32nd URSI GASS, Montreal, 19–26 August 2017

Directed propagation of electromagnetic waves in stratified periodic structures

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Abstract

The propagation of monochromatic electromagnetic waves in stratified periodic structures is studied both theoretically and numerically. The low frequency case is considered. The frequency should be close to the saddle point of the dispersion surface. The effect of directed propagation of waves in such structure is predicted theoretically and is confirmed by numerical simulation.

1 Introduction

Effects in the propagation of electromagnetic waves in periodic structures, named also photonic crystals [1], are studied by many researchers. The idea of approach to constructing localized near-the-straight-line waves in photonic crystals was suggested in [2]. We are interested in the propagation of beams in a periodic stratified medium and found that for some conditions the field can propagate only in some specific direction. A similar phenomenon in two-dimensional photonic crystals was found by numerical simulation in [3]. We deal with one-dimensional photonic crystals in the cases of two and three spatial dimensions. Our theoretical investigation is carried out by the asymptotic method of two scale asymptotic expansions, i.e., the field is oscillated with the period of the medium and has a slowly changing envelope. We introduce a small parameter, which is the ratio of the period of the medium and the scale of the envelope. The crucial assumption concerns the frequency of monochromatic waves, which should be close to the frequency of the saddle point of the dispersion surface. We obtained equations for the envelope of the field. These equations depend on the number of spatial dimensions. In the plane case (2D case), the envelope of beams of TM and TE satisfies independent wave equations, where the coordinate across the layers stands for time, see our paper [4], [5]. In the general three-dimensional case, the analysis of the field is more complicated. The envelopes of fields polarized in two orthogonal directions satisfy a system of differential equations, which cannot be separated. However if we introduce two new functions dependent on both envelope functions, we get for these new functions a system of independent wave equations, where the coordinate across the layers stands for time. The equation obtained enables us to analyze the transformation of polarisation of waves if we know the field on the surface of one layer.

2 Statement of the problem

A monochromatic electromagnetic field satisfies the Maxwell equations

$$\mathrm{rot}\mathbb{E} = ik\mu\mathbb{H}, \quad \mathrm{rot}\mathbb{H} = -ik\varepsilon\mathbb{E},$$

where $\varepsilon(z+b) = \varepsilon(z)$, $\mu(z+b) = \mu(z)$; ε and μ are piecewise smooth, $\varepsilon(z) \neq 0$, $\mu(z) \neq 0$.

By analogy with [6], we represent the Maxwell equations in matrix form

$$kP\mathbf{\Psi} = -i\widehat{\mathbf{\Gamma}}\cdot\nabla\mathbf{\Psi}, \quad \mathbf{\Psi} = \begin{pmatrix} \mathbb{E} \\ \mathbb{H} \end{pmatrix},$$

$$\hat{\mathbf{\Gamma}}\cdot\nabla \equiv \mathbf{\Gamma}_1\frac{\partial}{\partial x} + \mathbf{\Gamma}_2\frac{\partial}{\partial y} + \mathbf{\Gamma}_3\frac{\partial}{\partial z},$$

$$\mathbf{P} = \begin{pmatrix} \varepsilon I & 0 \\ 0 & \mu I \end{pmatrix}, \mathbf{\Gamma}_j = \begin{pmatrix} 0 & \boldsymbol{\gamma}_j \\ -\boldsymbol{\gamma}_j & 0 \end{pmatrix}, j = 1,2,3$$

$$\boldsymbol{\gamma}_1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{pmatrix}, \quad \boldsymbol{\gamma}_2 = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix},$$

$$\boldsymbol{\gamma}_3 = \begin{pmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

and $k/\sqrt{\varepsilon_{av}\mu_{av}} = \omega/c$, where c is the speed of light. The energy conservation law in matrix notation reads

$$\nabla\cdot(\mathbf{\Psi},\hat{\mathbf{\Gamma}}\mathbf{\Psi}) = 2\mathrm{Im}k(\mathbf{\Psi},\mathbf{P}\mathbf{\Psi}).$$

We introduce a small parameter

$$\chi = b/L \ll 1,$$

where b is the vertical period of the medium, L is the horizontal scale of the field.

There are waves of two polarizations in the layered medium: transverse electric and transverse magnetic ones. Each polarization of waves has its own multivalued dispersion function $\omega = \omega^H(\boldsymbol{p})$, $\omega = \omega^E(\boldsymbol{p})$. These functions coincide at the stationary points. We assume that the frequency ω is close to the frequency ω_* of the stationary point $\boldsymbol{p}_*$ of one of the sheets of the dispersion function $\omega = \omega^f(\boldsymbol{p})$, $f = H,E$, i.e., $\omega_* = \omega^E(\boldsymbol{p}_*) = \omega^H(\boldsymbol{p}_*)$, $\nabla\omega^f(\boldsymbol{p}_*) = 0$, $f = H,E$. We assume that

$$\omega = \omega_* + \chi^2\delta\omega, \quad \delta\omega \sim 1.$$

We assume also that there is one bounded and one unbounded Floquet-Bloch solution at the point $\boldsymbol{p}_*$. The stationary point is $p_{x*} = p_{y*} = 0$, $p_{z*} = \pm\pi/b$ or $p_{z*} = 0$.

3 The leading term of asymptotics

We seek the solution by the method of two scale asymptotic expansions $\boldsymbol{\Psi} = \boldsymbol{\Psi}(z, \boldsymbol{\rho})$, where $\boldsymbol{\rho} = (\xi, \eta, \zeta)$, $\xi \equiv \chi x$, $\eta \equiv \chi y$, $\zeta \equiv \chi z$ are slow variables. The leading term reads

$$(\mathbb{E}, \mathbb{H})^t\,(z, \boldsymbol{\rho}) = \alpha_1(\boldsymbol{\rho})\boldsymbol{\Phi}_*^X(z) + \alpha_2(\boldsymbol{\rho})\boldsymbol{\Phi}_*^Y(z) + O(\chi),$$

where $\boldsymbol{\Phi}_*^X = (E_0, 0, 0, 0, H_0, 0)^t|_{p_{z*}}$ and $\boldsymbol{\Phi}_*^Y = (0, -E_0, 0, H_0, 0, 0)^t|_{p_{z*}}$, are expressed via (E_0, H_0), which are the Floquet-Bloch solutions of the system

$$i\frac{dE_0}{dz} = -k\mu H_0; \quad i\frac{dH_0}{dz} = -k\varepsilon E_0.$$

The equations for the functions $\alpha_j(\boldsymbol{\rho})$, $j = 1, 2$, read

$$\begin{aligned}
\frac{\partial^2\alpha_1}{\partial\xi^2}\ddot{\omega}_{11*}^H + \frac{\partial^2\alpha_1}{\partial\eta^2}\ddot{\omega}_{11*}^E \quad &+ \quad \frac{\partial^2\alpha_1}{\partial\zeta^2}\ddot{\omega}_{33*}^0 + 2\frac{\delta\omega}{c}\alpha_1 \\
&- \quad \frac{\partial^2\alpha_2}{\partial\xi\partial\eta}(\ddot{\omega}_{11}^H - \ddot{\omega}_{11}^E) = 0, \\
\frac{\partial^2\alpha_2}{\partial\xi^2}\ddot{\omega}_{11*}^E + \frac{\partial^2\alpha_2}{\partial\eta^2}\ddot{\omega}_{11*}^H \quad &+ \quad \frac{\partial^2\alpha_2}{\partial\zeta^2}\ddot{\omega}_{33*}^0 + 2\frac{\delta\omega}{c}\alpha_2 \\
&- \quad \frac{\partial^2\alpha_1}{\partial\xi\partial\eta}(\ddot{\omega}_{11}^H - \ddot{\omega}_{11}^E) = 0.
\end{aligned}$$

The coefficients of the equations are the second derivatives of the dispersion functions $\omega = \omega^H(\boldsymbol{p})$, $\omega = \omega^E(\boldsymbol{p})$ calculated at the stationary points. For example, the Taylor formula for TM polarization near the stationary point reads:

$$\omega = \omega_* + \frac{1}{2}\ddot{\omega}_{11*}^H(p_x^2 + p_y^2) + \frac{1}{2}\ddot{\omega}_{33*}^0(p_z - p_{z*})^2 + \ldots.$$

The derivatives with respect to p_x and p_y are equal and depend on the type of polarization, the derivative with respect to p_z does not depend on the polarization type. The equations for $\alpha_j(\boldsymbol{\rho})$, $j = 1, 2$, can be simplified by introducing new functions.

If the field does not depend on one of the coordinates, for example, on η, the equations for the functions α_1 and α_2 can be separated without introducing additional functions and read

$$\begin{aligned}
\frac{\partial^2\alpha_1}{\partial\xi^2}\ddot{\omega}_{11*}^H + \frac{\partial^2\alpha_1}{\partial\zeta^2}\ddot{\omega}_{33*}^0 + 2\frac{\delta\omega}{c}\alpha_1 \quad &= \quad 0, \\
\frac{\partial^2\alpha_2}{\partial\xi^2}\ddot{\omega}_{11*}^E + \frac{\partial^2\alpha_2}{\partial\zeta^2}\ddot{\omega}_{33*}^0 + 2\frac{\delta\omega}{c}\alpha_2 \quad &= \quad 0.
\end{aligned}$$

If $\delta\omega = 0$ and $\ddot{\omega}_{33*}^0 < 0$, the equations transform into one-dimensional wave equations, where ζ stands for time, and the speed is the ratio of the second derivatives $\ddot{\omega}_{11*}^H/\ddot{\omega}_{33*}^0$, $\ddot{\omega}_{11*}^E/\ddot{\omega}_{33*}^0$. Properties of the wave equation yield the existence of undistorted beams, which can be inclined to oz axis only under a certain angle. The tangent of the angle is the ratio of second derivatives of the dispersion function. This effect was investigated analytically and numerically in [4]. The simplest model of the structure consisting of two alternating homogeneous dielectric layers was taken. The dispersion functions, their second derivatives and the Floquet-Bloch solutions were found analytically. Explicit formulas were used for the calculation of the beam propagation in the medium. The numerical simulation of beams propagation was carried out with the packet CST Microwave Studio. Results of calculations by explicit formulas and of numerical experiment are in good agreement.

The envelope functions dependent on three spatial coordinates are analyzed for the case of a half-space. The tangential components of magnetic fields are known on the boundary of the half-space. The condition of limiting absorption is implied. The transformation of modes of each polarization is investigated. If boundary data are well localized, the field of each polarization propagates in a cone, and the cone angle depends on the type of polarization. The change of the distribution of polarization of the field with z is studied theoretically and numerically.

4 Acknowledgements

This work was supported by RFBR grant 140200624 and by SPbGU grant 11.42.1073.2016.

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