



# Exact Solutions for Lens and Reflector Shaping

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## Abstract

We propose an exact solution technique for lens and reflector shaping, which determines the surface geometry of a reflector or a lens excited by a single feed element based on the specified far field information. Otherwise stated, given a desired far field pattern over a specified solid angle range, a single-reflector/single-feed antenna system can be obtained analytically via a closed form, exact solution. There is no need whatsoever to invoke optimization algorithms.

Given a required two-dimensional far field pattern, the surface of the reflector or lens, illuminated by a single feed, is generated via a closed form solution, a solution which satisfies Maxwell's equations and all relevant boundary conditions. For the purpose of demonstration, three reflector antennas, and one lens, each with a single feed, have been constructed based on three different far field contour beam patterns. In each such case, the CPU required to generate a thousand surface points amounted to just a few minutes of real time.

Solution verification was carried out by analytic examples, software simulations, and hardware measurement. Four analytic examples, including the standard planar, parabolic, elliptical, and hyperbolic reflectors, are presented herein. Also confirmed with commercial software was the performance of a shaped reflector which provides a specific far field contour beam. Additionally, a dish antenna measurement verified the perfect reconstruction of the reflector surface from its measured near field pattern. Excellent correlations were obtained in all demonstration cases.

## 1. Introduction and Technique

Shaped beam antennas provide arbitrary far field beam contours in radar detection and surveillance, and especially in satellite communications. Commonly encountered shaped beam antennas are those providing contour, annular, U-shaped, and flat-top beams. Two types of shaped beam antennas are often seen. One is the parabolic reflector excited by an array feed, the other a shaped reflector with a single feed. The focus of the present paper is the latter - a shaped reflector or a lens in tandem with a single feed.

While the determination of the local surface geometry of a reflector exposed to a single feed has been sought for decades, solutions have been found thus far only for special cases, such as the maximum efficiency antenna [1]. It is commonly understood and widely implemented that optimization algorithms must be used in conjunction with physical optics to determine each desired reflector geometry. In addition to issues of numerical stability and the choice of favorable initial conditions, the main difficulty in relying upon optimization is the large number of surface points which are sought as the unknown parameters. Any such optimization run requires intensive computing power and it may take days or weeks to achieve a satisfactory reflector geometry.

We propose herein an exact solution technique for reflector or lens shaping, which determines the reflector (or the lens) surface geometry when excited by a single feed element and responding to specified far field information. In a word, given a desired far field pattern over certain angles, a single-reflector/single-feed antenna system can be obtained analytically as a closed form exact solution. No optimization algorithms whatsoever are involved in producing this closed form solution.

Given a desired far field pattern over a certain  $(\theta, \phi)$  domain, or else the aperture distribution which results in the same far field data, coupled with a single feed pattern, the reflector or lens surface is generated as a closed form, analytic solution. This solution satisfies Maxwell's equations and all relevant boundary conditions. The CPU requirement to generate a thousand surface points is routinely met in just a few minutes of real time.

## 2. Design Examples

For the purpose of demonstration, one lens and three reflector antennas, each with a single feed, have been constructed based on three different far field contour beam patterns. In each such case, the CPU required to generate a thousand surface points amounted to just a few minutes of real time.

### A. Lens Antenna Design for South & Central America

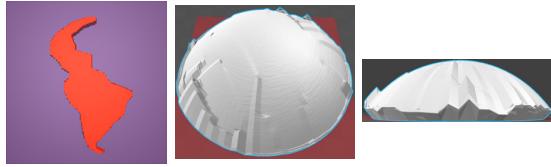


Fig. 1a *Left*: Desired far field contour pattern; 1b *Center*: 40" lens antenna fed by a single horn to provide the far field pattern at 11.811 GHz in 1a; 1c: *Right*: Side view of the lens geometry as in 1b.

### B. Shaped Reflector Antenna Design for South & Central America, the United States, & China

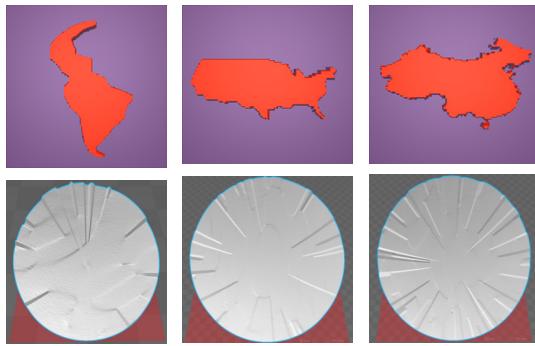


Fig. 2a *Top*: Three different desired far field contour patterns; 2b *Bottom*: Design of shaped reflector antennas fed by a single horn to provide the far field patterns respectively as in 2a.

### 3. Verification Test Cases

A verification of our closed form solution technique was carried out by four analytic examples, one software simulation, and one hardware measurement. Analytic examples include planar, parabolic, elliptical, and hyperbolic reflectors, in which the far field patterns or aperture distributions can be obtained analytically. The reflector surface geometries are then individually determined from these far field data banks. Next, given a desired contour beam covering the South and Central American continent from a geostationary satellite, a shaped reflector fed by a single feed horn was generated from the closed form solution. The far field pattern of this shaped reflector was calculated next using a commercial software simulation for comparison with the required contour beam. Finally, a hardware measurement of an existing, manufactured shaped reflector is reported herein by way of still further demonstration. The field of that shaped reflector was measured in its near zone so as to provide a numerical basis for obtaining its far field pattern, and a successful reconstruction of that selfsame dish was achieved based on this essentially measured far field.

Excellent test results are evident on the correlation between the original or required reflectors, and reflectors generated via the closed form solution.

### A. Analytic Examples

Cases 1-4 demonstrate the generation of reflector surfaces based on the analytic far field patterns from reflectors of planar, parabolic, elliptical, and hyperbolic surfaces respectively.

#### Case 1: Planar Reflector

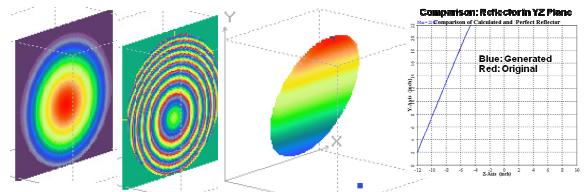


Fig. 3a *Left*: Calculated amplitude and phase on an aperture plane at  $z=10"$  from the center of a planar reflector fed with a horn at 11.811 GHz; 3b *Center*: Reflector generated by the closed form solution; 3c *Right*: Geometry comparison of generated and original reflectors.

#### Case 2: Parabolic Reflector

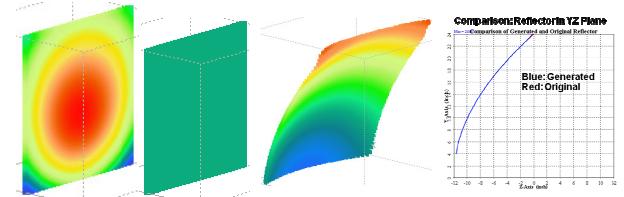


Fig. 4a *Left*: Calculated amplitude and phase on an aperture plane at  $z=10"$  from the center of a parabolic reflector fed with a horn at 11.811 GHz; 4b *Center*: Reflector generated by the closed form solution; 4c *Right*: Geometry comparison of generated and original reflectors.

#### Case 3: Elliptical Reflector

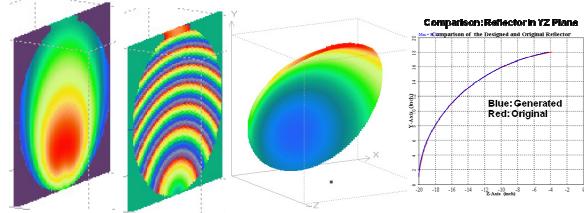


Fig. 5a *Left*: Calculated amplitude and phase on an aperture plane at  $z=10"$  from the center of an elliptical reflector fed with a horn at 11.811 GHz; 5b *Center*: Reflector generated by the closed form solution; 5c *Right*: Geometry comparison of generated and original reflectors.

#### Case 4: Hyperbolic Reflector

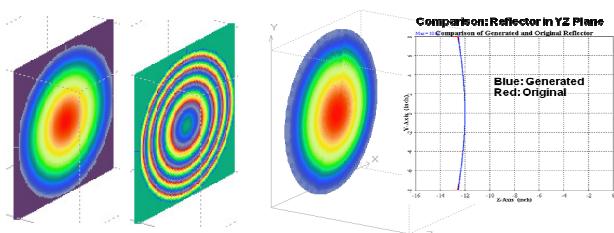


Fig. 6a *Left*: Calculated amplitude and phase on an aperture plane at  $z=10"$  from the center of a hyperbolic reflector fed with a horn at 11.811 GHz; 6b *Center*: Reflector generated by the closed form solution; 6c *Right*: Geometry comparison of generated and original reflectors.

## B. Software Simulation: South & Central America as Far-Field Target

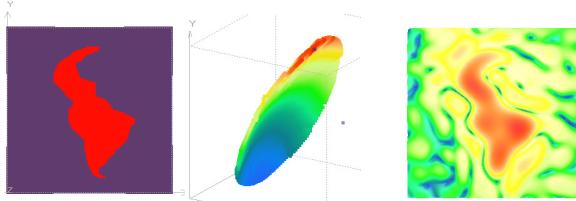


Fig. 7a *Left*: Required far field amplitude pattern at 11.811 GHz; 7b *Center*: Shaped reflector generated by the closed form solution; 7c *Right*: Far field pattern of the shaped reflector in 7b calculated by software simulations.

## C. Hardware Measurement

The far field pattern of an existing, candidate shaped dish, illuminated by a prescribed feed horn, was first obtained from a near field measurement acquired at 10 GHz in 201x201 grid points across an 80"x80" plane, and then numerically propagated toward infinity. A reflector surface associated with the given feed horn was then generated on the basis of this far field data via the algorithm under discussion. The generated reflector exhibits an excellent match with the dish under test. With due allowance for surface presentation as a mirror image across the y-axis, the geometric correlation offers a perfect reconstruction of the original dish reflector.

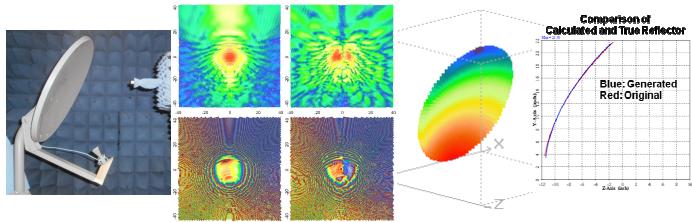


Fig. 8a *Left*: The dish antenna under test at 10 GHz; 8b *Center left*: Measured amplitude and phase of Ex and Ey respectively on an aperture plane at  $z=16"$  from the center of the dish; 8c *Center right*: Reflector generated based on 8b from the closed form solution; 8d *Right*: Geometry comparison of generated and original reflectors.

## 4. Conclusion

Given a far field target pattern, the determination of the required reflector surface takes only minutes for generating a thousand surface points since it is obtained from a closed form, analytic solution. The verification of this closed form solution was carried out by a) analytic examples, b) software simulations, and c) hardware measurements. All the test results show excellent correlations.

Future work will include the manufacture of three shaped reflectors and one lens intended for four different beam shapes in various applications, affording opportunities for still further validation.

## 5. References

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1. V. Galindo-Israel, R. Mittra, and A. Cha "Aperture Amplitude and Phase Control of Offset Dual-Reflectors," IEEE Trans. Antenna and Propagation, Vol. AP-27, No. 2, March 1979.