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Elimination of the Ill-Posedness in the Inverse Problem through Use of Focusing Properties of Maxwell Fish Eye Lens

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1 Introduction

Inverse source and scattering problems are ill-posed when stated in the form of first kind integral equation with the free space Green's functions. It can be shown that such ill-posedness can be eliminated if the imaging experiment is set up in a medium with focusing properties. Realization of such medium with Maxwell Fish Eye lens is considered in this work. The imaged object is taken to be a thin layer of non-magnetic inhomogenous dielectric featuring angular dependence of the relative permittivity and situated conformally with the surface of the lens. Angular discretization of the object with small elements followed by selection of the observation locations on the surface of the lens diametrically opposite to the centers of the elements lead to well-conditioned form of the discretized inverse source problem and accurate reconstruction of the permittivity distribution.

2 Abstract

Ability to reconstruct material properties of objects from the information on how they scatter electromagnetic fields enables important practical applications in remote sensing, biomedical imaging, non-destructive evaluation, and other areas. Mathematically such reconstruction is commonly formulated as first kind volume integral equation which is known to be ill-posed when it features a smooth kernel. Such smooth kernels, unfortunately, is a natural consequence in the formulation of the inverse problem when sensor locations are not allowed to be inside the imaged object. This is due to the fact that electromagnetic field radiated by a point source (aka Green's function) is a smooth function when observed far away from the source. For that reason the inverse problem in its commonly stated form constitutes an ill-posed problem.

We demonstrated in [1], however, that if the inverse problem is formulated in a medium with particular focusing properties its ill-posedness can be eliminated. In this work [2] we follow the principle proposed in [1] and consider as the focusing medium the Maxwell Fish Eye Lens. The electromagentic field produced by the source points situated near the surface of the lens features not only the singular behaviour near the source but also at the focal point situated diametrically opposite to the source. This property of the lens' fields can be utilized to eliminated undesired smoothness of the kernel in the first kind integral equation describing the inverse problem and lead to its unique solution. Numerically conducted imaging experiments in this work demonstrate validity of such approach to elimination of the inverse problem ill-posedness and show that material properties of the imaged objects can be accurately reconstructed when sufficient focusing is exhibited by the focusing medium.

References

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