Existence of Radially Symmetric Complex Surface Waves in Lossy Goubau Line

E. Kuzmina(1), Y. Shestopalov (2)
(1) Moscow Technological University (MIREA), Moscow, Russia,
ekaterina.kuzm@gmail.com
(2) University of Gävle, Gävle, Sweden

Abstract

We present a method and results of the analysis of the complex wave propagation in open metal–dielectric waveguides, including investigation of a homogeneous Goubau line (GL) covered with a lossy dielectric. A systematic analytical–numerical study is made of the dispersion equation (DE) for the GL surface waves in the complex domain with respect to the problem parameters. Several specific propagation regimes are identified, including anomalously small attenuation of the principal and higher-order complex surface waves.

1. Introduction

The GL [1, 2], which constitutes the simplest example of an open metal–dielectric waveguide, is a perfectly conducting cylinder of circular cross section covered by a concentric dielectric layer, a and b, $b > a$, denote the radii of the internal (perfectly conducting) and external (dielectric) cylinders. Main issues of the mathematical theory of surface waves in a lossless GL have been completed in [1, 2]. A general approach revealing occurrence of complex surface waves in a lossy GL was proposed in [3]. In this work, we develop an analytical–numerical technique [1–3] to study and describe fundamental properties of complex waves propagating in a GL with the aim to extend the methods to a wider family of open metal–dielectric waveguides.

2. Surface Waves in a Lossless GL

In [1, 2] we considered the propagation of symmetric surface eigenwaves in a lossless GL described in terms of nontrivial solutions to homogeneous Maxwell's equations. Let us give a brief summary of the statement and results. By applying the transmission (continuity) condition we obtain the dispersion equation (DE) $F_G(\bar{p}) = 0$, $\bar{p} = (a, b, \omega, \epsilon, \gamma)$, for surface waves in GL with a well-defined function F_G; here, ω, ϵ, and γ are, respectively, the field frequency, permittivity of the GL dielectric cover, and the longitudinal wavenumber of a symmetric surface wave (propagation constant). Using the analysis of the DE performed in [1, 2] we draw the following conclusions concerning basic properties of the spectrum of surface waves in a lossless GL: (a) the principal surface mode in GL exists for arbitrary values of permittivity of the cladding and radius of the inferred conductor (wire); (b) the spectrum of the GL surface waves with an infinitely thin dielectric coating is a perturbed set of zeros of a well-defined family of functions; (c) if the electrical cross-sectional dimension of the line is sufficiently large, then there exist several higher-order GL surface modes with the propagation constants γ_n $(n = 1, 2, \ldots N_0)$ located in the interval $(1, \sqrt{\epsilon})$ $(\epsilon > 1)$ each having a distinct cross-sectional structure in the dielectric layer, and the number of zeros (oscillations) of the potential function corresponding to the principal and the nth higher-order GL mode equals the index of the wave; (d) the spectrum of the GL surface waves is a regular perturbation of the surface wave spectrum of the fiber caused by an infinitely thin, perfectly conducting rod placed into the cross-sectional origin of the dielectric fiber.

In the next sections we make use of these properties of surface waves in a lossless GL and show that introduction of a small imaginary part of the permittivity of the GL dielectric layer shifts the surface wave spectrum to the complex domain. We will also describe more essential properties of the GL complex surface waves.

3. Complex Surface Waves

We prove the existence of radially symmetric complex surface waves in a GL using the parameter-continuation method. Namely, we determine the propagation constant γ of a radially symmetric complex surface wave as an implicit function specified by the DE [1, 2] obtained for lossless GL; consider the DE in the complex domain by constructing analytical continuation of the functions involved in the DE; and then build up a continuation of γ with respect to the imaginary part $t = \Im\epsilon$ of the permittivity of the dielectric cover using reduction to the Cauchy problem

$$\frac{d\gamma}{dt} = -\frac{\frac{\partial F_G}{\partial t}}{\frac{\partial F_G}{\partial \gamma}}; \qquad \gamma(0) = \gamma_n, \tag{1}$$

where $\gamma = \gamma(t)$ is the longitudinal wavenumber of a symmetric complex surface wave considered with respect to real parameter t.

Using the technique developed in [1, 2] we prove [3] that Cauchy problem (1) is uniquely solvable. This implies that complex zeros of $F_G = F_G(\gamma, t)$ constitute regular perturbations of its real zeros γ_n. Roots of DE become complex due to the presence of small imaginary part in parameter ϵ which causes small perturbation. The dependence $\gamma = \gamma(t)$, $t = \Im\epsilon$, is determined explicitly using implicit differentiation in the form of a segment of the Taylor series

$$\gamma_n(t) = \gamma_n + tD_1^n + O(t^2), \quad D_1^n = \left.\frac{d\gamma_n}{dt}\right|_{t=0}. \qquad (2)$$

The coefficient D_1^n which specifies, for each higher-order complex GL mode with the index $n = 2, 3, \dots$, the main contribution caused by the introduction of lossy dielectric, is a function of the problem parameters, $D_1^n = D_1^n(\bar{v})$, $\bar{v} = (a, b, \omega, \Re\epsilon)$. We show that for $n > 1$, the imaginary part of the longitudinal wavenumber of a symmetric complex surface wave (that is, contribution caused by the lossy dielectric which gives rise to the wave attenuation) is small and its magnitude is bounded uniformly with respect to the parameter vector $\bar{v}$. In fact, γ_n is real and attenuation is governed by $\Im D_1^n$. Performing analytical investigations and numerical simulations described briefly in the next section, we show that the imaginary part of the propagation constants of higher-order complex GL modes are small in a broad range if the problem parameters, as illustrated in Fig. 1.

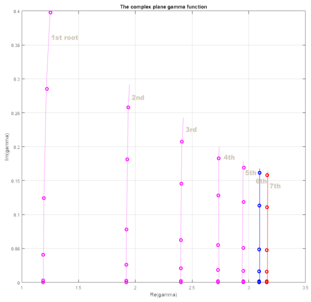

Figure 1. Values of $\boldsymbol{\gamma = \gamma(t)}$ on the complex plane $\boldsymbol{\gamma}$; circles denote points $\boldsymbol{\gamma_i = \gamma(t_i)}$, $\boldsymbol{t_i = \Im\epsilon_i = 0.0001, \dots, 1}$.

4. Analytical Properties and Specific Propagation Regimes

When losses of a GL dielectric cover are moderate (practically, the imaginary part of the permittivity does not exceed unity), we propose to study the properties of complex surface waves in a GL using an approach based on the analysis of the Taylor series (2) for the wave propagation constants. This method can be justified by the fact that we consider an *analytical* solution to the Cauchy problem (1) determined for the constructed well-defined analytical continuation of the right-hand side of the differential equation in (1). Remarkably, the coefficient $D_1^n(\bar{v})$ multiplying t in the segment of the Taylor series for $\gamma = \gamma_n(t)$ that solves Cauchy problem (1) can be determined explicitly. Particularly, it is not difficult to check using explicit forms of function F_G entering the DE and performing tedious algebra that partial derivatives of F_G are coupled by a linear relation,

$$\frac{\partial F_G}{\partial \gamma} = -\gamma F_{g1}; \qquad \frac{\partial F_G}{\partial t} = \frac{i}{2} F_{g1}, \qquad (3)$$

where F_{g1} is expressed explicitly as a rational function of the mixed products of cylindrical functions. This fact enables us to perform complete mathematical analysis of the coefficients entering the segment of the Taylor series in (2) and finally the complex surface wave propagation constants themselves. The volume of the present paper does not allow us to include all these bulky formulas; a special study will be prepared for publication providing a detailed analysis of these expressions.

Let us present a summary of the results of our analytical–numerical investigations: for small losses, the attenuation of the GL complex waves (i) is low and (ii) affected by the relative thickness *s = b/a* of the cover in an oscillatory manner: it may virtually vanish and attains a distinct maximum at almost periodically alternating values of *s*.

5. Conclusion

Introduction of a small imaginary part of the permittivity of the GL dielectric cover gives rise to complex surface waves with small attenuation characterized by a distinct anomalous behavior. The principal and higher-order waves of GL with a lossless dielectric are all transformed to complex waves.

The developed mathematical technique can be applied for the analysis of complex waves in a broad family of open metal-dielectric waveguides.

6. Acknowledgements

We acknowledge the support of The Swedish Institute, project Largescale, and colleagues from Moscow Technological University (MIREA).

7. References

1. Y. Shestopalov, E. Kuzmina, “Higher-order Surface Modes in the Goubau Line,” *Proc. Progress in Electromagnetics Research Symposium*, 2014, pp. 2605–2609.

2. Y. Shestopalov, E. Kuzmina, and A. B. Samokhin, “On a Mathematical Theory of Open Metal-Dielectric Waveguides,” *Forum for Electromagnetic Research Methods and Application Technologies*, 2014, Vol. 5.

3. E. Kuzmina, Y. Shestopalov, “Waves in a Lossy Goubau Line,” *The 10th European Conference on Antennas and Propagation EUCAP 2016*, April 2016, pp. 1-4, doi: 10.1109/EuCAP.2016.7481368