Bicontinuous DMRT Model Extracted from Multi-size QCA with Application to Terrestrial Snowpack

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Abstract

In terrestrial snow, the ice particles are irregular and densely packed together. At microwave frequencies (X-band to Ka-band), there are thousands of ice grains packed within one-wavelength cube. The interaction of the propagation wave and ice particles is coherent. The bicontinuous model is using computer-generated samples to construct the snow microstructures and numerically solve the Maxwell equations to include coherent wave and incoherent wave. There are two input parameters in the model, the scale parameter ζ and the size distribution parameter b. We related those two inputs with the ground measurement by comparing scattering coefficient of the bicontinuous model and multiple size spheres model. In the multi-size sphere model, the modified gamma distribution is used to describe size distribution of the different spheres. The scattering properties of the multi-size models are calculated by quasi-crystalline approximation (QCA). The incoherent wave is calculated with the distorted Born approximation in terms of the T-matrix and Fourier transform of the Percus-Yevick (PY) cross-pair distribution function. The scale parameter is related to the mean size of the ice particles. Therefore we can extract the size distribution parameter b of the bicontinuous model from the multi-size QCA by comparing the scattering coefficients.

1. Introduction

Recently the bicontinuous model [1-2] has drawn considerable attention due to its ability of representing the complex structure of the snow sample. In previous papers [1-2], we discussed the generation of the bicontinuous microstructure for snow sample, the statistical characterization of the bicontinuous media and the electromagnetic scattering properties. This capability of computer generation of snow samples and corresponding distinct scattering properties made the bicontinuous model useful to handle a variety of snow types, from fresh snow to depth hoar. The microstructures of bicontinuous media resemble that of real snow and can be characterized by correlation functions [3] as in random media theory and also by specific surface area [1,4]. The backscattering of the layer of snow was calculated by combining the bicontinuous model phase matrix and extinction with the dense media radiative transfer (DMRT) theory [2,5]. The distinct scattering properties using bicontinuous model area as follows. First, the cross-polarization phase matrix is non-zero, because the near-field interaction between the grains is included. Secondly, due to the clustering properties intrinsic in the microstructures, the frequency dependence of the extinction can be as low as 1.7 which is much lower than the classical Rayleigh theory of 4^{th} power. The weaker frequency dependence is consistent with the experiment data. Thirdly, the bicontinuous model has strong forward scattering with mean cosine $\bar{\mu}$ larger than zero.

1. Methodology

In this paper, we establish the connection between the input parameters for bicontinuous model and the multi-size sphere model. To generate the bicontinuous medium by computer, we divided the sample into cubic grids. For each realization, the bicontinuous medium is generated by the following algorithm,

$$S(\bar{r}) = \frac{1}{\sqrt{N}} \sum_{n=1}^{N} \cos(\bar{\zeta}_n \cdot \bar{r} + \varphi_n) \tag{1}$$

where $\bar{r}$ is the position variable for the center of each cubic grid of the bicontinuous medium. The random function $S(\bar{r})$

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at each grid $\bar{r}$ is generated stochastically by choosing a large number N. In Eq. (1), the wave vector $\bar{\zeta}_n$ and the phase φ_n are independent random variables. The phase φ_n is uniformly distributed between 0 and 2π, and the unit vector $\bar{\zeta}_n$ is uniformly distributed on a unit spherical surface. The magnitude of the wave vector ζ_n is distributed between 0 and $+\infty$ and has a probability density function $p(\zeta)$. A variety of distributions can be used. We choose to use the Gamma distribution:

$$p(\zeta) = \frac{1}{\Gamma(b+1)} \frac{(b+1)^{b+1}}{\langle\zeta\rangle} \left(\frac{\zeta}{\langle\zeta\rangle}\right)^{b} \exp\left[-(b+1)\frac{\zeta}{\langle\zeta\rangle}\right] \tag{2}$$

After all the random variables are generated, the random function $S(\bar{r})$ is calculated at each grid point $\bar{r}$ by Eq. (4). Then a cutting level α is introduced to determine the space filling in each cube of the bicontinuous medium.

$$\alpha = \mathrm{erf}^{-1}(1 - 2f_V) \tag{3}$$

Where $\mathrm{erf}^{-1}(\cdot)$ is the inverse error function. For every point $\bar{r}$

$$\Theta_\alpha(\bar{r}) = \begin{cases} 1, S(\bar{r}) > \alpha \\ 0, S(\bar{r}) < \alpha \end{cases} \tag{4}$$

$\Theta_\alpha(\bar{r}) = 1$ corresponds to filling with ice in the cube where $\bar{r}$ locates, while $\Theta_\alpha(\bar{r}) = 0$ corresponds to filling with air in the cube.

In the QCA multi-size sphere model [6], we used the Percus-Yevick sphere approximation to calculate cross-pair distribution functions for multiple sizes. The analytical method of Quasi-crystalline Approximation (QCA) was then used to calculate the scattering properties of extinction and phase matrix. An advantage of the multi-size model is that in the ground measurements, different snow types have snow grain size profile with a range of the grain sizes. Thus the size distribution can be extracted in principle from ground measurements.

As we described aboved, we establish the connection between the input parameters for bicontinuous model and the multi-size sphere model. The bicontinuous model is generated by superimposing a large number of stochastic waves, which are generated by the defined random function. There are three parameters in the random function. First one is the cutting level, which is directly related to the snow density. The other two parameters of the gamma distribution function in the wavenumbers of the bicontinuous stochastic waves are related to snow grain size: the scale parameter $\langle\zeta\rangle$ and the size distribution parameter "b". The scale parameter $\langle\zeta\rangle$ is related the SSA and the correlation length [7]. However, the correlation length and the SSA are weakly dependent on the size distribution parameter "b". In the past, snow sections were used to extract the "b" parameter [8]. In this paper, we make use of the multi-size sphere model. Using the multiple size QCA model, we calculate the mean cosine $\bar{\mu}$ of the co-polarization phase matrix, which indicates the strength of the forward scattering versus the backscattering. The mean cosine $\bar{\mu}$ describes the angular distribution of the phase matrix. Next we generate a list of bicontinuous snow sample by using various size distribution parameters "b" By solving the Maxwell equation numerically using DDA [1-2], we calculate the mean cosine factor. By comparing the mean cosine factor for the two models, we set up the connection between the size distribution parameter "b" of the bicontinuous model and the grain size distribution from the multi-size QCA model. We also check agreement of the auto correlation functions with that of the bicontinuous medium. Numerical results of scattering properties of the two models are also illustrated and compared.

There are a few distinct scattering properties using bicontinuous model. First, the cross-polarization phase matrix is non-zero, because the near-field interaction between the grains is included. Secondly, due to the flexibility of modeling the various snow structure, the frequency depending of the extinction can be as low as 1.7, which is consistent with the experiment data.

2. References

[1] KH Ding, X Xu and L Tsang, “Electromagnetic Scattering by Bicontinuous Random Microstructures With Discrete Permittivities,” *Geoscience and Remote Sensing, IEEE Transactions on* , vol.48, no.8, pp.3139-3151, Aug. 2010.

[2] X. Xu, L. Tsang, and S. Yueh,, “Electromagnetic Models of Co/Cross-polarization of Bicontinuous/DMRT in Radar Remote Sensing of Terrestrial Snow at X- and Ku-band for CoReH2O and SCLP Applications,” *Selected Topics in Applied Earth Observations and Remote Sensing, IEEE Journal of* , vol.5, no.3, pp.1024-1032, June 2012

[3] F. Vallese, & J. A. Kong, “Correlation function studies for snow and ice”. *Journal of Applied Physics*, 52(8), 4921-4925. 1981.

[4] M. Kerbrat, B. Pinzer, T. Huthwelker, H. W. Gäggeler, M. Ammann, & M. Schneebeli, “Measuring the specific surface area of snow with X-ray tomography and gas adsorption: comparison and implications for surface smoothness”. *Atmospheric Chemistry and Physics*, *8*(5), 1261-1275. 2008

[5] L. Tsang; KH Ding; S. Huang and X. Xu, "Electromagnetic Computation in Scattering of Electromagnetic Waves by Random Rough Surface and Dense Media in Microwave Remote Sensing of Land Surfaces," *Proceedings of the IEEE ,* vol.101, no.2, pp.255,279, Feb. 2013.

[6] K.H. Ding, C.E. Mandt, L. Tsang, J.A. Kong, "Monte Carlo Simulations of Pair Distribution Functions of Dense Discrete Random Media With Multiple Sizes of Particles" , *Journal of Electromagnetic Waves and Applications*, Vol. 6, Iss. 7, 1992

[7] W. Chang etal “Dense Media Radiative Transfer Applied To SnowSAR and SnowScat” submitted to IEEE Journal of Selected Topics in Applied Earth Observations and Remote Sensing”, 2014.

[8] C. Xiong; J. Shi; M. Brogioni, and L. Tsang, "Microwave snow backscattering modeling based on two-dimensional snow section image and equivalent grain size,"Geoscience and Remote Sensing Symposium (IGARSS), 2012 IEEE International , vol., no., pp.150,153, 22-27 July 2012 doi: 10.1109/IGARSS.2012.6351615