Novel CAD Models of Propagation Characteristics for Non-Planar CPS

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Abstract

The paper presents CAD models for computing dispersive and conductor thickness based line parameters of semi-elliptical/circular cylindrical coplanar striplines (CPS) on single-layered dielectric substrates. Analysis of the non-planar CPS has been performed using conformal mapping technique (CMT). The developed models are verified and validated for wide range of line geometries from 1 GHz - 60 GHz. All the models are within 2.4% of average deviation against reference data.

1. Introduction

Transmission lines used to feed printed antennas wrapped around different geometric dielectric layers such as elliptical or cylindrical surfaces for different non-conventional surfaces in aircraft, missiles and mobile communication applications have recently become very popular [1-5]. The construction of the elliptical and cylindrical transmission lines during the fabrication requires suitable flexible dielectric materials. The determination of their line parameters is quite important concept in design steps of practical applications [5-8]. Thus there is an engineering need for CAD models of different configurations of lossy non-planar transmission lines that can compute line parameters accurately for the design and analysis based applications. To date, many authors [2-3,9] have investigated the electrical parameters of elliptical (ECPS) and cylindrical coplanar strip lines (CCPS) using full-wave approach incorporating a moment-method calculation and CMT. However, to the author's best knowledge, in the literature computation of dispersive propagation characteristics of non-planar CPS with different finite ground planes and finite conductor thickness have not been investigated yet.

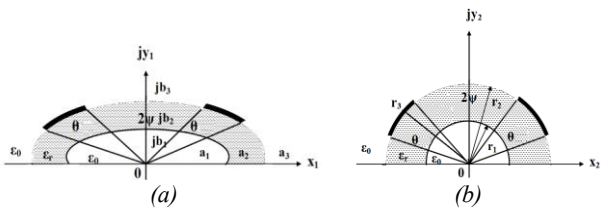

Figure 1. CPS on the curved surfaces: (a) Semi-ellipsoidal CPS (SECPS) and (b) Semi- circular cylindrical CPS (SCCPS).

The present work reports the CMT analysis and closed-form models to compute conductor thickness-based dispersive propagation characteristics of semi-elliptical (SECPS) and semi-cylindrical coplanar strip lines (SCCPS) on a finite-thickness dielectric substrate, shown in Fig.1. The available static closed-form models are modified empirically, to take into account the finite strip conductor thickness and dispersion on the circular and elliptical cylindrical surfaces, by adopting the expressions used for the planar single-layered CPS [9]. The models are compared against two 3D - EM Simulators- Ansoft HFSS [10] and CST Microwave Studio [11]. The developed models are verified and validated for wide range of line geometries with permittivity range of $2.5 \le \varepsilon_r \le 20$, conductor thickness range from 0.25μm - 9μm and the frequency range from 1 GHz - 60 GHz.

2. Closed-form Models for SECPS and SCCPS

Fig.1 presents the CPS on the semi- ellipsoidal and semi-circular cylindrical surfaces. Fig. 1(a) shows cross section of the SECPS which is formed by two confocal semi - ellipses. Their semi major axis and semi minor axis are a_1, a_2, b_1 and b_2 respectively. The 3rd semi-ellipse, not shown in Fig.1, with dimensions a_3 and b_3 considers the finite strip conductor thickness.

The arc of strip width is θ and the gap between the two strips at the centre is 2ψ. For SECPS and SCCPS, ground plane width is (π-2θ-2ψ). In Fig. 1(a), the conductor thickness for semi-major and semi-minor axes are $(a_3 - a_2)$ and $(b_3 - b_2)$, respectively. Moreover, the dielectric substance thickness for the semi-major and semi-minor axes are $(a_2 - a_1)$ and $(b_2 - b_1)$, respectively. The relative dielectric constant is ε_r in both the cases.

The structures are analyzed for the computation of the propagation characteristics for which effective relative permittivity ε_{eff} and characteristic impedance Z_0 have been computed with the help of conformal mapping method [2-4,12]. It has been assumed that all conductors are infinitely thin and perfectly conducting. The dielectric substrate is assumed to be isotropic and lossless medium. The boundary between air and dielectric medium is assumed to be a perfect magnetic wall. The SECPS is first transformed to the SCCPS. The SCCPS is then transformed to the planar CPS line on the finite thickness substrate. Finally the planar CPS, with the help of Schwarz- Cristoffel transformation, is transformed to the parallel plate capacitor [5]. Due to the invariance of the electrostatic

energy, the capacitance of the parallel plate capacitor gives the line capacitance of the SECPS [8,12].

As a result, ε_{eff} and Z_0 of SECPS and SCCPS using the conformal mapping method [2-4] can be written as:

$$\varepsilon_{eff} = 1 + \frac{\varepsilon_r - 1}{2} \frac{K(k_0)}{K(k_0')} \frac{K(k_1')}{K(k_1)} \quad (1) \qquad Z_0 = \frac{120\pi}{\sqrt{\varepsilon_{eff}}} \frac{K(k_0)}{K(k_0')} \quad (2)$$

where, modulus k_0 and k_1 and their complimentary modulus k_0' and k_1' involved in the complete elliptic integral of the first kind are defined, in terms of the structural parameters, as follows [4]:

$$k_0 = \sqrt{\frac{\psi^2}{(\theta+\psi)^2} \frac{\left(\pi^2 - [2(\theta+\psi)]^2\right)}{\left(\pi^2 - (2\psi)^2\right)}} \quad (i) \qquad k_0' = \sqrt{1-k_0^2} \quad (ii) \quad (3)$$

$$k_1 = \sqrt{\frac{x_\psi^2}{x_{\theta+\psi}^2} \frac{\left(x_{\pi/2}^2 - x_{\theta+\psi}^2\right)}{\left(x_{\pi/2}^2 - x_\psi^2\right)}} \quad (i) \; k_1' = \sqrt{1-k_1^2} \quad (ii) \quad (4)$$

also,

$$x_{\pi/2} = \left[\frac{exp\left(\pi^2/2H\right) - 2}{exp\left(\pi^2/2H\right) + 2}\right]^2 \quad ,1 < \frac{\pi}{2H} < \infty \quad (i)$$

$$x_{\pi/2} = \sqrt{1 - \left[\frac{exp(2H) - 2}{exp(2H) + 2}\right]^4} \quad ,0 < \frac{\pi}{2H} < 1 \quad (ii) \quad (5)$$

where H term is dependent on dimensional parameters of SECPS and SCCPS, and is expressed by,

$$H = \ln\left(\frac{r_2}{r_1}\right) = \ln\left(\frac{a_2+b_2}{a_1+b_1}\right) \quad (6)$$

x_ψ and $x_{\theta+\psi}$ can be found using the relations given in [5].

The cross-section of the original semi-ellipsoidal cylinder, shown in Fig.1(a), is located in the ζ -plane. The focal distance between three confocal ellipses is

$$c = \sqrt{a_1^2 - b_1^2} = \sqrt{a_2^2 - b_2^2} = \sqrt{a_3^2 - b_3^2} \quad (7)$$

The confocal semi-ellipsoidal cylinders are transformed into the semi-circular cylinders in the *Z*-plane on using the following mapping function [2-3]:

$$z = \frac{1}{c}\left[\zeta \mp \sqrt{(\zeta^2 - c^2}\right] \quad (8)$$

The above mapping function provides the concentric semi-circular cylinders of radius r_1, r_2 and r_3

$$r_1 = \sqrt{\frac{a_1+b_1}{a_1-b_1}} \quad (i) \qquad r_2 = \sqrt{\frac{a_2+b_2}{a_2-b_2}} \quad (ii) \qquad r_3 = \sqrt{\frac{a_3+b_3}{a_3-b_3}} \quad (iii) \quad (9)$$

The CPS on the concentric semi-circular cylinders is transformed into the planar CPS using the following mapping function [2-3]:

$$u = j\ln\left[\frac{z}{r_2}\right] + \frac{\pi}{2} \quad (10)$$

The above mapping function provides the following conducing slot width (*s*), strip width (*w*), substrate thickness (*h*) and strip conductor thickness (*t*) of the transformed SECPS into the corresponding planar CPS:

$$s = 2\Psi \quad (i) \qquad w = \theta \quad (ii) \qquad h = \ln\frac{r_2}{r_1} = \ln\frac{a_2+b_2}{a_1+b_1} \quad (iii) \quad (11)$$

$$t = \ln\frac{r_3}{r_2} = \ln\frac{a_3+b_3}{a_2+b_2} \quad (iv)$$

When $a_1 = b_1 = a$, $a_2 = b_2 = b$, $a_3 = b_3 = d$ i.e., $c = 0$, h becomes $h = ln\ (b/a)$ and t becomes $t = ln\ (d/b)$, then equation-(11) gives the structural parameters for the SCCPS, as shown in Fig. 1(b).

In order to account for the strip conductor thickness for these line structures, the equivalent strip width (θ_{eq}) and equivalent slot width (ψ_{eq}) for the SECPS / SCCPS are given as

$$\psi_{eq} = \psi - \Delta\theta \quad (i) \qquad \theta_{eq} = \theta + \Delta\theta \quad (ii)$$

$$where,\ \Delta\theta = \frac{1}{2\pi\varepsilon_r}\left(\ln\frac{a_3+b_3}{a_2+b_2}\right)\left[1 + \ln\left(\frac{16\pi\psi}{\ln\frac{a_3+b_3}{a_2+b_2}}\right)\right] \quad (iii) \quad (12)$$

The ε_{eff} of the SECPS / SCCPS with conductor thickness is computed from the following empirical relation [5]:

$$\varepsilon_{eff}(f=0,t) = \varepsilon_{eff}(t=0) - \frac{1.4[\varepsilon_{eff}(t=0)-1]\left(\ln\left(\frac{a_3+b_3}{a_2+b_2}\right)\Big/2\psi\right)}{[K(k_{0,t}')/K(k_{0,t})] + 1.4\left(\ln\left(\frac{a_3+b_3}{a_2+b_2}\right)\Big/2\psi\right)} \quad (13)$$

where, the modulus k_0 and k_1 will be modified into $k_{0,t}$ and $k_{1,t}$ along with their complementary modulus, in which ψ and θ will be replaced by using equation-(12) into ψ_{eq} and θ_{eq} respectively. To account for dispersion, skin-depth and conductor thickness are incorporated with static ε_{eff} from equation-(1). Thus, the final closed-form expressions of non-planar CPS with ground plane width (π-2θ-2ψ), for dispersion with skin-depth dependent factor (*S*) and finite strip thickness are summarized below:

A. Effective relative permittivity

$$\varepsilon_{eff}(f,t,\delta) = \left(\sqrt{S\times\varepsilon_{eff}(f=0,t)} + \frac{\left[\sqrt{\varepsilon_{req}} - \sqrt{S\times\varepsilon_{eff}(f=0,t)}\right]}{1+m(f/f_{TE})^{-r}}\right)^2 \quad (a)$$

$$where\ Skin-effect\,factor,\ S = \left(\frac{K(k_{0,\delta}')}{K(k_{0,\delta})}\right)\left(\frac{K(k_{0,t})}{K(k_{0,t}')}\right) \quad (i)$$

Due to the field penetration in the conductor by $\delta_s/2$ all around, θ decreases by δ_s and ψ increases by δ_s.

$$k_{0,\delta} = \sqrt{\frac{(\psi+\delta_s)^2}{(\theta+\psi)^2}\frac{\left(\pi^2-[2(\theta+\psi)]^2\right)}{\left(\pi^2-(2\psi+2\delta_s)^2\right)}} \quad (ii) \qquad k_{0,\delta}' = \sqrt{1-k_{0,\delta}^2} \quad (iii)$$

In this expression, $\varepsilon_{eff}(f=0,t)$ is the conductor thickness dependent static effective relative permittivity which is computed using equation-(13), f *is* the frequency as independent variable and f_{TE} is the surface wave TE_1– mode cut-off frequency[9,12].

B. Characteristic impedance

$$Z_0(f,t) = \frac{120\pi}{\sqrt{\varepsilon_{eff}(f,t)}} \times S \quad (i) \quad where \quad S = \left\{\frac{K(k_{0,\delta}')}{K(k_{0,\delta})}\frac{K(k_{0,t})}{K(k_{0,t}')}\right\} \quad (ii) \qquad (b)$$

C. Wheeler's incremental inductance rule

$$\alpha_c = \frac{\pi}{\lambda_0}\sqrt{\varepsilon_{eff}\left[\varepsilon_r,\theta,2\psi,\ln\frac{a_2+b_2}{a_1+b_1},f,\ln\frac{a_3+b_3}{a_2+b_2}\right]}\left\{\frac{\Delta Z\left[\varepsilon_r=1,\theta,2\psi,\ln\frac{a_2+b_2}{a_1+b_1},f,\ln\frac{a_3+b_3}{a_2+b_2},\delta_s\right]}{Z_0\left[\theta_{eq},\psi_{eq},\ln\frac{a_2+b_2}{a_1+b_1},f,\ln\frac{a_3+b_3}{a_2+b_2},\varepsilon_r=1\right]}\right\} \text{ Np/m } (c)$$

D. Improved Holloway and Kuester (IHK)

$$\alpha_c = \frac{R_{sm}(\theta+\psi)^2}{16Z_0K^2(k_0')\theta\cdot(\theta+2\psi)}\cdot\left\{\begin{array}{l}\frac{1}{\Psi}ln\left[\left(\frac{2\Psi}{\Delta}+1\right)\left(\frac{\theta-\Delta}{\theta+2\Psi+\Delta}\right)\left(\frac{\pi+2\psi-2\Delta}{\pi-2\psi-\Delta}\right)\right]\\ +\frac{1}{(\theta+\psi)}ln\left[\left(\frac{2(\theta+\psi)}{\Delta}-1\right)\left(\frac{\theta-\Delta}{\theta+2\Psi+\Delta}\right)\right]\end{array}\right\} \text{ Np/m } (d)$$

E. Dielectric loss

$$\alpha_d = 27.29\frac{\varepsilon_r}{\sqrt{\varepsilon_{eff}(f,t)}}\left[\frac{\varepsilon_{eff}(f,t)-1}{\varepsilon_r-1}\right]\frac{\tan\delta}{\lambda_0} \quad \text{dB/m} \quad (e) \quad (14)$$

in which the frequency and conductor thickness dependent $\varepsilon_{eff}(f,t)$ is computed using equation-(14a).

3. Discussion of Results

The accuracy of the closed-form models developed for propagation characteristics of non-planar CPS are tested against the results obtained from EM-simulators- HFSS and CST.

Fig. 2 presents comparisons of performances of CPS on the circular and semi-circular cylindrical surfaces in respect of effective relative permittivity, characteristic impedance and losses. It also presents such comparisons for the CPS on the elliptical and semi- ellipsoidal cylindrical surfaces. The results are obtained over the frequency range, 1 GHz – 60 GHz. For simulation, the substrate with ε_r=2.5, θ = 40°, ψ = 25°, h = 635 µm and t = 3 µm has been considered. For ECPS, the ellipticity c = 0.7 is considered. With increase in the ellipticity and decrease in the ground width, there is increase in both ε_{eff} and Z_0. So by controlling ellipticity, ε_{eff} and Z_0 of the line can be controlled as well. We have observed that in both HFSS and CST, increase in the value of ε_{eff} starts after 100 MHz; however, the closed-form model shows continuous increase after 10 GHz. The experimental results are not traceable to verify the nature of dispersion. The model has an average deviation of 2.9% against the results of both HFSS and CST over whole frequency range.

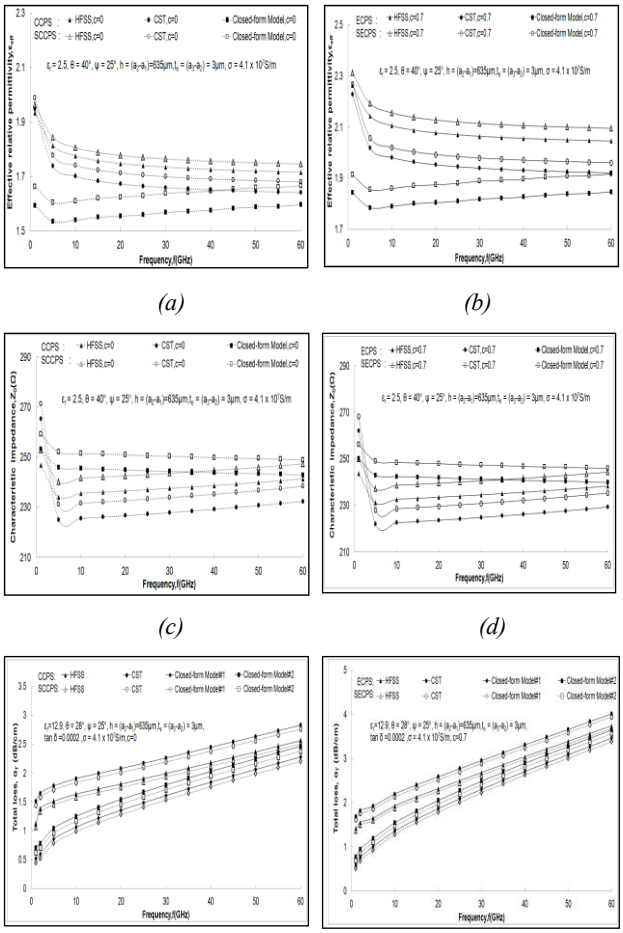

Figure 2. Comparison of different line parameters of non-planar CPS w.r.t. frequency.

It has been observed, ε_{eff} increases for the semi- circular and semi-ellipsoidal CPS as compared against the CPS on the circular and elliptical surfaces. Then, Z_0 of semi - circular and semi- ellipsoidal cases are also higher as compared to the CPS on the circular and elliptical surfaces. The total loss α_T of non-planar CPS is computed for ε_r = 12.9, t = 3µm, θ = 28°, ψ = 25°, c = 0.7, h = 635 µm, tan δ = 0.0002 and σ = 4.1 x 10^7 S/m between frequency range 1 GHz – 60 GHz. On comparing, closed-form model #1 (= Wheeler + α_d) and closed-form model #2 (= IHK + α_d)

have % average and % maximum deviation of (6.5%, 16.7%) and (3%, 8%) respectively against both the EM-simulators, excluding results at 1 GHz which show large deviations. The closed-form model # 2 is in close agreement with HFSS at higher frequencies. The losses and nature of dispersion are almost identical for both $(2\pi-2\theta-2\psi)$ and $(\pi-2\theta-2\psi)$ ground plane widths of non-planar CPS. The results are summarized in Table- I. The above models can be useful for MIC, MMIC, slotlines, antenna and integrated optical applications.

Table-I: % Change in characteristics of models w.r.t. simulators on non-planar CPS with different ground-plane widths

(a): % change in characteristics of semi-ellipsoidal CPS over elliptical CPS.

Characteristics	Closed-form Model		HFSS		CST	
	% av.	% max.	% av.	% max.	% av.	% max.
Increase in ε_{eff}	3.6	3.9	2.3	2.5	2.0	2.3
Increase in Z_0	2.4	2.7	2.5	2.7	2.5	2.8
Decrease in α_T	4.5	10.8	2.5	4.1	2.6	4.1

(b): % change in characteristics of semi-circular CPS over circular cylindrical CPS.

Characteristics	Closed-form Model		HFSS		CST	
	% av.	% max.	% av.	% max.	% av.	% max.
Increase in ε_{eff}	4.4	4.7	1.7	1.8	2.3	2.5
Increase in Z_0	2.4	2.6	2.5	2.6	2.5	2.9
Decrease in α_T	5.5	15.5	2.8	4.9	2.8	4.6

4. Conclusions

The current work presents the CMT analysis and CAD models for the conductor thickness dependent dispersive line parameters for SECPS and SCCPS on single-layer substrates. The expressions are simple and accurate for microwave circuit designs and are applicable to both thin and thick conductors. Comparisons with 3D-EM simulators are presented to validate the derived expressions for the frequency range from 1 GHz - 60 GHz. All the models are within 2.4% of average deviation against reference data and thereby proving its applicability.

5. Acknowledgements

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6. References

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