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Analysis and Synthesis of Fields in Modulated Metasurfaces

G. Minatti[(1)], F. Caminita[(2)], E. Martini[(2)] and S. Maci[(1)]
(1) Department of Information Engineering and Mathematics, University of Siena, Siena, Italy {minatti, macis}@dii.unisi.it
(2) Wave-Up S.r.l., Florence, Italy, {francesco.caminita, enrica.martini}@wave-up.it

Abstract

This paper concerns the analysis of fields and currents on modulated metasurfaces (MTSs) and the synthesis of MTSs implementing a given field distribution. The work herein described has been developed to set up an effective design process for modulated MTSs realizing antennas with customizable pattern. Here we give a brief description of the process for the analysis, synthesis and implementation of fields in modulated MTSs in the framework of planar leaky wave antennas. As support to the discussion, numerical results are presented for highly directive beam antennas.

1. Introduction

Planar structures based on metasurfaces (MTSs) have become popular in the recent years due to their appealing characteristics of low profile, manufacturing simplicity and deep control of the electromagnetic fields [1]. At the microwave regime, the basic structure is a dielectric slab, textured with a distribution of electrically small metal elements, tightly placed on a regular lattice. MTSs found applications as screens for controlling the reflection and transmission properties of impinging waves or as devices for controlling the dispersion properties of surface waves (SWs). In this latter application, the dielectric slab is usually backed by a ground plane and the MTS is used for changing the dispersion properties of a guided SW, modifying its propagation path [2] or transforming it into a radiative leaky wave (LW) [3].

This paper deals with the analysis of fields and currents in MTSs and with the synthesis of MTSs in the framework of planar LW antennas. The earliest work on this concern are quite recent and their main scope was to demonstrate the feasibility of the concept [4], [5]. A comprehensive analysis of the antenna performances and their design has appeared only recently [3], [6]. Planar LW antennas based on MTSs result in a low profile device, with a high degree of customization of the performances in terms of beam shape, phase and polarization. The concept is that the same overall structure can be used for several antenna designs with very different performances, leading to an attractive solution also for dynamically reconfigurable devices.

We have structured the paper in the following way. Section II describes the radiation mechanism in antennas based on modulated MTSs, introducing a global description of fields and currents in this kind of structure. Section III discusses the synthesis of a MTS for implementing a target aperture field distribution and controlling the antenna performances. Section IV deals with the implementation of the MTS by means of a thin layer of metallic patches. Section V describes a fast, efficient, full-wave solver for the analysis of the antenna Layout. Concluding remarks are given in Section VI.

2. Fields and Currents on Modulated Metasurfaces

Here we consider a structure composed of a grounded dielectric slab printed with a dense texture of electrically small metal patches having different size and shape. The metallic cladding forms the MTS, which is modelled as a continuous layer of capacitive, anisotropic reactance denoted as "transparent reactance" $\underline{\underline{\mathbf{X}}}$. This latter relates the tangential electric field $\mathbf{E}_t$ to the current flowing in the patches, which means:

$$\mathbf{E}_t = j\underline{\underline{\mathbf{X}}}\cdot\mathbf{J} \qquad (1)$$

The patch cladding implements a sampling of the boundary conditions (BCs) that the MTS has to impose to change the dispersion properties of a SW excited on it. The SW is provided by a TM point source, usually embedded in the MTS, and, in its simplest configuration, it is constituted by a vertical monopole (Figure 1). The SW can be transformed into a LW by modulating in a periodic or quasi-periodic fashion the MTS-BCs interacting with the SW.

Fields and currents in strictly periodic environments are rigorously described by the Floquet wave (FW) theory. However, in our MTSs, the modulation might not be strictly periodic, as it might have a not uniform period and amplitude to control the beam shape and polarization. Despite this, an adiabatic FW expansion associated with the local periodicity of the surface is still a valid tool for describing fields and currents: fields and currents are approximated as the ones of a locally planar wavefront on an infinite uniform modulation with amplitude and period matched to the local periodicity [3].

In addition, using the same locally planar approximation, fields and currents are related each other through the Green's function of the grounded dielectric slab, evaluated at each local FW wavenumber. Thus, we can set up an adiabatic local dispersion equation for determining the local value of the wavenumber of the SW on the modulated MTS.

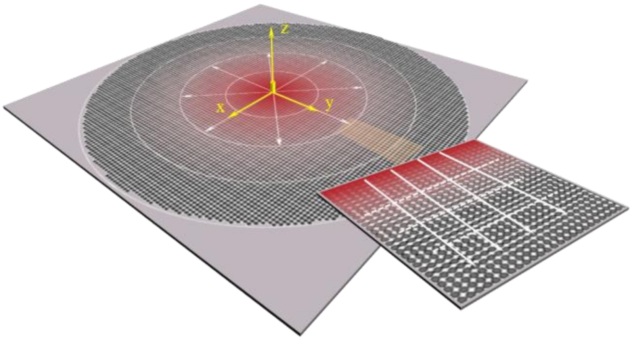

Figure 1 Sketch of the reference structure for a MTS of printed patches on a grounded dielectric slab, fed by a TM point source at the origin of the reference system. The inset sketches the local periodic approximation of the MTS for the adiabatic formulation of the FW theory.

3. Synthesis of the Metasurface

The problem of the synthesis of the MTS is the determination of the modulation parameters (namely phases and amplitudes of the tensorial reactance X modulation) such that the radiative aperture field matches a target aperture field [6]. To set up the synthesis process, first we have to recognize that not all the FWs that describe the current on the MTS produce a radiation. Naming $\beta^{(n)}$ the n-th FW wavenumber, only the ones with $|\mathrm{Re}\{\beta^{(n)}\}|\leq k$ (being k the free space wavenumber) produce radiation controlled by the MTS, whilst all the other radiate only by diffractive phenomena from the edge of the MTS.

By matching the radiative modes with the target aperture distribution, it is possible to set up an iterative procedure that gives the parameters of the modulation. During each iteration, the process estimates amplitude and phase of the entries of the reactance $\underline{\underline{\mathbf{X}}}$ as functions of space. Next, the complex propagation wavenumber on the surface is determined by solving the adiabatic dispersion equation. The process reaches convergence when the change of the modulation parameters becomes smaller than a given threshold. Roughly speaking, the local period of the modulation rules the interaction with the SW, determining the phase of the radiative field. The depth of the modulation controls the amplitude of the aperture field and the amount of energy spilled from the SW and transformed into a LW. The polarization of the aperture field is controlled by the relative phases between the entries of the reactance tensor $\underline{\underline{\mathbf{X}}}$.

As numerical examples, we show in Figure 2 (a)-(c), the X_{11} component of the impedance tensor $\underline{\underline{\mathbf{X}}}$, as a function of space, for impedance surfaces producing (a) a broadside beam, (b) a 25° off-axis beam and (c) a 50° off-axis beam. Each surface has a diameter of 16λ at 26.65 GHz, on a substrate with thickness 0.508 mm and relative permittivity 9.8, and it is excited by a vertical monopole. Figure 3 shows the relevant radiation patterns obtained by a full-wave solver for continuous impedance BCs [7]. Peak directivities for the three beams of Figure 3 are 32.9 dBi for the broadside beam 32.3 dBi for 25° off- axis beam and 30.76 dBi for the 50° off-axis beam, thus resulting in efficiencies respectively around 77%, 74.3%, and 73.4 %. The apertures producing the three beams in Figure 3 only differ for the BCs imposed by the MTS, which have been designed using the iterative procedure based on the asymptotic FW expansion herein introduced.

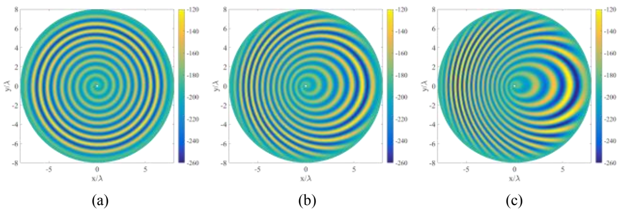

Figure 2 X_{11} component of the tensor $\underline{\underline{\mathbf{X}}}$ as a function of space, for MTS antennas producing (a) a broadside beam, (b) a 25° off-axis beam and (c) a 50° off-axis beam.

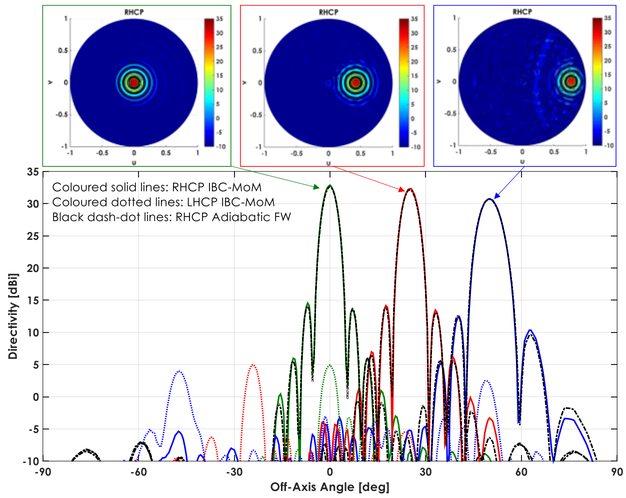

Figure 3 Example of radiation patterns that can be obtained from the same aperture (16λ of diameter), with the same feeding system by reshaping the BCs imposed by the MTS.

4. Implementation of the Metasurface

A common way to implement the MTS is through a texture of electrically small patches, tightly placed on a regular lattice (typically a Cartesian lattice). Inside the lattice, the geometrical parameters of the patches gradually change so that they implement a thin layer of capacitive, anisotropic reactance which represents a sampling of the continuous impedance BCs $\underline{\underline{\mathbf{X}}}$ provided by the synthesis process. Each impedance sample is implemented printing inside the corresponding lattice cell a metallic patch, whose geometrical parameters are found within a Data Base. The Data Base is constructed analyzing several elliptical patches by a periodic full-wave solver. This means that a given elliptical patch is characterized in terms of tensorial impedance by analyzing it in a periodic environment within a uniform lattice, for several incidence directions of planar surface waves.

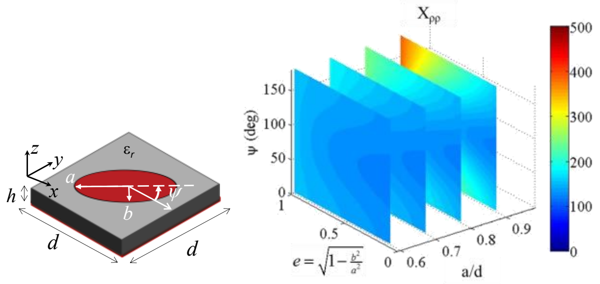

Figure 4 Example of impedance map produced by patches of elliptical shape, excited by a SW propagating along the x-direction.

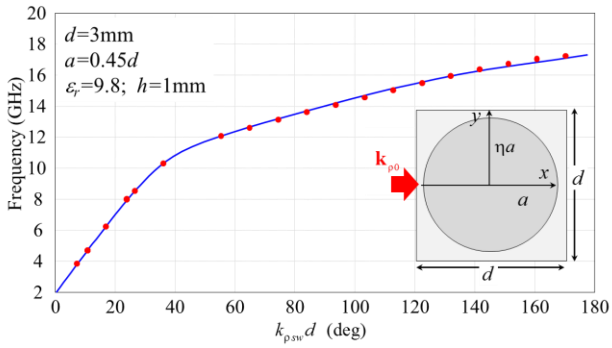

Figure 5 Comparison of the dispersion curves provided by CST (red dots) and a periodic MoM solver based on the entire domain basis functions (blue solid line) described in [8].

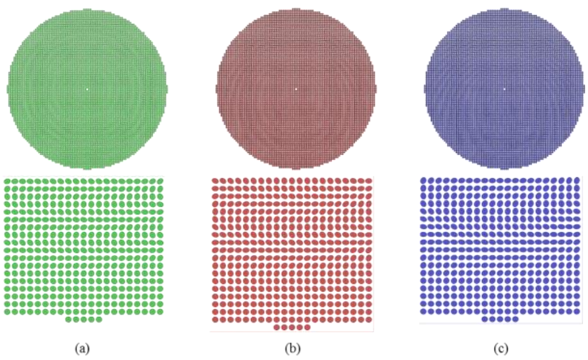

Figure 6 Layouts of the impedance surfaces shown in Figure 2 implemented by a texture of elliptical patches producing (a) a broadside beam, (b) a 25° off-axis beam and (c) a 50° off-axis beam.

From the periodic full-wave analysis, maps are constructed that link an elliptical geometry to an impedance tensor, for a given surface wave incidence direction (Figure 4). Among the possible choices of patch shapes (for instance rectangular, circular slotted patch, elliptical patches… etc.), the elliptical shape provides several advantages with respect to others from the computational point of view. Indeed, as described in [8], currents on elliptical patches can be effectively described by only three entire domain basis functions, whose spatial and spectral form are known in a closed form. This represents a significant advantage in a full-wave formulation to analyze the performance of the textured surface, as the number of unknowns can be drastically reduced with respect to a mesh based on RWG basis functions. Figure 5 compares the dispersion curves obtained from CST and an in-house built periodic MoM solver based on the entire domain basis functions described in [8], showing a very good agreement between the results. We have synthesized the impedance surfaces designed in Figure 2 by a texture of elliptical patches, placed on a Cartesian lattice with lattice size of 1.9 mm. The layouts are shown in Figure 6 together with relevant details of the surface.

5. Full-Wave Analysis of the Metasurface

An efficient, effective MoM solver can be set up using the entire domain basis functions described in [8]. The electric field integral equation, in its linear system formulation reads as

$$\underline{\underline{\mathbf{Z}}} \cdot \underline{\mathbf{I}} = \underline{\mathbf{V}} \qquad (2)$$

where $\underline{\mathbf{V}}$ is the known vector term depending on the source, $\underline{\mathbf{I}}$ is the unknown vector and $\underline{\underline{\mathbf{Z}}}$ is the interaction matrix. In its simplest implementation, a vertical elemental dipole at the center of the surface, can be used to feed the MTS antenna. A more elaborated feeding system can be employed to increase the accuracy of the simulation. The proper Green's function is used to construct both $\underline{\mathbf{V}}$ and $\underline{\underline{\mathbf{Z}}}$ accounting for the presence of the grounded slab.

The computational efficiency of the solver can be further increased implementing the classical Fast Multiple Algorithm (FMA) described in [9] and [10], representing $\underline{\underline{\mathbf{Z}}}$ as a superposition of a near interaction matrix $\underline{\underline{\mathbf{Z}}}_{\text{near}}$ and a far interaction matrix $\underline{\underline{\mathbf{Z}}}_{\text{far}}$. The 2D-FMM version reported in [11]-[12] is used to accelerate the matrix-vector product relevant to $\underline{\underline{\mathbf{Z}}}_{\text{far}}$. The near interactions matrix $\underline{\underline{\mathbf{Z}}}_{\text{near}}$ is build using a spectral MoM formulation, implementing the efficient procedure in [13] and taking advantage of the knowledge of the closed-form expression of the spectra of the basis functions [8]. Using the scheme herein briefly introduced, it is possible to set up a fast, accurate MoM solver and hence to simulate also very large MTS antenna structures, which would be unfeasible with a conventional MoM solver based on RWG basis functions. Figure 7 shows the results obtained from the FMM-MoM, for a MTS antenna of 48λ of diameter, composed of more than 45000 elliptical patches, radiating a 30° off-axis pencil beam. The antenna gain is 41.5 dBi thus resulting in an efficiency of about 71.73%.

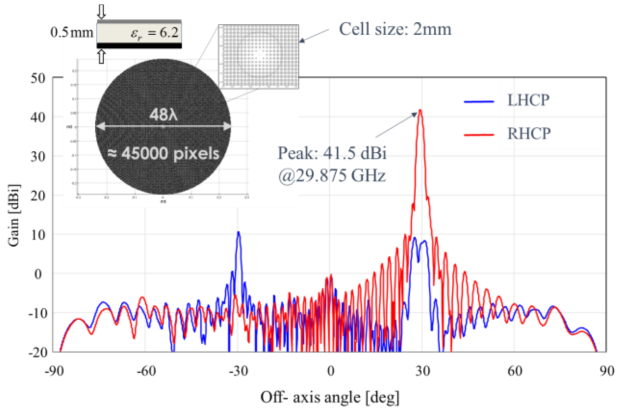

Figure 7 Far field pattern of a MTS antenna of 48λ of diameter, realized with more than 45000 metal patches. Results have been obtained by a MoM solver based on entire domain basis functions [8], implementing a 2D- Fast multiple algorithm.

5. Conclusions

We have provided a global picture of synthesis, analysis and implementation of fields in modulated MTSs realizing planar LW antennas. The synthesis is based on an asymptotic form of the Floquet wave theory, adiabatically matched to the locally periodic problem associated to curvilinear BCs. Despite the Floquet wave theory is rigorously valid in strictly periodic problems, its application in the frame of modulated MTS antennas allows to set up an effective synthesis procedure for making them radiate a desired field. The MTS is implemented by a texture of electrically small patches. When they have elliptical shapes, three entire domain basis functions are enough for describing the behavior of the current on them. These entire domain basis functions, whose spatial and spectral form are known in a closed form, have been effectively used to implement a MoM solver based on a Fast Multiple Algorithm to analyze the full patched layout. Numerical results have been given to prove the effectiveness of the concepts introduced.

7. References

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