On the Accuracy of Friis' Transmission Formula at Short Range

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Abstract

This paper presents a numerical investigation on the accuracy of Friis' transmission formula for short distances between the transmitting and receiving antennas. It addresses the accuracy of the formula around the far-field distance where effects due to multiple reflections may be of significance.

1. Introduction

Friis' transmission formula [1] is a corner stone of radio science; it relates the available power at the terminal of a receiving antenna P_{av} to the accepted power at the terminal of the transmitting antenna P_{acc} for a time-harmonic, free-space radio system and is thus central to establishing the link budget. Using the gains of the transmitting and receiving antennas, G_t and G_r, the distance R between these, and the wavelength λ, a simple version of the formula reads

$$P_{av} / P_{acc} = G_t G_r (\lambda / 4\pi R)^2 \tag{1}$$

This can readily be extended to account for polarization mismatch and medium loss, as well as to express the received power in terms of the available power of the transmitter by including impedance mismatch factors. The formula is based on far-field antenna parameters, it assumes power density to decrease with the inverse of distance squared, and it neglects multiple reflections between the antennas; hence, its accuracy obviously increases with increasing distance R.

Friis' transmission formula is acceptably accurate for most practical radio systems as these operate over large distances. However, it is of interest to clarify its accuracy at short distances, e.g. for short-range communication and for antenna measurements. Indeed, already Friis [1] discussed this and stated that the formula is accurate within a few percentages for identical ideal aperture antennas at distances larger than the Rayleigh distance.

This work investigates the accuracy of Friis' transmission formula versus distance for three antennas with each their far-field condition; this includes effects of multiple reflections between the antennas. In addition, the choice of proper reference points defining the distance will be addressed; obviously; the distance is ambiguous for antennas of non-zero size at finite distance.

2. Far-field Conditions

The derivation of the far-field radiation integral, the basis for calculating antenna far-field parameters, employs two conditions for the distance r from the geometrical center of the antenna to the observation point; in addition, the Rayleigh distance is often used as a far-field condition. With d being the largest linear dimension of the antenna, these three conditions can be expressed as

$$r / \lambda >> 1/(2\pi) \tag{2a}$$

$$r / \lambda >> d /(2\lambda) \tag{2b}$$

$$r / \lambda > 2(d / \lambda)^2 \tag{2c}$$

Depending on the electrical size of the antenna, d/λ, either of these can be the most stringent; if ">>" is, somewhat arbitrarily, chosen as a factor of 10, (2b) is largest in the interval $1/\pi < d/\lambda < 5/2$, while (2a) is largest below, and (2c) above, this interval; see Figure 1.

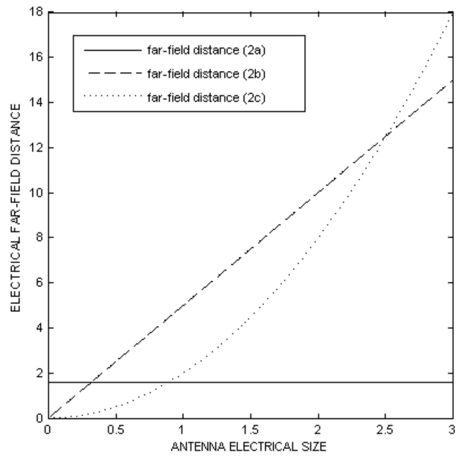

Figure 1. Electrical far-field distance r/λ vs. electrical antenna size d/λ; see Eqs. (2a)-(2c).

3. Test-Case Antennas

Three test-case antennas having each their far-field condition will be investigated. The first two are non-resonant, center-fed linear dipoles with lengths equal to 0.1λ and 1.25λ, respectively, while the third is a resonant, center-fed half-wave dipole backed by 3λ x 3λ square reflector at a distance of 0.24λ behind the dipole; see Table 1. The frequency is chosen to be 300 MHz.

Table 1. Test-case antennas gain, input impedance, and far-field distances

	Antenna 1	Antenna 2	Antenna 3
Size d/λ	0.100	1.250	4.243
Gain (dB)	1.771	5.002	7.829
Impedance (Ω)	1.686 -j1063	109.81 -j429.45	76.43 -j1.404
Distance r/λ (2a)	1.592	1.592	1.592
Distance r/λ (2b)	0.500	6.250	21.21
Distance r/λ (2c)	0.020	3.125	36.00

4. Radio System Modelling and Simulation

A radio system consisting of two identical antennas, of each of the three test-case antennas, is modelled in the integral-equation / method of moment software WIPL-D [2] with both antennas fed by 50 Ω delta-gap generators. From the calculated S-parameters the ratio of available power to accepted power is determined as

$$P_{av} / P_{acc} = |S_{21}|^2 / (1-|S_{11}|^2)(1-|S_{22}|^2)\,. \qquad (3)$$

The Friis' transmission formula (1) is compared to this rigorous expression (3) for varying distance between the two antennas; the gains of the three antennas being calculated in separate simulations. For the first two antennas, the feed point is the obvious reference point for the distance R of Friis' transmission formula, but for the third antenna the reference point is less obvious and the choice of this will affect the accuracy of the transmission formula. For ease of graphical comparison, we plot the ratio between (1) and (3), since this will approach 1 at large distance, against the distance normalized by the relevant far-field condition; see Figures 2a, 2b, and 2c.

5. Discussion

It is seen that Friis' transmission formula is accurate within 0.5 dB at the far-field distance and within 0.1 dB at three times this distance for all test-case antennas; for the 0.1λ dipole even much more accurate. However, it is clearly important to apply the correct far-field distance to achieve this accuracy. For the 0.1λ dipole the multiple reflections are negligible, but these become noticeable for the 1.25λ dipole and dominating for the 3λ x 3λ conductor-backed half-wave dipole; in the latter case, the "average" power ratio is within 0.1 dB at the far-field distance (see insert of Fig. 2c) but the multiple reflections cause oscillations of about 0.4 dB. For distances less than the correct far-field distance the accuracy quickly deteriorates. The accuracy of Friis' transmission formula around the far-field condition is challenged not only by the finite distance but also by the multiple reflections.

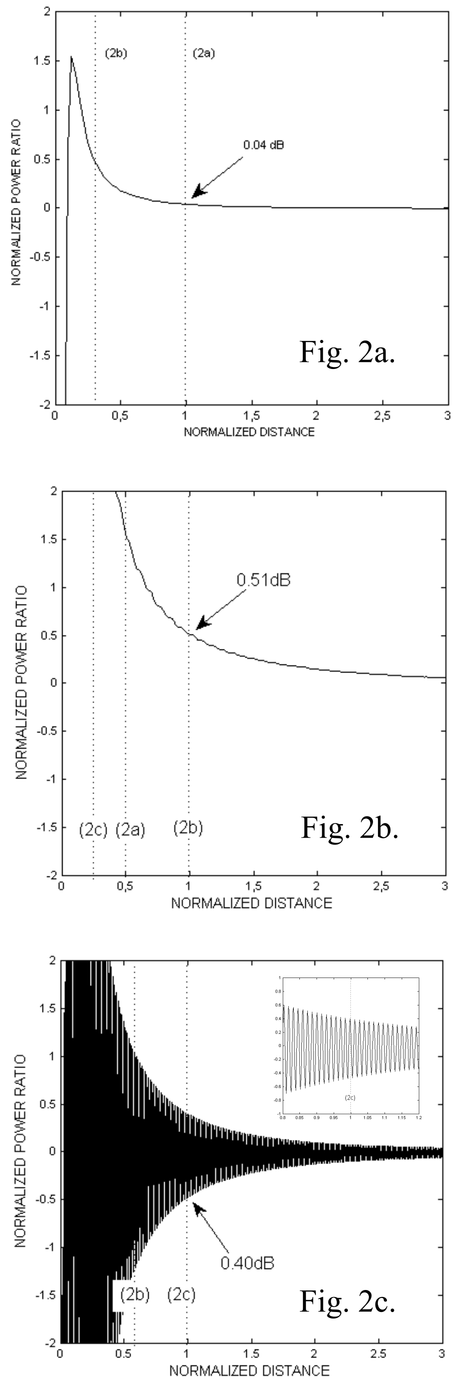

Figure 2. Normalized power ratio vs. normalized distance for system of two test-case antennas of type 1 (Fig. 2a), 2 (Fig. 2b), and 3 (Fig. 2c); see Table 1.

6. References

[1] H.T. Friis, "A Note on a Simple Transmission Formula", *Proceedings of the I.R.E. and Waves and Electrons*, pp. 254-256, May 1946.
[2] WIPL-D homepage *www.wipl-d.com*, February 2017.