Diffraction of TE Polarised Electromagnetic Waves by a Nonlinear Inhomogeneous Metal-Dielectric Waveguide

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Abstract

An analytical-numerical approach is proposed, justified and applied for the solution to the problem of diffraction of an electromagnetic TE wave by a metal-dielectric waveguide filled with nonlinear inhomogeneous medium.

1 Introduction

The electromagnetic wave diffraction by homogeneous (see [1] where early and classical results for circular geometries are summarized) or inhomogeneous (see e.g. [2]) cylindrical metal-dielectric bodies filled with linear medium has been a subject of intense studies since the late 1940s.

However, the case of nonlinear inhomogeneous filling still constitutes to a big extent an open problem, both in view of mathematical justification and creation of efficient numerical techniques. A progress here is associated with recently developed techniques [3] for the analysis of nonlinear boundary value problems for the Maxwell and Helmholtz equations.

This study opens a series of works that have an objective to fill these gaps in the electromagnetic theory and numerical modeling and methods aimed at efficient solution to the problems of diffraction by cylinders filled with nonlinear inhomogeneous media.

2 Statement of the Problem

The problem of diffraction of a monochromatic polarized electromagnetic TE wave by a metal-dielectric waveguide Σ filled with nonlinear inhomogeneous medium is considered. The waveguide is located in three-dimensional space $\mathbb{R}^3$ (with a cylindrical coordinate system $O\rho\varphi z$) and described as follows

$$\Sigma := \{(\rho,\varphi,z) : r_0 \leqslant \rho \leqslant r,\ 0 \leq \varphi < 2\pi\}.$$

We will consider TE-polarized waves in the monochromatic mode

$$\mathbf{E}e^{-i\omega t} = e^{-i\omega t}\left(0, E_\varphi, 0\right)^T, \ \mathbf{H}e^{-i\omega t} = e^{-i\omega t}\left(H_\rho, 0, H_z\right)^T,$$

with the components

$$E_\varphi = \mathrm{E}_\varphi(\rho)e^{i\gamma z}, H_\rho = \mathrm{H}_\rho(\rho)e^{i\gamma z}, H_z = \mathrm{H}_z(\rho)e^{i\gamma z}. \quad (1)$$

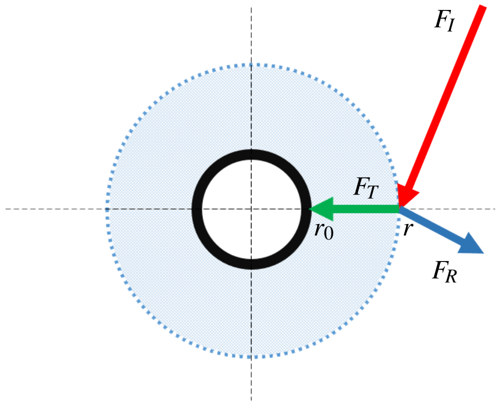

Figure 1. Waveguide Σ, where r_0 and r are the radii of the internal (perfectly conducting) and external (dielectric) cylinders, respectively.

Complex amplitudes of the electromagnetic field $\mathbf{E},\mathbf{H}$ satisfy the Maxwell equations

$$\begin{cases} \mathrm{rot}\mathbf{H} = -\mathrm{i}\omega\varepsilon\mathbf{E}, \\ \mathrm{rot}\mathbf{E} = \mathrm{i}\omega\mu_0\mathbf{H}, \end{cases} \quad (2)$$

subject to the following boundary conditions: the tangential components of the electric field vanish on the metal surface $\rho = r_0$ and the tangential field components are continuous on the media interface $\rho = r$.

The dielectric permittivity in the whole space has the form $\varepsilon = \widetilde{\varepsilon}\varepsilon_0$, where

$$\widetilde{\varepsilon} = \begin{cases} \varepsilon(\rho) + \alpha f\left(|\mathbf{E}|^2\right), & r_0 \leqslant \rho \leqslant r, \\ 1, & \rho > b, \end{cases} \quad (3)$$

where $\varepsilon(\rho) \in C^1[R_1,R_2]$, $\varepsilon(\rho) > 0$ and α is a real constants.

Let $k_0^2 = \omega^2\mu_0\varepsilon_0$. Substituting complex amplitude $\mathbf{E}$ and $\mathbf{H}$ with components (1) into equations of system (2) and introducing the notation $u(\rho) := \mathrm{E}_\varphi(\rho)$, we obtain

$$\left(\rho^{-1}(\rho u)'\right)' + \left(k_0^2\widetilde{\varepsilon} - \gamma^2\right)u = 0 \quad (4)$$

where u is a sufficiently smooth real function,

$$u \in C^1 [r_0, +\infty) \cap C^2 (r_0, r) \cap C^2 (r, +\infty) . \qquad (5)$$

The tangential components E_φ and H_z are continuous at the interface:

$$[u]|_{\rho=r} = [u']|_{\rho=r} = 0, \qquad (6)$$

$$u|_{\rho=r_0} = 0, \qquad (7)$$

where $[v]|_{\rho=s} = \lim\limits_{\rho\to s-0} v(\rho) - \lim\limits_{\rho\to s+0} v(\rho)$ is the jump of the limit values of the function at a point s defined by (7).

The solution of equation (4) in the domain $\rho > r$ have the form

$$u(\rho) = F_I I_1 (k\rho) + F_R K_1 (k\rho) , \qquad (8)$$

where $k^2 := \gamma^2 - k_0^2$ and I_1, K_1 are the modified Bessel functions. Constants F_I and F_R are called an amplitudes of incident and reflected fields, respectively.

Solution of equation (4) inside the layer $r_0 < \rho < r$ we will called the transmitted field.

Formulate *Problem P: Find the value of the amplitude of reflected field F_R such that for a given value of the amplitude of incident field F_I, there exists a non-trivial function u such that for $\rho \in [r_0, +\infty]$ u satisfies the equation* (4)*, transmission conditions* (6) *and* (7)*.*

3 Numerical method

For the numerical solution of Problem P a method based on the solution to the auxiliary Cauchy problem is proposed-which makes it possible in particular to determine and plot the amplitude of the reflected field, F_R, with respect to the amplitude of the incident field, F_I.

Consider the Cauchy problem for the equation

$$u'' = -\rho^{-1}u' + \rho^{-2}u - \left(k_0^2\varepsilon(\rho) - \gamma^2\right) u - \alpha f(u^2)u \quad (9)$$

with the following initial conditions

$$\begin{aligned} u(r) &= F_I I_1 (kr) + F_R K_1 (kr), \\ u'(r) &= kF_I I_1' (kr) + kF_R K_1' (kr). \end{aligned} \qquad (10)$$

To justify the solution technique, we use classical results of the theory of ordinary differential equations concerning the existence and uniqueness of the solution to the Cauchy problem and continuous dependence of the solution on parameters.

Using the transmission condition on the boundary $\rho = r_0$ (7), we obtain the following dispersion equation

$$\Delta (F_I, F_R) \equiv u(r_0) = 0, \qquad (11)$$

where $\Delta (F_I, F_R)$ is determined explicitly and quantity $u(r_0)$ is obtained from the solution to the Cauchy problem for fixed values of F_I and F_R.

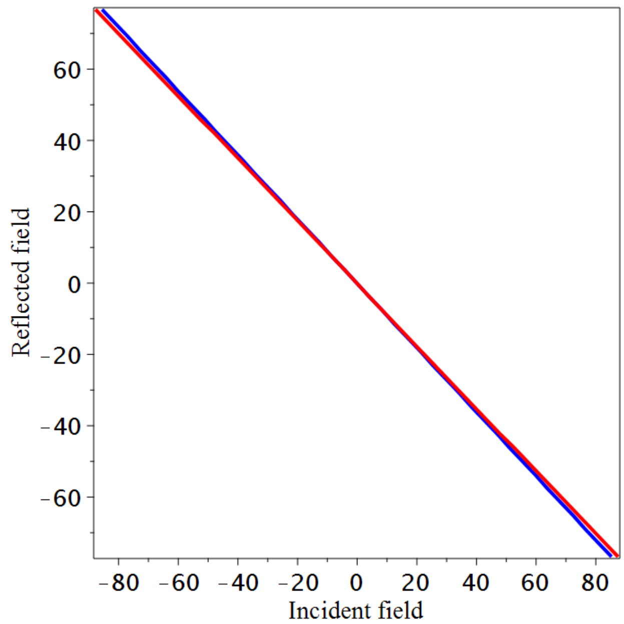

Figure 2. Amplitude of the reflected field F_R vs amplitude of the incident field F_I in the linear ($\alpha = 0$, blue) and nonlinear (red) cases. The values of parameters are $\gamma = 1.05$, $r_0 = 2$, $r = 4$, $\varepsilon_c = 4$, $\alpha = 10^{-3}$, $\beta = 10^{-2}$.

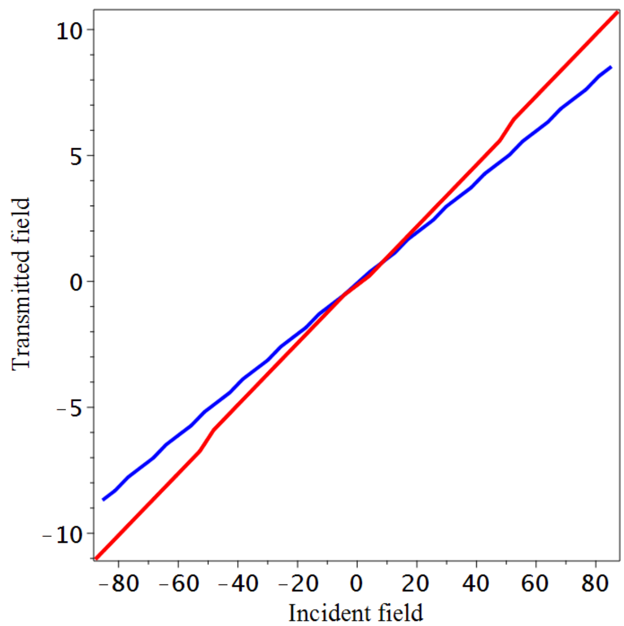

Figure 3. Amplitude of the transmitted field F_T vs amplitude of the incident field F_I in the linear ($\alpha = 0$, blue) and nonlinear (red) cases. The values of parameters are $\gamma = 1.05$, $r_0 = 2$, $r = 4$, $\varepsilon_c = 4$, $\alpha = 10^{-3}$, $\beta = 10^{-2}$.

Thus for fixed value of F_I, when the number $F_R = \widetilde{F}_R$ is such that $\Delta(F_I, F_R) = 0$, then F_R is the solution of problem P which corresponds to the value of F_I.

Numerical experiments are carried out for the nonlinearity with saturation

$$\varepsilon = \varepsilon_c + \frac{\alpha u^2}{1+\beta u^2},$$

where ε_c is a positive real constant.

In Figs. 2–5 the amplitudes of the reflected, F_R , and transmitted, F_T, fields calculated with respect to the amplitude of the incident field F_I are shown.

These simulation results describe the essential relationships between linear and nonlinear problems. Namely, the nonlinear reflected field can be predicted from that obtained from the linear problem using the perturbation theory method (for small value of nonlinearity coefficient α). Uniqueness of the solution to the nonlinear problem is preserved. Note that the curves in Figs. 2, 3 and 4, 5 have different qualitative features, namely, different inclination directions depending on the character of the field: reflected or transmitted.

4 Conclusion

We have developed an analytical-numerical approach for the analysis of electromagnetic wave diffraction by a metal-dielectric waveguide of circular geometry filled with nonlinear inhomogeneous medium. The approach employs analysis and numerical solution to nonlinear boundary value problems for the Helmholtz equations and is implemented as a program code.

5 Acknowledgements

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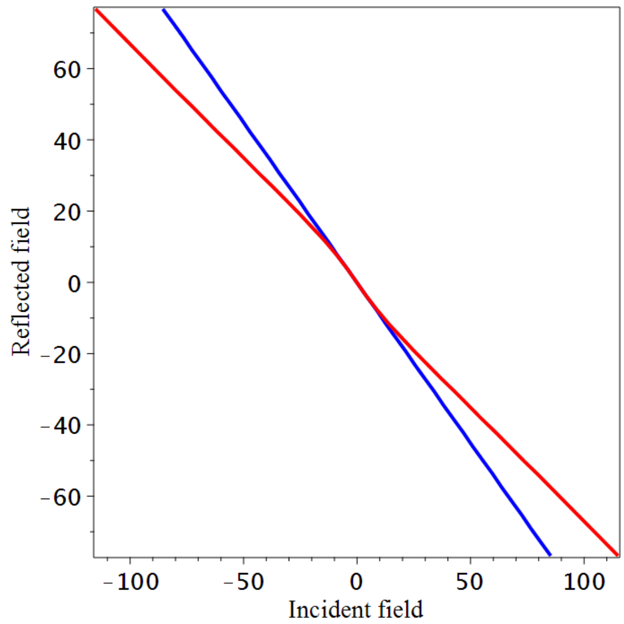

Figure 4. Amplitude of the reflected field F_R vs amplitude of the incident field F_I in the linear ($\alpha = 0$, blue) and nonlinear (red) cases. The values of parameters are$\gamma = 1.05$, $r_0 = 2$, $r = 4$, $\varepsilon_c = 4$, $\alpha = 10^{-2}$, $\beta = 10^{-2}$.

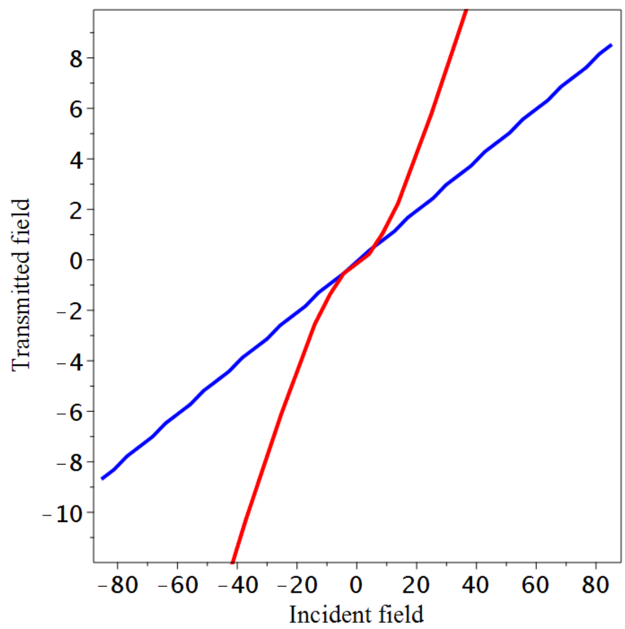

Figure 5. Amplitude of the transmitted field F_T vs amplitude of the incident field F_I in the linear ($\alpha = 0$, blue) and nonlinear (red) cases. The values of parameters are$\gamma = 1.05$, $r_0 = 2$, $r = 4$, $\varepsilon_c = 4$, $\alpha = 10^{-2}$, $\beta = 10^{-2}$.