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An $\mathscr{H}$-Matrix Accelerated Method of Moments Solution of a New Single-Source Integral Equation for Scattering Problems

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A new single-source integral equation (SSIE) has been recently proposed for solution of scattering problems of homogenous and layered medium (V. Okhmatovski, et.al., URSI-EMTS, 2016). Unlike the previously known SSIEs, the new equation has only electric-field-type of Green's function instead of both electric and magnetic field Green's functions. However, due to the complexity and computational cost of the dense matrix arising from the naive MoM solution, all previous works were limited to small problems. To mitigate this well-known computational challenge, the hierarchical matrix ($\mathscr{H}$-Matrix) framework (W. Hackbusch, Computing, 62, 89-108, 1999) is applied to the new SSIE to accelerate matrix operations, factorization, and back-substitution which is considered as a fast direct and iterative solution for a large scale 3D scattering problem.

To create all three blocks of integral operators (surface-to-surface, surface-to-volume, and volume-to-surface) of the new SSIE, $\mathscr{H}$-matrix based solver starts from the hierarchical division of the geometry of interest into the small subdomains. Each block of $\mathscr{H}$-matrix is then in correspondence to the interaction between two subdomains of the observer and the source (τ and σ) which for the well-separated subdomains can be approximated as a low rank block of Rank-k matrix (Rk-matrix) and is represented in the form of $Z_{\tau\times\sigma} \approx AB^H$. The accuracy of this approximation is dependent on the low-rank factorization strategies, such as singular value decomposition (SVD), adaptive cross approximation (ACA) and grid based compression algorithms. These compressed blocks result in significant reduction in memory storage and evaluation time of matrix operations which are dependent on low-rank decomposition method and partitioning strategy. In the case of a dense impedance matrix of $N \times N$, comparing with the traditional direct solver, the memory storage of $\mathscr{H}$ -matrix solver is reduced from $O(N^2)$ to $O(KNlogN)$, the time cost is reduced from $O(N^3)$ to $O(K^2NlogN) + O(NK^3)$ for decomposition and from $O(N^2)$ to $O(K^2NlogN) + O(NK^3)$ for inversion approaches of the forenamed matrix where k is the maximum rank among all rank deficient blocks.

At the conference, the capability of the proposed method in terms of the accuracy, computational time, and compression (memory storage) will be discussed. To verify it, the final accelerated results of the new SSIE for the solution of the large scale 3D problem will also be shown.