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# Analysis of the Composite Exit-hole Effect on the Seawater Dielectric Measurements

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## Abstract

The resonant cavity technique is widely used for accurately determining the complex dielectric constants of liquids at a single frequency. The liquid is introduced into the cavity through a capillary glass tube. In order to insert the tube into the cavity, an exit-hole is drilled on the center of the top and bottom endplates of the cavity. Since the cross section of the cavity and the exit-hole region contains several different dielectric layers, the problem is called the "composite" exit-hole effect. Researchers have found these exit-holes have a frequency-pulling effect on the measurements, which results in a bias in the real part of the measured dielectric constant. The George Washington University (GW) has been making accurate measurements of the seawater dielectric constant at L-band (1.413 GHz) by using this cavity technique. Based on the experimental setup of GW seawater dielectric measurements, this paper describes the analysis of the composite exit-hole effect and the bias in the real part of the measured dielectric constant results.

## 1. Introduction

The effect of the exit-holes on the cavity endplates has concerned investigators for years. Ho and Hall [1] used an approximate method derived by Estin and Bussey [2] showing that the error introduced in the measurement of the dielectric constant was less than 0.1% for a cavity at 1.43 GHz. However, in the calculation provided by Estin and Bussey, only $TM_{010}$ mode is considered in the tube region. In addition, their calculation only considers the case of the exit-hole filled with solid sample material. More recently, Risman and Wappling-Raaholt [3] have performed an FDTD computer simulation of the cavity with exit holes. They found that the exit hole did provide a large enough frequency shift to be important in the computation of the real part of the dielectric constant.

Accurate measurements of seawater dielectric constants have been made by The George Washington University at L-band [4]. To examine whether biases exist in the measurements, the composite exit-hole effect has been investigated by GW group recently.

This paper is structured as follows: In Section 2, the seawater dielectric measurements and the previous work on the exit-hole effect are introduced briefly. Section 3 gives the analysis of the composite exit-hole effect. In addition, the bias on the dielectric constant results due to this effect is discussed. Finally, the conclusion and the future work are presented in Section 4.

## 2. Seawater Measurement and the Previous Work on the Exit-Hole Effect

At GW's microwave laboratory, a brass microwave cavity operating at L-band (1.413 GHz) has been employed to determine the seawater dielectric constant. The seawater is introduced into the brass microwave cavity through a capillary glass tube having an inner diameter of 0.1 mm. The seawater that enters the capillary tube then perturbs the field inside the cavity causing a change in both the resonant frequency $f$ and the cavity quality factor, $Q$. The real and imaginary part of the seawater dielectric constant can be obtained by a perturbation relation, which is given by

$$\varepsilon' - 1 = 2C\Delta f / f_0 \quad , \quad \Delta f = f_0 - f \tag{1a}$$

$$\varepsilon'' = C\Delta(1/Q) \quad , \quad \Delta(1/Q) = 1/Q - 1/Q_0 \tag{1b}$$

where $\varepsilon'$ and $\varepsilon''$ are the real and imaginary parts of the relative dielectric constant of the seawater sample and $C$ is a calibration constant. The variables $f_0$, $Q_0$ and $f$, $Q$ are the resonant frequency and the quality factor of the cavity before and after the sample solution has been introduced respectively. The calibration constant $C$, appearing in the algorithm, is determined by using a reference solution with a known dielectric constant. Thus, $C$ can be determined by inverting eq. (1). In this work, the reference solution is chosen to be methanol based on the measurements made by Gregory and Clark [5].

Due to the complexity of the exit-hole problem containing a capillary tube, a waveguide analysis for a simpler case -- empty cavity with only a small hole was performed previously. A special design for a small hole of adjustable length has been completed and the experimental results were compared with the theoretical results. They had a good agreement and the difference was less than 2%. The details of this previous work were presented in [6].

## 3. Analysis of the Composite Exit-Hole Effect

Similar to the derivation presented by [6], the composite exit-hole problem is equivalent to a step-cylindrical waveguide problem. Based on the actual experimental setup, the geometry of the transverse plane of the composite exit-hole is shown in Figure 1.

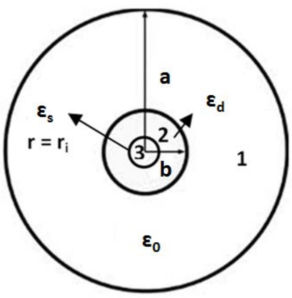


**Figure 1.** Geometry of the transverse plane of the cavity with composite exit-hole

In the figure, region 1 is the cylindrical layer of the empty cavity that is filled with air ($\varepsilon_0$); region 2 is the cylindrical layer of the quartz tube that has a dielectric value of $\varepsilon_d$ and region 3 is the cylinder of the sample solution that has a dielectric value of $\varepsilon_s$. The radii of region 1,2 and 3 are respectively $a$,$b$ and $r_i$.

Figure 2 shows a side view of the equivalent step-cylindrical waveguide. In the figure, the cavity region is represented by A and the exit-hole region is represented by B.

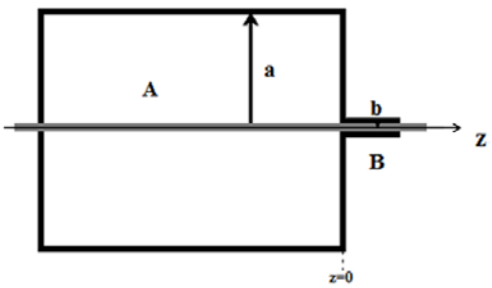


**Figure 2.** Geometry of the longitude plane of the cavity with composite exit-hole

In region A, only the dominant mode ($TM_{010}$) can propagate while the higher order modes are cut-off. All the modes in region B are cut-off. Thus, the electric fields in both regions can be written as,

$$E_{\rho A} = \mathbf{e}'_{A1}(\rho)\cdot(e^{-j\kappa_{A1}z} + \Gamma_1 e^{+j\kappa_{A1}z}) + \sum_{m=2}^{\infty}\mathbf{e}'_{Am}(\rho)\cdot\Gamma_m e^{+\alpha_{Am}z}, \quad z \le 0 \quad (2a)$$

$$E_{\rho B} = \sum_{n=1}^{\infty}\mathbf{e}'_{Bn}(\rho)T_n e^{-\alpha_{Bn}z}, \quad z \ge 0 \quad (2b)$$

where subscribe $m$ and $n$ represent the order numbers; $\Gamma_m$ and $T_n$ are the reflection and transmission coefficients due to the composite exit-hole effect; $\kappa_{A1}$ is the propagation constant of the dominant mode in region A; $\alpha_{Am}$ and $\alpha_{Bn}$ are the attenuation constants in region A and B with the order of $m$ and $n$; $\mathbf{e}_{Am}$ and $\mathbf{e}_{Bn}$ are the mode functions in region A and B with the order of $m$ and $n$. Note that at z=0, all higher order modes are excited and have to be taken into analysis. The mode functions $\mathbf{e}_{Am}$ and $\mathbf{e}_{Bn}$ are different in different layers. They are given by,

$$\mathbf{e}'_{Am}(\rho) = \begin{cases} [A'_{fm}J_1(p'_{Afm}\rho) + \bar{A}'_{fm}Y_1(p'_{Afm}\rho)]\hat{\rho}, & b \le \rho \le a \\ [A'_{dm}J_1(p'_{Adm}\rho) + \bar{A}'_{dm}Y_1(p'_{Adm}\rho)]\hat{\rho}, & r_i \le \rho \le b \\ A'_{sm}J_1(p'_{Asm}\rho)\hat{\rho}, & 0 \le \rho \le r_i \end{cases} \quad (3a)$$

$$\mathbf{e}_{Bn}'(\rho) = \begin{cases} [B'_{dn}J_1(p'_{Bdn}\rho) + \bar{B}'_{dn}Y_1(p'_{Bdn}\rho)]\hat{\rho}, & r_i \le \rho \le b \\ B'_{sn}J_1(p'_{Bsn}\rho)\hat{\rho}, & 0 \le \rho \le r_i \end{cases} \quad (3b)$$

where $p'_{Afm}$, $p'_{Adm}$ and $p'_{Asm}$ are the eigenvalues of the cavity region in the layers of the air, the quartz tube and the sample solution; $p'_{Bdn}$ and $p'_{Bsn}$ are the eigenvalues of the composite exit-hole region in the layers of quartz tube and sample solution.

Due to the inhomogeneous structure of the cavity, the eigenvalues can be determined by matching the boundary conditions on the corresponding transverse cross-sections. In region A, the magnetic field in $\phi$ direction, $H_\phi$, and the electric field in z direction, $E_z$, are matched on the interfaces at $\rho = r_i$, $b$ and $a$. Note that the outer wall of the cavity is assumed to be lossless since the loss on the wall has little effect on the resonant frequency. Similar process can be used to determine the eigenvalues in region B. The equations of the boundary conditions also reveal the relationships between the coefficients of the mode functions (e.g. $A'_{fm}$ and $\bar{A}'_{fm}$ in eq. (3)). Thus, the expressions of the mode functions in both regions are obtained.

The expressions for the reflection coefficient $\Gamma_m$, shown in eq. (2), can be determined by matching the boundary condition at the interface between the exit-hole and cavity regions. The frequency shift can then be calculated by solving the resonance condition of the equivalent transmission line with and without the reflection coefficient, $\Gamma_1$. Note that although only $\Gamma_1$ is used for determining the frequency shift, it depends on all the higher order modes of the reflection coefficients.

Based on this method, the frequency shifts due to the composite exit-hole effect are calculated by choosing the value of $\varepsilon_s$ to be the value of the dielectric constant for seawater, methanol and air in the algorithm; these frequencies shifts are written as $\Delta f_{es}$, $\Delta f_{em}$ and $\Delta f_{e0}$, respectively.

The bias on the real part of the measured seawater dielectric constant, which is denoted as $\delta\varepsilon_s'$, can be obtained by modifying eq.(1a) as,

$$\varepsilon_s' + \delta\varepsilon_s' - 1 = 2(C + \Delta C)[\Delta f_s + \Delta f_{es} - \Delta f_{e0}] / f_0 \qquad (4)$$

where, $C + \Delta C = (\varepsilon_m' - 1) f_0 \big/ 2(\Delta f_m + \Delta f_{em} - \Delta f_{e0})$

In eq.(4), $\varepsilon_s'$ is the real part of the measured seawater dielectric constant, $\Delta f_s$ is the frequency change due to the presence of seawater, $\Delta C$ is the difference in the calibration coefficient by taking the exit-hole effect into account. All the other parameters are defined previously in this article.

It can be seen from eq. (4) that the bias on the real part of the measured dielectric constants is cancelled out mostly by the presence of $\Delta C$. In the other words, the frequency pulling due to the exit-hole effect in the seawater measurements is cancelled out by the frequency pulling that exists in the calibration measurements. More details and results will be presented at the meeting.

# 4. Conclusion

In summary, this article presents a waveguide method to analyze the composite exit-hole effect on the seawater dielectric measurements using cavity technique. It is shown that the bias is less than 0.1% for the real part of the measured dielectric constant. This is mainly due to the fact that the calibration measurements cancel out the frequency pulling. If the calibration measurements are made with a different tube or a different system, the bias is expected to be much greater.

# 5. Reference


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