Choosing Frequency Dependent Moment-Method Basis Functions for Antenna Problems

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Abstract

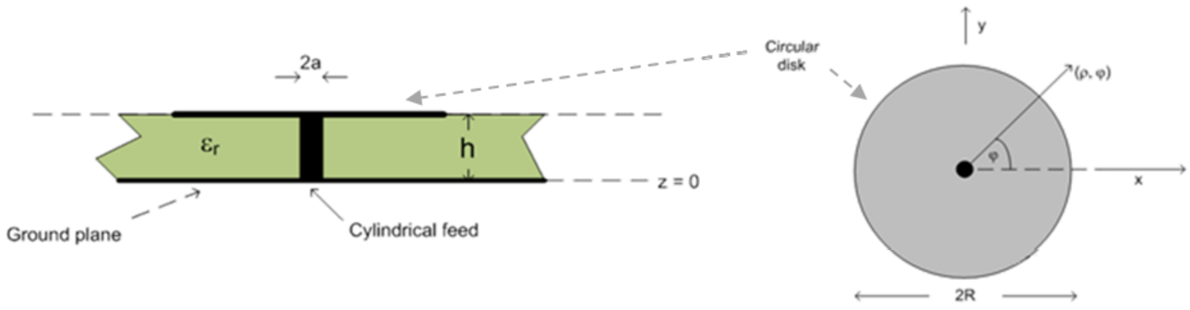
A method for selecting frequency dependent moment-method basis functions is proposed. These entire-domain, continuous frequency-dependent basis functions describe field components outside the antenna, and correctly obey the edge condition near the wedge of the antenna, and the behaviour of the fields at infinity. The centre-fed microstrip disk antenna serves as a toy model for presenting the method. Moreover, the (un-tabulated) Fourier transforms of the proposed basis functions are obtained in terms of fast converging sums. The agreement between the numerically and analytically calculated transforms is very good.

1. Introduction

Moment Method (MM) is frequently used in solving electromagnetic problems [1]. Different basis and test functions were checked during the last years in order to get more accurate and fast converging solutions. However, there is no clear guidance for choosing appropriate basis functions for a specific electromagnetic problem and the most popular way for choosing test functions is the Galerkin method. Several papers [2-3] addressed the importance of including the edge behaviour of the fields in the basis functions in order to get more accurate and fast converging solutions. Fuzzy basis functions which can approximate any function to an arbitrary degree of accuracy are applies in [4], while in this method the user's insight can be taken into account easily and systematically resulting in a better approximation. Recently, large complex structures are solved by the MM by dividing the structure to sub-domains, where few basis functions are needed for each sub-domain. Use of these basis functions leads to a significant reduction in the number of unknowns, and results in a substantial size reduction of the MM matrix [5-6]. Several more papers [7-10], dealt with electrostatic cases, addressed the importance of having the basis functions to exactly obey the preliminary known physical behaviour of the unknown function, in order to obtain a more accurate and fast converging solution, as compared to the case when the basis functions do not obey the preliminary known physical behaviour. By physical behaviour we mean the symmetry and accurate behaviour of the unknown function at the edges of the definition domain. The solutions proposed by these papers had an excellent agreement with the analytical solutions.
The common assumption was that MM is only efficient for solving problems of electrically small structures [11-13], however recently few papers [14] introduced methods for increasing the efficiency of MM in high frequencies by incorporating frequency-dependent expressions in the basis functions. In [14], the authors used spheroidal wave functions (SWF) as basis functions, and it was shown that under proper conditions, these SWF's evolve to provide an approximation to the current variation with frequency. In [15-16], the authors created frequency-varying basis functions by incorporating the phase information into them and consequently managed to reduce the computation time and memory requirements of the MM for high frequencies. The selected entire-domain, continuous frequency-dependent basis functions, exactly contained the physical behaviour of the fields near the wedge and at infinity, therefore produced a more accurate solution for the given centre-fed disk antenna structure. Also, It was shown that only 3 frequency-dependent field basis functions were needed in order to describe the first 4 resonances, while 3 frequency-independent surface current basis functions, were only enough to describe the first resonance. For the next resonances, one had to add more surface current basis functions, for example, for the 2nd resonance one needed 4 frequency-independent surface current basis functions, and so on. However, the Fourier-Bessel transforms of the frequency-dependent field basis functions were numerically calculated, requiring a long computation time.
In this paper, we explain how the basis functions for the disk antenna case were selected, by presenting the expansion of the fields near the wedge of the disk and at infinity. In addition, we efficiently calculate the Fourier-Bessel transforms of the basis functions in order to shorten the computation time.

2. Choosing the Basis Functions

A flat, infinitely thin, ideal conducting circular disk antenna of radius R, fed by a vertical cylindrical current source of radius a (a<<R) is shown in figure 1; Substrate is air.



(a) (b)

Fig. 1: The center-fed microstrip disk antenna. (a) Side view. (b) Upper view.

Our goal in this chapter is to find the behaviour of E_ρ on the plane $z = h$ outside the disk, for ρ near R and at infinity, in order to choose proper basis functions in the range $R < \rho < \infty$.

The behavior of E_ρ at $\rho \to R$ is known from the electrostatic case and is given by:

$$E_\rho(\rho) \approx \alpha_1(\rho - R)^{-1/2} + \alpha_2(\rho - R)^{1/2} + \alpha_3(\rho - R)^{3/2} + \dots \quad , \quad \rho \to R \tag{1}$$

where α_n are constants.

In order to find the behavior of E_ρ at infinity we use the intermediate zone expansion for the vector potential:

$$\mathbf{A}(\mathbf{x}) = \mu_0 ik \sum_{l,m} h_l^{(2)}(kr) Y_{lm}(\theta,\varphi) \int \mathbf{J}(\mathbf{x}') j_l(kr') Y_{lm}^*(\theta',\varphi') d^3x' \tag{2}$$

where $h_l^{(2)}(z)$ is the spherical Hankel function of the second kind, $j_l(z)$ is the spherical Bessel function of the first kind and $Y_{lm}(\theta,\varphi)$ are the Laplace spherical harmonics which are given by:

$$Y_{lm}(\theta,\varphi) = \sqrt{\frac{2l+1}{4\pi} \cdot \frac{(l-m)!}{(l+m)!}} P_l^m(\cos\theta) e^{im\varphi} \tag{3}$$

where $P_l^m(z)$ is the associated Legendre polynomial.

The vector potential becomes:

$$A_\rho(\rho, z) = \sum_l a_l h_l^{(2)}\left(k\sqrt{\rho^2 + z^2}\right) P_l\left(\frac{z}{\sqrt{\rho^2 + z^2}}\right) \quad , \quad l = 1,3,5,\dots \tag{4}$$

Since the vector potential is in the radial direction and azimuthal independent, we get:

$$B_\varphi(\rho,z)=\frac{\partial A_\rho}{\partial z}=\sum_l a_l \frac{\partial}{\partial z}\left[h_l^{(2)}\left(k\sqrt{\rho^2+z^2}\right)P_l\left(\frac{z}{\sqrt{\rho^2+z^2}}\right)\right] \quad , \quad l=1,3,5,... \tag{5}$$

Next, we calculate the electric field $\mathbf{E}$ using:

$$\nabla\times\mathbf{B}=\mu_0\varepsilon_0\frac{\partial\mathbf{E}}{\partial t} \;\Rightarrow\; \mathbf{E}=-\frac{i}{kc}\nabla\times\mathbf{B} \tag{6}$$

The magnetic flux density is in the azimuthal direction (B_φ), thus the electric field has two components (E_ρ & E_z). Since we're only interested in the radial direction, we have:

$$E_\rho(\rho,z)=\frac{i}{kc}\sum_l a_l \frac{\partial^2}{\partial z^2}\left[h_l^{(2)}\left(k\sqrt{\rho^2+z^2}\right)P_l\left(\frac{z}{\sqrt{\rho^2+z^2}}\right)\right] \quad , \quad l=1,3,5,... \tag{7}$$

It can be shown [22] that the behavior of the electric field at $z=h$ and $\rho\to\infty$ is given by:

$$E_\rho(\rho)\approx\exp\left(-ik\sqrt{\rho^2+h^2}\right)\cdot\left(\frac{\delta_1(l)}{\rho^3}+\frac{\delta_2(l)}{\rho^4}+\frac{\delta_3(l)}{\rho^5}+...\right) \;,\; l=1,3,5,... \;,\; \rho\to\infty \tag{8}$$

where $\delta_n(l)$ are constants.

We can now describe the electric field component E_ρ as a set of basis functions:

$$\begin{aligned} f_n(\rho)&=\frac{\exp(-ik_0\rho)}{\rho^n\sqrt{\rho^2-R^2}} \quad , \quad n=2,3,4,... \\ g_n(\rho)&=\frac{\exp(-ik_0\rho)\sqrt{\rho^2-R^2}}{\rho^n} \quad , \quad n=4,5,... \end{aligned} \tag{9}$$

which exactly contain the edge condition and the behavior of the field at infinity as a function of frequency.

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