32nd URSI GASS, Montreal, 19–26 August 2017

Realizability of Structures for Dual Band Complementary Reflection/Transmission Using a Convex Optimization Approach

Daniel Sjöberg[(1)]
(1) Electrical and Information Technology, Lund University, P.O. Box 118, 221 00 Lund, Sweden,
http://www.eit.lth.se, daniel.sjoberg@eit.lth.se

1 Introduction

Recently, there has been an interest in using dual band circular polarization selective structures in satellite communication, *i.e.*, panels that reflect left hand circular polarization (LHCP) and transmit right hand circular polarization (RHCP) in one band, and *vice versa* in another [1]. A typical specification is to have the first band around 20 GHz and the second band around 30 GHz, using the panel as a diplexer to enable two clusters of feeding horns with complementary circular polarization to access the same reflector. This setup leads to the theoretical problem of characterizing how closely a pass band and a stop band can be controlled next to each other. We study this in the context of convex optimization using a representation formula for the equivalent surface admittance of the reflection and transmission coefficients.

2 Representation formula and convex optimization

The co-polarized reflection and transmission coefficients r and t of any panel can be transformed to a positive real admittance function $Y_r = (1-r)/(1+r)$ and $Y_t = (1-t)/(1+t)$. A positive real function can be represented as follows (where H is the Hilbert transform) [2]

$$Y = \frac{1}{\mathrm{j}\omega L} + G + \mathrm{j}\omega C + \sum_{n=1}^{N} x_n \left(f_n(\omega) - \mathrm{j}[\mathrm{H} f_n](\omega)\right), \quad \mathrm{H} f = \frac{1}{\pi}\mathrm{p.v.} \int_{-\infty}^{\infty} \frac{f(x')}{x-x'}\,\mathrm{d}x' \tag{1}$$

Here, $f_n(\omega) = f(\omega/\Delta\omega - n) + f(\omega/\Delta\omega + n)$, with $f(x)$ being the triangular pulse function $f(x) = 1 - |x|$ when $|x| < 1$ and 0 elsewhere, and $\Delta\omega$ represents the discretization of the representation. In order for Y to be a positive real function, each entry in the parameter vector $x = \{L^{-1}, G, C, \{x_n\}_{n=1}^{N}\}$ is positive. Using L^{-1} as a parameter instead of L makes Y a linear function of x, and we can formulate a number of convex optimization problems on how to fit $Y(x;\omega)$ to a target function $Y_\mathrm{t}(\omega)$, *i.e.*, minimize $\|Y(x;\omega) - Y_\mathrm{t}(\omega)\|$ for a suitable norm subject to various convex constraints on the parameters x, which need to be carefully tailored to the design specification at hand.

With the dual band circular polarization selective structure concept proposed in [3, 4], it is clear that the low frequency asymptote in reflection and transmission are $Y_r \sim 1/(\mathrm{j}\omega L) + \mathrm{O}(\omega)$ and $Y_t \sim 1 + \mathrm{O}(\omega)$, respectively. This restricts the choice of free parameters in the representation of Y in (1), and some parameters can be directly linked to physical quantities. For instance, in the reflection case, the inductance L is proportional to the thickness of the structure [2]. Thus, from the unique solution to the convex optimization problems we can evaluate how closely the design specifications for pass band, stop band, and the transition between them, can be met depending on crucial design parameters such as the thickness of the panel. Several examples will be given in the presentation, the simplest of which shows that for the first pass band the product of return loss in dB and the band width is bounded by a factor proportional to the thickness of the panel, similar to Rozanov's limit for PEC backed absorbers.

References

[1] "Appendix 1 to ESA ITT AO/1-8608/16/NL/AF, Dual-Band Polarising Surfaces for Single Feed per Beam Broadband Antennas," *European Space Agency*, 2016.

[2] Nordebo, Sven, et al. "Optimal realizations of passive structures." *IEEE Transactions on Antennas and Propagation*, **62**(9), pp. 4686–4694, 2014.

[3] Sjöberg, Daniel, "A dual band complementary circular polarization selective structure." *IEEE Antennas and Propagation Symposium*, Vancouver, July 19–24, 2015.

[4] Sjöberg, Daniel, and Andreas Ericsson. "A multi layer meander line circular polarization selective structure (MLML-CPSS)." *Antennas and Propagation (EuCAP), 8th European Conference on*, 2014.