sqrt{k_0^2 - \xi^2}, \quad q_2(\xi) = \sqrt{k_0^2 n_2^2 - \xi^2} \tag{4}$$

Since the refractive index difference of an optical fiber δn^2 is very small, we use only the first term of the right-hand side of Eq.(2) to calculate the scattered field.

The incident wave is expressed as

$$E_y^i = e^{-jpx + jq_0(p)z} \tag{5}$$

where p is the incident wave number ($|p| > k_0$) and $Im q_0(p) < 0$. The reflected and transmitted waves are

$$E_y^r = R(p)e^{-jpx-jq_0(p)z} \tag{6}$$

$$E_y^t = T(p)e^{-jpx+jq_2(p)z} \tag{7}$$

where R and T are the reflection and transmission coefficients, respectively,

$$R(p) = \frac{q_0(p)-q_2(p)}{q_0(p)+q_2(p)}, \quad T(p) = \frac{2q_0(p)}{q_0(p)+q_2(p)} \tag{8}$$

By substituting Eqs.(3) and (7) into Eq.(2) and executing the integral the scattered wave is given by

$$E_y^s = \sum_m D_m(p)e^{-jp_mx-jq_0(p_m)z} \tag{9}$$

where $D_m(p)$ is the m-th order diffraction amplitude,

$$\begin{aligned} D_m(p) = \; & -\frac{k_0^2\delta n^2 K_h T(p)}{4\pi} \\ & \times \frac{2}{q_0(p_m)+q_2(p_m)} \frac{\tilde{\phi}_a(mK_h)}{q_2(p)+q_2(p_m)} \end{aligned} \tag{10}$$

and $p_m = p+mK_h$, $K_h = 2\pi/h$ where h is the spacing between cores (a period). $\tilde{\phi}_a$ is defined by

$$\tilde{\phi}_a(p) = 2\frac{\sin pa}{p} \tag{11}$$

where a is a half of the core width.

The average power flowing into the z direction is given by

$$\begin{aligned} P_p \; & = \frac{1}{h}\int_0^h \frac{1}{2} E \times H^* \cdot \bar{z} dx \\ & = \frac{1}{2\omega\mu_0}\{-q_0(p)^* e^{-2Im[q_0(p)]z} \\ & \quad -j2q_0(p)^* Im[D_0(p)+R(p)] \\ & \quad +\sum_m q_0(p_m)^*|D_m(p)+\delta_{m0}R(p)|^2 e^{2Im[q_0(p_m)]z}\} \end{aligned} \tag{12}$$

The first term of the second line is the reactive power of the incident evanescent wave. The third term is the power of the m-th order diffracted wave. The power is effective for $|p_m| < k_0$ and reactive for $|p_m| > k_0$. The effective power is independent of z and the reactive power decays in the z direction. The second term is the effective power caused by the interference between the incident and scattered waves. That is a measure of the power of an evanescent wave and coincides with the result in Ref.1. However, the origin of the power is unclear. From a consideration on reflection and transmission of an evanescent wave by the interface between two media it is expected that the continuity of the power flow including the reactive power is held at the interface and the power generated by the interference is transferred to the diffracted waves. From Eqs.(8) and (10) $Im[D_0+R] < 0$ for $k_0 < |p| < k_0n_2$ and the second term gives the effective power flowing into the $-z$ direction. However, $Im[D_0+R] = 0$ for $|p| > k_0n_2$ and the second term vanishes.

3 Beam wave scattering

We consider the following incident wave,

$$E_y^i = \frac{w_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-\frac{w_0^2p^2}{2}-j(p+p_0)x+jq_0(p+p_0)z}dp \tag{13}$$

where w_0 is the spot size and $|p_0| > k_0$. By using a paraxial approximation the incident wave is

$$E_y^i = \frac{w_0}{w_i(z)}e^{-\frac{(x+j\bar{q}_0'(p_0)z)^2}{2w_i(z)^2}-jp_0x+\bar{q}_0(p_0)z} \tag{14}$$

where

$$w_i(z) = \sqrt{w_0^2-\bar{q}_0''(p_0)z} \tag{15}$$

$$\bar{q}_0(p_0) = \sqrt{p_0^2-k_0^2} \tag{16}$$

$$\bar{q}_0'(p_0) = \frac{p_0}{\bar{q}_0(p_0)} \tag{17}$$

$$\bar{q}_0''(p_0) = -\frac{k_0^2}{\bar{q}_0(p_0)^3} \tag{18}$$

Although the incident wave propagates in the x direction, the field concentrates in a region near $x = 0$. The reflected wave is

$$E_y^r = \frac{w_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty} R(p+p_0)e^{-\frac{w_0^2p^2}{2}-j(p+p_0)x-jq_0(p+p_0)z}dp \tag{19}$$

The m-th order diffracted wave is given by

$$\begin{aligned} E_y^{sm} = \; & \frac{w_0}{\sqrt{2\pi}}\int_{-\infty}^{\infty} D_m(p+p_0) \\ & \times e^{-\frac{w_0^2p^2}{2}-j(p+p_{0m})x-jq_0(p+p_{0m})z}dp \end{aligned} \tag{20}$$

where

$$p_{0m} = p_0+mK_h \tag{21}$$

For $|p_{0m}| < k_0$ the m-th order diffracted wave is an ordinary beam wave,

$$E_y^{sm} = D_m(p_0)\frac{w_0}{w_b(z)}e^{-\frac{(z_m\sin\vartheta_m)^2}{2w_b(z)^2}-jk_0x_m} \tag{22}$$

where

$$w_b(z) = \sqrt{w_0^2-j\frac{z}{k_0\sin^3\vartheta_m}}, \tag{23}$$

and ϑ_m is the diffraction angle,

$$p_{0m} = k_0\cos\vartheta_m \tag{24}$$

x_m and z_m are the local coordinates associated with the m-th order diffracted plane wave. The diffracted beam wave propagates in the x_m direction. For $|p_{0m}| > k_0$ the m-th order diffracted wave is an evanescent wave,

$$E_y^{sm} = D_m(p_0)\frac{w_0}{w_s(z)}e^{-\frac{(x-j\bar{q}_0'(p_0)z)^2}{2w_s(z)^2}-jp_{0m}x-\bar{q}_0(p_{0m})z} \tag{25}$$

where

$$w_s(z) = \sqrt{w_0^2+\bar{q}_0''(p_{0m})z} \tag{26}$$

The diffracted field concentrates in a region near $x = 0$ and decays into the z direction.

The total power flowing into the z direction is given by

$$
\begin{aligned}
P_b &= \int_{-\infty}^{\infty} \frac{1}{2} E \times H^* \cdot \bar{z} dx \\
&= \frac{\sqrt{\pi}}{2\omega\mu_0} \{ -\frac{w_0^2}{w_i(z)} q_0^*(p_0) e^{\frac{(\bar{q}_0'(p_0)z)^2}{w_i(z)^2} + 2\bar{q}_0(p_0)z} \\
&- w_0 \sum_{|p_{0m}|>k_0} j2q_0^*(p_{0m}) Im[D_m(p_0) + \delta_{m0}R(p_0)] \\
&\times e^{-\frac{(mK_h w_0)^2}{4}} \\
&+ w_0 \sum_{|p_{0m}|<k_0} q_0^*(p_{0m}) |D_m(p_0) + \delta_{m0}R(p_0)|^2 \\
&+ \sum_{|p_{0m}|>k_0} \frac{w_0^2}{w_s(z)} q_0^*(p_{0m}) |D_m(p_0) + \delta_{m0}R(p_0)|^2 \\
&\times e^{-\frac{[\bar{q}_0'(p_{0m})z]^2}{w_s(z)^2} - 2\bar{q}_0(p_{0m})z} \}
\end{aligned}
\tag{27}
$$

The first term of the second line is the reactive power of the incident wave. The third term is the effective power of the diffracted beam wave and the fourth term is the reactive power of the evanescent beam wave. The second term is the effective power caused by the interference between the incident and scattered waves. The second term has the interference terms with the high order diffracted waves. For a sufficiently large spot size only the interference with the zero order diffracted wave remains.

4 Conclusion

The scattering of an evanescent wave from the end-face of an ordered waveguide system composed of cores of equal space and a cladding has been treated by the perturbation method and the scattered field has been analytically derived. The complex power of the total field has been calculated and it has been shown that the effective power incident on the end-face is generated by the interference between the incident and scattered waves. It is expected that the power generated by the interference is transferred to the diffracted waves at the end-face.

References

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