*32nd URSI GASS, Montreal, 19–26 August 2017*

# The Antenna Correlation Coefficient in Wireless Sensor Networks

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## Abstract

Diversity antennas are for improving communications and sensing systems. The concept has been around for well over half a century and overlaps with sensor array design. Diversity in communications has come to the fore because most of our radio links are now mobile, seldom having a line-of-sight but being rich in multipath where the signals from single port antennas are degraded. It was originally developed for the receiving case, but recent advances, driven by consumer demand for better performance, has seen diversity deployed at both ends, allowing multiple input, multiple output (MIMO) communications. With wireless sensor networks (WSNs) gaining broader interest - i.e., systems other than cellular and wifi networks, antenna diversity techniques take on a broader interest for improving WSN sensor node and network performance. The most important parameter in diversity design is the *correlation coefficient*, but this is not widely understood. We present an update which aims to clarify some common misconceptions. The various definitions of correlation coefficients for similar elements, are shown analytically to be the same and this is demonstrated for practical antennas for WSN nodes. Finally, it is clarified why the ensemble signal correlation coefficient, or its estimate, is often different from that commonly calculated in commercial antenna simulators.

## 1 Introduction

A WSN usually consists of sensor nodes that measure an aspect of their environment and communicate this information with other nodes in the network or a base station/gateway. They have drawn increasing attention for a vast range of applications such as monitoring health, environment, infrastructure integrity; coordinating smart manufacturing; and target detection, tracking, localization, etc. A multipath environment degrades the WSN (communications between the nodes, or between a node and the base station) under nearly all definitions of WSN performance. Antenna diversity has been extremely effective in dealing with similar problems in mobile communications, and can be applied to improve WSNs. The diversity action mitigates the fading of multipath links, improves energy efficiency, increases communication range, reduces outage; and reduces the variance in link gain. As such it can also improve other performance aspects such as location accuracy estimates that use RSSI (signal strength).

In the design of diversity antennas, it is known that the antenna elements should be "uncorrelated", but the meaning of this term or how to design for it on an electrically compact platform is not widely understood. The original meaning of the correlation stems from the ensemble correlation of the signals representing narrow-band channel transfer functions imposed by multipath. For narrow-band channels, this means a spatial (or time, for a moving terminal) series correlation between the Rayleigh-like fading envelopes of continuous-wave signals (i.e. narrow-band) transmitted via the multipath. The envelopes were used because receivers often have a real-time RSSI voltage signal proportional to the channel gain or its log), which could be recorded and processed for correlation estimation. But such a time series estimate is difficult in the field and can have questionable meaning and accuracy owing to the typically non-stationary channels and difficult-to-repeat measurements. Instead, using sufficiently simplified models of a complicated physical situation, the correlation coefficient can be expressed as an antenna parameter, viz., the normalized mutual resistance (or the equivalent expressed as scattering parameters), or an inner product of the patterns [1]. Such a parameter can be estimated/measured in the comfort of a laboratory or computer. But it has to be remembered that the time series is still the bottom line for communications and sensing performance. Good diversity performance relies on local, i.e. instantaneously, uncorrelated channel degradation, rather than some ensemble estimate which may not reflect all the local behaviour. The link between these correlation measures remains largely unexplored. A practical design approach is to step back from the mutual resistance or its equivalent scattering parameter expression, and model the propagation scenario statistically as directional rather than omnidirectional; and use models or estimates of the element patterns to estimate the correlation between the elements. A "low" correlation is best of course, although for small-dimensioned antennas (just a couple of elements), a "high" correlation coefficient (well over 0.5) gives strong performance improvement for most applications. For large numbers of antenna elements - a key to future communications systems - even low correlations degrade the potential communications performance quickly [2]. The following Sections review the correlation coefficient in its many forms as used by designers, and Section 4 shows their equivalence.

## 2 Multipath environment and the antenna voltage correlation coefficient

The correlation coefficient was developed for mobile communications, see [3] and the references therein. Fig. 1

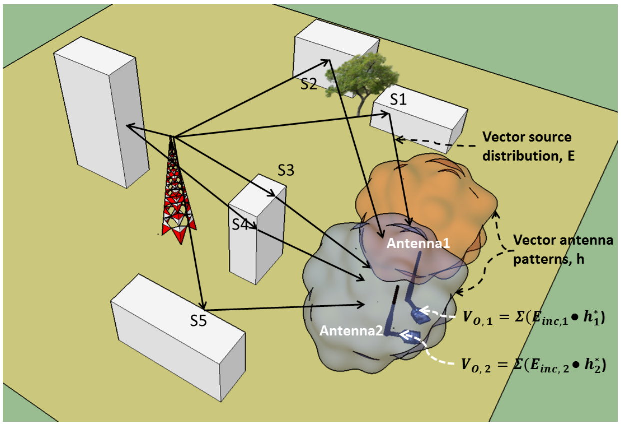


**Figure 1.** *Antennas transform vector waves to scalar voltages which the designer strives to be uncorrelated.*

depicts the conversion by the sensor elements of incoming waves to signals. The starting point is the definition of a vector receiving pattern $h_O^*(\theta,\phi)$ from the scalar open circuit voltage from a vector incident wave, $V_O(\theta,\phi) = E_{inc}(\theta,\phi) \bullet h_O^*(\theta,\phi)$. (A *pattern* refers to the transmitting case, and the transmit pattern is $h_O(\theta,\phi)$.) The multipath is accounted for by integrating the incoming waves at the local position, e.g. [4]. So for the $i$th element of a diversity antenna, here with the pattern frequency dependence introduced for the antenna but not for the incident wave distribution (only because the incident wave distribution is later idealized in a statistical treatment - see below),

$$V_{O,i}(\omega) = \int_0^{2\pi}\int_0^{\pi} E_{inc,i}(\theta,\phi,\omega) \bullet h_i^*(\theta,\phi,\omega)\sin\theta d\theta d\phi \tag{1}$$

where $h_i^*$ is an *embedded* (meaning mounted on the platform of the terminal) - vector receiving pattern of the $i$th antenna element. Here the embedded pattern takes the meaning that it is the open circuit pattern measured when all other antenna elements of the diversity system are present and open circuited. The *loaded circuit voltage* results when the elements are terminated in their loads. When there is mutual coupling - almost always the case - the embedded loaded element patterns are in general different to the embedded open circuit patterns. Equation (1) shows the inseparable nature of the antenna pattern and the incident waves in defining the channel. Measurements taken with one antenna cannot characterize the channel for using a different antenna.

The amplitudes, and in particular the phases, of the incoming waves depend on the antenna position (or time, for a moving terminal), so averaging is required over space for estimating power and, from a statistical approach, other moments. The spatially averaged power available at the antenna's terminals is proportional to the spatial autocorrelation of the open circuit voltage:

$$\begin{aligned}\langle |V_{O,i}|^2\rangle &= \left\langle \left(\oiint_{4\pi} E_{inc,i}(\theta,\phi)\bullet h_i^*(\theta,\phi)\sin\theta d\theta d\phi\right)\right.\\ &\quad\times\left.\left(\oiint_{4\pi} E_{inc,i}(\theta,\phi)\bullet h_i^*(\theta,\phi)\sin\theta d\theta d\phi\right)^*\right\rangle\\ &= \oiint_{4\pi}\left\langle |E_{inc\theta,i}(\theta,\phi)|^2\right\rangle |h_{\theta,i}(\theta,\phi)|^2\sin\theta d\theta d\phi\\ &\quad+\oiint_{4\pi}\left\langle |E_{inc\phi,i}(\theta,\phi)|^2\right\rangle |h_{\phi,i}(\theta,\phi)|^2\sin\theta d\theta d\phi\,.\end{aligned} \tag{2}$$

in which major assumptions are clearly evident. This power can be viewed as from an (angularly) matched filter between a pdf of the incoming wave power - one for each polarization - and the receiving power pattern in each polarization. The cross-correlation coefficient is

$$\begin{aligned}\rho_{O,ij} &= \frac{\langle V_{O,i}V_{O,j}^*\rangle}{\sqrt{\langle |V_{O,i}|^2\rangle\langle |V_{O,j}|^2\rangle}}\\ &= \frac{E\left\{(V_{O,i}-\widetilde{V_{O,i}})(V_{O,j}-\widetilde{V_{O,j}})^*\right\}}{\sqrt{E\left\{|V_{O,i}-\widetilde{V_{O,i}}|^2\right\}E\left\{|V_{O,j}-\widetilde{V_{O,j}}|^2\right\}}}\,,\end{aligned} \tag{3}$$

where the second line uses an expectation notation to reflect that a local estimate of the mean, denoted by a tilde (~), is required. Using these standard relations, the correlation matrix of a diversity antenna system can be estimated from observations of the voltage signals.

The alternative antenna parameter approaches ([4]-[7]) for estimating the correlation coefficient are now discussed.

## 3 Correlation coefficient as the inner product of element patterns

It is known that under several assumptions, the correlation coefficient between the $i$th and $j$th elements of a diversity antenna system can be couched in terms of their embedded far-field patterns, with the inner product weighting given by the distribution of incident power ([4, 5]) –

$$\rho_{ij} = \frac{\oiint_{4\pi} H_{i,j}(\theta,\phi)\,d\Omega}{\sqrt{\oiint_{4\pi} H_{i,i}(\theta,\phi)\,d\Omega}\sqrt{\oiint_{4\pi} H_{j,j}(\theta,\phi)\,d\Omega}} \tag{4}$$

where

$$H_{i,j}(\theta,\phi) = XPR\cdot h_{\theta,i}h_{\theta,j}^*P_\theta + h_{\phi,i}h_{\phi,j}^*P_\phi$$

$\Omega$ denotes the solid angle with $d\Omega = \sin\theta d\theta d\phi$; $XPR$ is the cross-polarization ratio of mobile communications (different to the XPR of satellite communications); $h_{\theta,i}(\theta,\phi)$ and $h_{\phi,i}(\theta,\phi,)$ is the $i$th element receiving pattern; and $P_\theta \equiv P_\theta(\theta,\phi)$ and $P_\phi \equiv P_\phi(\theta,\phi)$ are the $\theta$- and $\phi$- polarization pdfs of the incoming waves respectively.

An underlying condition is that the electrical size of the diversity antenna system is small enough to consider all the elements see the same set of incoming waves. Another assumption is that the incoming waves are angularly uncorrelated, and uncorrelated between polarizations. Few attempts have been made to experimentally test these assumptions, and it is far from straightforward to establish these assumptions experimentally.

The gaussian basis from many incoming waves means that the envelope correlation coefficient, ECC ($\rho_e$), can be approximated by the correlation coefficient of the magnitude square of the complex correlation coefficient, which is the power signal, easily obtained from the RSSI:-

$$\rho_{e,ij} \approx |\rho_{ij}|^2 = \rho_{e^2,ij}. \tag{5}$$

In practice, estimates of these will not be equal owing to the finite sample size of an estimation. The estimate using the complex version is the more accurate because no information is discarded before the correlation calculation.

The pattern inner product method of (4) is convenient because it is analytic, repeatable, and allows the sensitivity to different propagation conditions and models to be established. Figure 2 shows examples for a small Internet-of-Things device with two inverted-F antennas, spaced and rotated to provide diversity action [8, 9], which emphasizes how a correlation coefficient depends on two well-known propagation models. This is the major reason why estimates of the time series correlations are seldom the same as those from the "patternless" calculations given below.

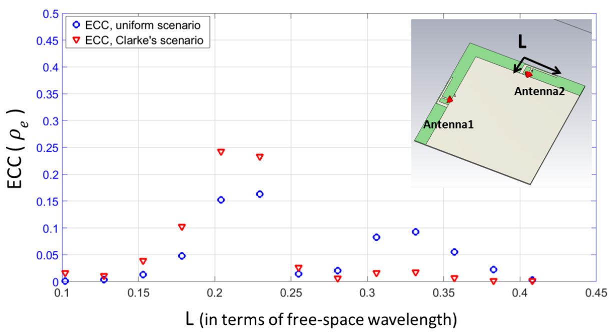


**Figure 2.** Plots of ECC for different wave distributions.

## 4 Similarity of expressions for the correlation coefficient

The pattern inner product method requires the antenna patterns. With a further major simplification of the propagation model - a uniform incoming wave distribution from all directions, and for (lossless) minimum scattering antennas, the patterns can be removed from the formation. The result is the normalized mutual resistance of the antennas [4]:

$$\rho_{O,ij} = \frac{R_{ij}}{\sqrt{|R_{ii}||R_{jj}|}}, \tag{6}$$

where $R$ denotes the real part of the antenna impedances. It can be negative which is the nature of a mutual impedance where the relation between the real and imaginary parts set the phase of the coupling. This mutual resistance, as a simple antenna parameter, offers a very convenient estimation for the correlation coefficient.

The loaded circuit voltage correlation ($\rho_L$) can be related to open circuit voltage correlations ($\rho_O$) using [1]:

$$\rho_{L,ij} = F\,\rho_{O,ij}\,F^H, \tag{7}$$

where $F = \mathscr{Z}_{\mathscr{L}}(\mathscr{Z}_{\mathscr{A}} + \mathscr{Z}_{\mathscr{L}})^{-1}$. $\mathscr{Z}_{\mathscr{A}}$ is the antenna impedance matrix, and $\mathscr{Z}_{\mathscr{L}}$ denotes the loading circuit impedance matrix and superscript $H$ is hermitian transpose. With $\rho_O$ related to the available power and $\rho_L$ to the received power at the antennas' terminals, $\rho_L$ will in general be unnormalized.

A physical measurement of the antenna resistance usually involves a classical conversion from the scattering parameters. For two identical antenna elements (i.e., $\mathscr{Z}_{A11} = \mathscr{Z}_{A22} = Z_{11}$ and considering reciprocal system, i.e., $\mathscr{Z}_{A12} = \mathscr{Z}_{A21} = Z_{12}$), terminated in $\mathscr{Z}_L$ which can be limited to a diagonal form (i.e., $\mathscr{Z}_{L11} = \mathscr{Z}_{L22} = Z_L$ and $\mathscr{Z}_{L12} = \mathscr{Z}_{L21} = 0$); then the self- and cross- terms of (7) simplify to:

$$\rho_{L,11} = \alpha - (\beta + \beta^*)\rho_{O,12}, \tag{8}$$
$$\rho_{L,12} = -2Re\{\beta\} + (\gamma + \kappa)\rho_{O,12}, \tag{9}$$

where,

$\alpha = \Delta \cdot (|Z_{12}|^2 + |Z_{11} + Z_L|^2)$, $\quad \beta = \Delta \cdot (Z_{12}^*(Z_{11} + Z_L))$,
$\gamma = \Delta \cdot |Z_{11} + Z_L|^2$, $\quad \kappa = \Delta \cdot |Z_{12}|^2$,

$$\Delta = \frac{|Z_L|^2}{((Z_{11}+Z_L)^2 - Z_{12}^2)((Z_{11}+Z_L)^2 - Z_{12}^2)^*},$$

so the loaded circuit correlation coefficient (normalized) between two lossless identical antennas can be expressed as:

$$\rho_{L,12} = \frac{\rho_{L,12}}{\rho_{L,11}} = \frac{-2Re\{(1+Z_{11n})Z_{12n}^*\} + (|1+Z_{11n}|^2 + |Z_{12n}|^2)\rho_{O,12}}{-2Re\{(1+Z_{11n})Z_{12n}^*\}\rho_{O,12} + (|1+Z_{11n}|^2 + |Z_{12n}|^2)}, \tag{10}$$

where $Z_{ijn} = \dfrac{Z_{ij}}{Z_L}$. Conversion from Z- to S- parameters [10], and some manipulations result in

$$\rho_{L,12} = -\frac{2Re\{S_{11}^* S_{12}\}}{1 - |S_{11}|^2 - |S_{12}|^2}. \tag{11}$$

It was independently shown in [6] and [7] that for any two antennas in a uniform propagation scenario, the pattern inner product expression of (4) can be expressed as:

$$\rho_{L,12} = -\frac{S_{11}^* S_{12} + S_{21}^* S_{22}}{\sqrt{1 - |S_{11}|^2 - |S_{21}|^2}\sqrt{1 - |S_{22}|^2 - |S_{12}|^2}}, \tag{12}$$

which for identical antennas ($S_{11} = S_{22}$), reduces to (11). So, under the several assumptions (uniform scenario, lossless elements etc.) all three formulations (4), (6) and (12) are equivalent. This equivalence does not appear to have been derived previously.

From (11) and (12), the designer's job appears easy: just match the antennas for a zero correlation! These expressions have also encouraged designers to simply plumb for low coupling, i.e., a low $S_{21}$, etc. But this is misleading, because a poor match will also create low coupling under this definition. This misinterpretation has lead many designers down the garden path. A few numerically solved antenna examples with lossless structures were undertaken to verify the above, summarized in the self-explanatory Figures 3-5. In Fig. 3 and Fig. 4, the antennas are identical. In Fig. 5 it can be seen that even for the case where antennas are non-identical ($S_{11} \neq S_{22}$), the different representations (from antenna patterns, and circuit parameters) for correlation coefficient still hold up well. Further examples and pitfalls will be discussed during the presentation.

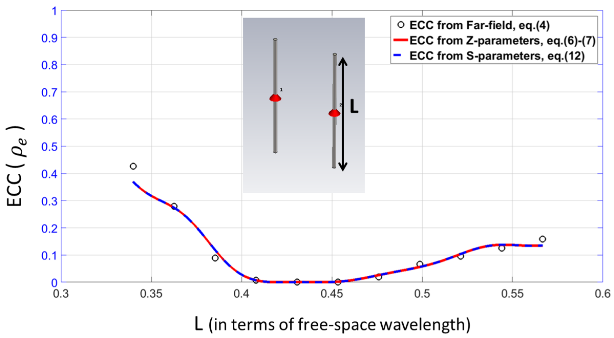


**Figure 3.** Example of two identical dipoles.

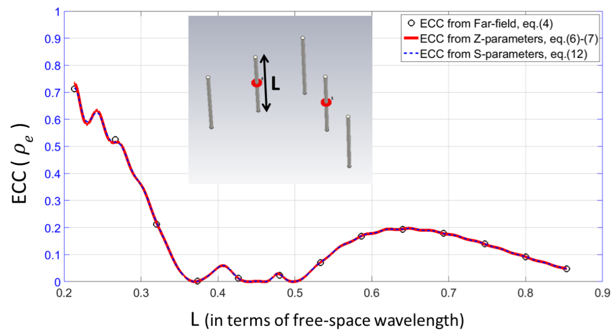


**Figure 4.** Example of two identical Yagis.

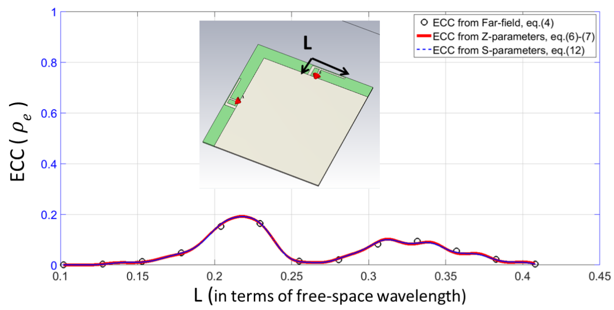


**Figure 5.** Example of two non-identical IFAs on PCB.

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