



## Electric Polarizability Estimation For Planar Frequency Selective Arrays

Andrei Ludvig-Osipov\* and B.L.G. Jonsson

KTH Royal Institute of Technology, School of Electrical Engineering, 100 44 Stockholm, Sweden

### 1 Extended Abstract

Planar frequency selective structures have practical applications in a number of electromagnetic devices such as filters, antennas, absorbers, polarizers, reflection/transmission arrays and metamaterials. Moreover, they may also possess the property of extraordinary transmission. In this presentation we propose a computationally effective method to estimate the electric polarizability for such structures. A polarizability tensor is defined as a connection between an external applied field and a corresponding induced dipole moment in the structure. Estimates of polarizabilities give physical bounds on the transmission bandwidth for certain radiation properties and it is thus very important in the design process [1]. The here considered structures are two-dimensional PEC patch arrays optionally backed up by a dielectric substrate.

The proposed method is based on the result of [2] showing a relation between  $xx$ -component of low-frequency reflection coefficient  $R$  and electric polarizability per unit area ( $\gamma_{xx}/A$  as shown for  $xx$ -tensor component below)

$$R(k) = ik \frac{\gamma_{xx}}{2A} + o(k), \quad (1)$$

where  $k$  is a free-space wavenumber. The equation (1) is valid for any low-pass planar periodic structure, and given a reflection coefficient at low frequency we can estimate electric polarizability. Towards this goal we employ a multimodal equivalent circuit approach, presented in [3] that gives an analytically derived network model for planar periodic structures with dielectric layers. It provides an accurate result in a wide range of frequencies, including low-frequency limit. A distinct feature of the approach is that it provides a deep insight into the physics of a problem. This advantage is inherited in our proposed polarizability estimation method.

In the proposed method we extract a low-frequency asymptote from the result of [3] to obtain a polarizability estimate. As an illustrative example of our method we show a polarizability estimate for a periodic array of arbitrarily shaped metal patches printed on a dielectric substrate of a thickness  $d$  and with permittivity  $\epsilon$

$$\frac{\gamma_{xx}}{2A} = \left( \sum_{\substack{m,n=-\infty \\ (m,n) \neq (0,0)}}^{\infty} \left| \frac{\tilde{J}(k_{xn}, k_{ym})}{\tilde{J}(k_{x0}, k_{y0})} \right|^2 \frac{k_{xn}^2}{\sqrt{k_{xn}^2 + k_{ym}^2}} \cdot \left\{ \frac{2(\epsilon + \tanh[d\sqrt{k_{xn}^2 + k_{ym}^2}])}{2\epsilon + (1 + \epsilon^2) \tanh[d\sqrt{k_{xn}^2 + k_{ym}^2}]} \right\} \right)^{-1} + \frac{(\epsilon - 1)d}{2}, \quad (2)$$

where  $\tilde{J}(k_{xn}, k_{ym})$  is a Fourier transform of an estimated electric current profile induced on a patch,  $k_{xn} = 2\pi n/P_x$  and  $k_{ym} = 2\pi m/P_y$  are projections of modal wavenumbers with  $P_x \times P_y$  being a unit cell size. The second term here can be identified as a polarizability of a free-standing dielectric slab, while the multiple in curly brackets shows how the presence of a substrate affects the evanescent fields associated with patch elements. Thus, the proposed method provides an insight on an explicit interaction between the patch and the dielectric layer. Our results are validated by comparison with (1) applied to full-wave simulations. An ongoing development of our method focuses on its applications to sum rules and optimization of electrical currents to maximize polarizability.

### References

- [1] A. Bernland et al., "Sum rules and constraints on passive systems," *J. Phys. A: Math. Theor.*, **44**, 145205, 2011.
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- [3] R. Rodríguez-Berral et al., "Analytical Multimodal Network Approach for 2-D Arrays of Planar Patches/Apertures Embedded in a Layered Medium," *IEEE Trans. Antennas Propag.*, **63**, 5, 2015.