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An Explicit Time Marching Scheme for Solving Surface Integral Equations of Acoustics

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1. Extended Abstract

Transient acoustic scattering from rigid objects can be analyzed by solving the time dependent Kirchhoff surface integral equation (TD-KSIE) [1]. Marching-on-in-time (MOT)-based schemes, which are developed for solving the TD-KSIE, expand the unknown acoustic potential in terms of spatial and temporal basis functions. This expansion is inserted into the TD-KSIE, and the resulting equation is spatially tested at discrete times to yield a matrix system. Then a time marching scheme is used to solve this system for the unknown coefficients of the expansion. Depending on types of the spatial and temporal basis functions, spatial testing functions, and the time-step size, the MOT scheme can be either implicit or explicit [1]. Unlike their implicit counterparts, explicit MOT schemes do not call for a matrix inversion at every time step. However, they require a smaller time-step size to ensure stability.

To alleviate this disadvantage of the explicit MOT schemes, a novel quasi-explicit MOT scheme has been developed in [2] to solve the time domain magnetic field integral equation (TD-MFIE) of electromagnetics. First, the TD-MFIE is discretized using (zeroth/first order) spatial basis and testing functions [3]. The resulting matrix system is cast in the form of a first order ordinary differential equation, which is then integrated numerically in time to yield the coefficients of the basis functions expanding the unknown (current density in electromagnetics). This quasi-explicit scheme solves a matrix system involving a Gram matrix (between testing and basis functions) at every time step. However, unlike the implicit solvers, the sparsity level of this system does not depend on the time-step size. As a result, it can be solved very efficiently using an iterative method even when the time-step size is large. Additionally, numerical results have shown that, unlike the traditional explicit MOT schemes, this solver's time-step size can be as large as that of its implicit counterparts without sacrificing accuracy or stability.

In this work, a fully explicit time marching scheme is developed to solve the TD-KSIE for analyzing transient acoustic scattering from rigid objects. Unlike the above quasi-explicit solver, TD-KSIE is spatially discretized using a higher-order Nyström method [4]. Since the Nyström method requires only samples of the acoustic potential on discretization elements without using a basis function, which extends over two or more discretization elements, the Gram matrix resulting from the spatial discretization is block diagonal. This Gram matrix is inverted (to yield a block diagonal inverse) and stored before the time marching starts. This makes the time marching very efficient since the "matrix inversion" needed by the quasi-explicit solver is now replaced by a single block-diagonal matrix-vector multiplication. Additionally, the explicit solver developed here is higher-order accurate both in space and time. Numerical results, which demonstrate the proposed solver's accuracy, efficiency, and applicability, will be presented.

2. References

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