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Combining Microscopic and Macroscopic Poynting Theorems to Find Positive Energies

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1 Extended Abstract

Microscopic and macroscopic Poynting theorems are combined to derive positive semi-definite expressions for macroscopic time-domain energy density in "unconditionally passive", frequency dispersive, but spatially nondispersive, dipolar media [1]. The derivation proceeds from the underlying microscopic Maxwell equations and microscopic Poynting theorem satisfied by the microscopic fields of the electric charge and current that comprise the distribution of discrete, bound, dipolar molecules or inclusions of the media. The macroscopic Maxwell equations and macroscopic Poynting theorem are connected to their microscopic counterparts by showing that the normal macroscopic and microscopic Poynting vectors are equal across a thin free-space gap left by the removal of a thin slice of the dipolar material.

The bound microscopic electric dipoles are assumed to have either zero dipole moments before any external fields are applied or randomly oriented, fixed-magnitude dipole moments that can be aligned by the external fields. In both cases, the applied fields can extract no energy from the initial electric dipoles and thus the electric dipoles are "unconditionally passive".

The combined microscopic and macroscopic Poynting theorems are used to derive two distinct positive semi-definite (nonnegative) macroscopic time-domain energy expressions, one that applies to diamagnetic media and another that applies to paramagnetic media, which includes ferro(i)magnetic and antiferromagnetic materials. The diamagnetic dipoles are unconditionally passive because their Amperian magnetic dipole moments are zero in the absence of applied fields. The analysis of the paramagnetization, produced by the alignment of randomly oriented permanent (yet slightly changeable in magnitude as they rotate in an external magnetic field) Amperian magnetic dipole moments that dominate any induced diamagnetization, is greatly simplified by first proving that the microscopic power equations for rotating "permanent" Amperian magnetic dipoles (which are not unconditionally passive because energy can be extracted from their initial magnetic dipole moments) reduce effectively to the same power equations obeyed by rotating unconditionally passive magnetic-charge magnetic dipoles.

The difference between the paramagnetic and diamagnetic energy is equal to a "hidden energy" that parallels the hidden momentum often attributed to Amperian magnetic dipoles [2, eq. (2.161)–(2.162)]. It is noteworthy that the microscopic derivation reveals that this hidden energy is supplied by the change of inductive energy in the Amperian magnetic dipole moments as the "permanent" dipoles align in an applied field, even though the magnitudes of the induced magnetic dipole moments are negligible compared to the magnitudes of the total magnetic dipole moments.

Metamaterials produced by inclusions comprised of dielectrics and closed wire loops, or open wire loops such as split ring resonators or chiral omega-shaped wires, have fields that satisfy the macroscopic diamagnetic energy inequalities. Metamaterials characterized by paramagnetic (ferro(i)magnetic/antiferromagnetic) inclusions producing a macroscopic paramagnetization that dominates over any diamagnetization resulting from the inclusion's conduction and electric polarization currents have fields that satisfy the macroscopic paramagnetic inequalities.

References

[1] A.D. Yaghjian, "Classical power and energy relations for macroscopic dipolar continua derived from the microscopic Maxwell equations," *PIER B*, **71**, pp. 1–37, November 2016.

[2] T.B. Hansen and A.D. Yaghjian, *Plane-Wave Theory of Time-Domain Fields: Near-Field Scanning Applications*, Wiley/IEEE Press, New York, 1999.