On the scattering by a pulsed source with the CWA

Cristina Ponti, Massimo Santarsiero and Giuseppe Schettini
"Roma Tre" University, Department of Engineering, via Vito Volterra 62, 00146, Rome, Italy

Abstract

The time-domain Cylindrical Wave Approach has been implemented to simulate the interaction of a pulsed source with buried perfectly conducting or dielectric cylinders. The solution is developed on an analytical basis with the Cylindrical Wave Approach, i.e., suitable cylindrical waves used to describe the scattered field contributions. A spectral approach is employed, as the frequency spectra of the fields are evaluated, and time-domain waveforms are derived through an Inverse Fast Fourier Transform. The method can simulate the A-scan and B-scan radargrams returned by the Ground Penetrating Radar.

1. Introduction

Several techniques have been proposed in the literature for the simulation of 2D scattering by buried targets [1]-[11], with applications to the Ground Penetrating Radar (GPR) [12]. The plots returned by the GPR are in the form of A-scan or B-scan radargrams, with the scattered field given as a function of time at one or more receiver points, respectively. Time-domain scattering from pulsed sources can be simulated in a straightforward way with the Finite-Difference Time-Domain (FDTD) method, as it is developed directly in the time-domain [1]-[4]. Another advantage of the FDTD technique is in the possibility to simulate an arbitrary 2D or 3D scenario with targets of general shape. However, for targets with simple shape, that may simulated with canonical scatterers, like cylinders, the FDTD is very time consuming, compared to an analytical solver.
In this work, an analytical solution is given to the scattering by cylindrical targets, perfectly conducting or dielectric, based on the Cylindrical Wave Approach (CWA) [8]-[11]. The CWA solution is applied at the frequency spectra of the fields, that, considered as the superposition of monochromatic contributions, are suitable sampled. Then, the time-domain pulses are derived through an Inverse Fast Fourier Transform. The source of the scattering problem is a pulsed plane-wave, with arbitrary frequency spectrum. In the numerical results, typical pulses used in the GPR applications are simulated, as the first derivative of the Gaussian pulse, i.e., the Gaussian monocycle, or its second derivative, i.e., the Gaussian doublet.

2. Theoretical analysis

The geometry of the problem is given in Fig. 1: N perfectly conducting or dieletric cylinders are placed below a flat interface, in a semi-infinite medium. In the approach, the total fields is decomposed into several fields contributions: the source of the problem, i.e., a pulsed plane-wave, the plane waves reflected and transmitted by the interface, and the scattered fields. The latter, highlighted in Fig. 1, are the scattered field by the cylinders in the lower medium V_s, the scattered-reflected field V_{sr}, and the scattered-transmitted field in air V_{st}.
The incident plane-wave is given by the following expression:

$$V_i[\xi(f),\zeta(f)] = V_0^i(f) e^{i\{n_\perp[\xi(f)-\xi_0(f)]+n_\parallel[\zeta(f)-\zeta_0(f)]\}} \tag{1}$$

where the function $V_0^i(f)$ is the freqeuncy spectrum assigned to the source, with arbitrary shape, $\xi = k_0 x$, $\zeta = k_0 z$ are the normalized coordinates, with $k_0 = 2\pi f/c$ the vacuum wavenumber.
As to frequency-domain spectra of the scattered fields, the expressions given in [8] are used, evaluated on the whole frequency spectrum.
The scattered field V_s is expanded into cylindrical waves CW_m, proportional to first kind Hankel functions:

$$V_s\left[\xi(f),\zeta(f)\right] = \sum_{q=1}^{N}\sum_{m=-\infty}^{+\infty} c_{qm}(f) CW_m\left[n_1\xi_q(f), n_1\zeta_q(f)\right] \tag{2}$$

As to the scattered-reflected field, reflected cylindrical waves RW_m are used as basis function

$$V_{sr}\left[\xi(f),\zeta(f)\right] = \sum_{\ell=-\infty}^{+\infty} J_\ell\left[n_1\rho_p(f)\right] e^{i\ell\theta_p} \sum_{q=1}^{N}\sum_{m=-\infty}^{+\infty} c_{qm}(f) \times RW_{m+\ell}\left\{-n_1[\chi_q(f)+\chi_p(f)], n_1[\eta_q(f)-\eta_p(f)]\right\} \tag{3}$$

In a similar way, suitable transmitted cylindrical waves TW_m are introduced to be used as basis functions of the scattered transmitted field

$$V_{st}\left[\xi(f),\zeta(f)\right]=\sum_{q=1}^{N}\sum_{m=-\infty}^{+\infty}c_{qm}(f)TW_m\left[\xi_q(f),\zeta_q(f),\chi_q(f)\right] \tag{4}$$

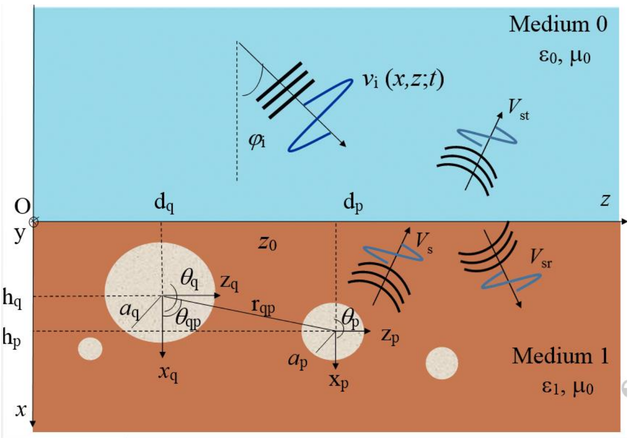

Figure 1. Geometry of the problem.

The unknown expansion coefficients $c_{qm}(f)$ are found applying the monochromatic approach proposed in [8] at each frequency of the spectrum. Once the spectra of all the fields contributions have been found, the relevant time-domain field is calculated through an Inverse Fast Fourier Transform:

$$v_s(x,z;t)=2\mathrm{Re}\left\{IFFT\left[V_s(f_k)\right]\cdot\Delta f\right\} \tag{5}$$

where Δf is the sampling step.

3. Numerical results

In the numerical results, the technique is employed to simulate applications of the GPR. The spectrum of the field $V_0^i(f)$ introduced in (1) is implemented with the typical spectra of the pulses used in the Ground Penetrating Radar technique, as the Gaussian monocycle

$$V_i[\xi(f),\zeta(f)]=K_1\sigma\sqrt{\pi}(j2\pi f)e^{-(\pi\sigma f)^2}e^{j2\pi fT} \tag{6}$$

or the Gaussian doublet

$$V_i[\xi(f),\zeta(f)]=-K_2\sigma\sqrt{\pi}(j2\pi f)^2e^{-(\pi\sigma f)^2}e^{j2\pi fT} \tag{7}$$

with $\sigma=1/\sqrt{2}\pi f_0$, and T=1/f_0 the time delay.

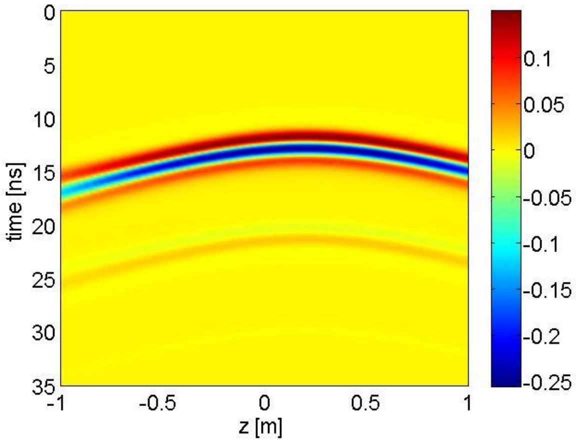

Figure 2. B-scan radargram with a buried PEC cylinder.

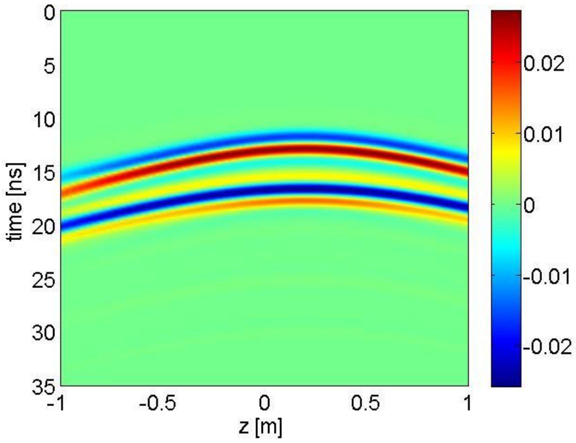

Figure 3. B-scan radargram with a buried dielectric cylinder.

Two examples of B-scan are reported in Figs. 2-3, relevant to the simulation of a perfectly conducting (PEC) and dielectric cylinder simulation, respectively. Excitation is a Gaussian-doublet pulse with central frequency f_0 = 250 MHz. The radius of the cylinder is a = 125 cm, depth 75 cm. The dielectric cylinder has refraction index ε_{r1} = 6, and conductivity σ = 0.015. N = 60 receivers are aligned along a line in x_0 = -5 cm. In Fig. 2, relevant to the PEC cylinder, the B-scan has two hyperbola: the direct scattering from the cylinder, followed by a later hyperbola of smaller intensity, accounting for a second reflection of the scattered field by the interface. In the case of the dielectric target, shown in Fig. 3, two close hyperbola are visible: the first is the scattering from the top of the cylinder, and it is followed by the scattered field from the backside.

4. References

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