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Remembering Staffan and electromagnetic scattering: Corrections to classical mixing formulae

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Abstract

This presentation will discuss the manner how electrodynamic and scattering effects can be incorporated into quasistatic-based mixing rules and homogenization principles, thus extending their range of applicability into higher frequencies.

1 Introduction

Staffan Ström was a scientist of radio science and a man of human touch. He was one of my predecessors as the Chairman of URSI Commission B (Fields and Waves) and also a long-serving president of the Swedish URSI member committee (SNRV, Svenska Nationalkommittén för RadioVetenskap) with lasting contributions to radio science. Much of his influence to our community has been beautifully documented in the "Festskrift" for Staffan for his retirement in 1999 [1]; in this presentation I wish to provide an image of how I personally feel his effect on my own radio science research.

One of Staffan's research foci was electromagnetic scattering. Scattering refers to the way waves become distorted by encounters with obstacles. To write a full description of an electromagnetic scattering process is a formidable task in the case of objects with arbitrary shapes and constellations. Staffan formulated an analysis for this general problem by extending the pioneering work by Peter Waterman of the T-matrix approach [2] into many-particle environments—this was the topic in his widely-cited article [3].

2 Homogenization principles

In the electromagnetic modeling heterogeneous media, the main interest is not on the detailed description of the particuliarities of the behavior of the scattered field. Rather, the focus is on the *homogenization* problem: how can one wash out all the structural microsopic details of a sample but yet save the essentials of the effective electric response within the macroscopic description and effective parameters. This is the domain of mixing formulas [4]: the recipes for an effective permittivity of a mixture, given its component permittivities and the structural composition.

A mixing rule, in its simplest form, gives the effective permittivity $\varepsilon_{\rm eff}$ of a mixture as function of the permittivities of the constituents and their volume fractions. Often considered as the most classical one, the Maxwell Garnett formula [5]

$$\varepsilon_{\rm eff} = \varepsilon_{\rm e} + 3\varepsilon_{\rm e} p \frac{\varepsilon_{\rm i} - \varepsilon_{\rm e}}{\varepsilon_{\rm i} + 2\varepsilon_{\rm e} - p(\varepsilon_{\rm i} - \varepsilon_{\rm e})} \qquad (1)$$

predicts the homogenized permittivity $\varepsilon_{\rm eff}$ for a heterogeneity where (spherical) inclusions with permittivity $\varepsilon_{\rm i}$ occupy a volume fraction p in the environment of permittivity $\varepsilon_{\rm e}$. A simple formula as such, it beautifully satisfies the limiting cases of vanishing inclusions phase ($p = 0 \rightarrow \varepsilon_{\rm eff} = \varepsilon_{\rm e}$) and the "fully occupied environment" ($p = 1 \rightarrow \varepsilon_{\rm eff} = \varepsilon_{\rm i}$), and has also been shown to explain properties of many types of dielectric mixures, with unexpected quantitative accuracy [6].

The simple appearance and consequent easiness to use of the basic Maxwell Garnett rule (and its generalizations [4]) arises from assumptions on which it has been derived, in particular on the quasistatic character. Neither the frequency nor the wavelength of the elecromagnetic field does appear in the formula. The implicit limitation is that the applicability of the formula is restricted to low frequencies, in other words the size of the inclusions in the mixture (or more generally, the scale of the inhomogeneity in the case of more continuous random media) has to be much smaller than the wavelength of the incident field. (It is a constantly-appearing topic of discussion in the literature of electromagnetic homogenization how much smaller the scatterer size has to be compared to the wavelength for a mixing formula to be applicable; often the criterion of an order of magnitude is applied.)

3 Towards higher frquencies

However, to upgrade the effectiveness of mixing rules (like the one in Equation (1)) to cover frequencies that surpass the quasistatic limit, basic scattering mechanisms can be modelled and included. As is known, for small scatterers the scattering cross section is very strongly dependent on the relative size of the scatterer (the phenomenon called as Rayleigh scattering [7]), increasing in the fourth power in the size parameter. Compared to homogeneous lossless media where a wave propagates without attenuation, scattering processes inevitably correspond to decrease of the coherent incident radiation, and can be therefore affiliated to effective losses. There are several approaches how this phenomenon, *radiative damping* as it is sometimes called [8], can be connected with the imaginary part of the effective permittivity. The main effect and contribution due to the scattering term on the imaginary part can be included into the imaginary part of the effective permittivity in the following manner (to the first order in the volume fraction p):

$$\varepsilon'_{\text{eff}} - \mathrm{j}\,\varepsilon''_{\text{eff}} = \varepsilon_{\text{e}} + 3\varepsilon_{\text{e}} p \frac{\varepsilon_{\text{i}} - \varepsilon_{\text{e}}}{\varepsilon_{\text{i}} + 2\varepsilon_{\text{e}}} \left[1 - \mathrm{j} \frac{2x^3}{3} \frac{\varepsilon_{\text{i}} - \varepsilon_{\text{e}}}{\varepsilon_{\text{i}} + 2\varepsilon_{\text{e}}} \right] \quad (2)$$

where $x = k_0 a = 2\pi a/\lambda$ is the size parameter of the inclusion spheres (with radius a) in the mixture.

The various ways of including scattering effects to arrive in the result (2) will be discussed in the presentation.

References

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