Investigation into the comparative amplitudes of phase and amplitude scintillation indices at low/equatorial and high latitudes

Hal J. Strangeways

School of Electrical and Electronic Engineering, University of Newcastle, Newcastle upon Tyne, NE1 7RU, UK.

email: Hal.Strangeways@ncl.ac.uk

Abstract

It is well known that it is usual for ground based receivers at high latitudes to see significantly larger phase than amplitude scintillation for transionospherically VHF/UHF signals from satellites whereas the opposite of larger amplitude than phase scintillation is generally seen at low or near equatorial latitudes. There could be a number of reasons for this including geometrical, lower cut-off frequency, detrending and Fresnel filtering effects or a combination of two or more of these or other effects such as different properties and/or heights of the irregularities in the two regions. In order to determine the relative importance of all these effects, in this paper the ratio of the S4 index to the phase scintillation index will be determined to see which factors or parameters have the greatest effect in altering it, looking in particular for an effect that will minimise this ratio at low latitudes and maximise it at high latitudes and including variation with elevation, dip and irregularity properties.

1. Introduction

Scintillation is a problem for global satellite systems such as GPS or Galileo as it can degrade the positioning, cause cycle slips, or result. in the worst case scenario of strong scintillation, in the loss of lock of the receiver PLL. This problem is important for low latitudes (particularly in the equatorial anomaly regions) and at high (particularly polar and auroral) latitudes. Understanding the physical mechanisms of scintillation better and their effect on GNSS receivers should lead to improved mitigation strategies which to be optimum may well vary between low and high latitude applications. It is well known that ground based receivers at high latitudes see significantly larger phase than amplitude scintillation for transionospherically propagated VHF/UHF signals whereas the opposite of larger amplitude than phase scintillation is generally seen at low or near equatorial latitudes. It is desirable to be able to understand this but there are a number of possible reasons. Firstly, scintillation indices are larger for satellite to receiver path aligned close to the geomagnetic field direction as the irregularities causing scintillation are commonly aligned in this direction. This is a then a geometric effect [1]. This path alignment will occur for paths in or near the magnetic meridian and close in elevation to the dip which then have high elevations at high latitudes and low elevations at low latitudes. At the average latitude of the equatorial anomaly regions, where scintillation generally maximises, the dip angle would be about 20º, an elevation for which the scintillation would also be significantly increased due to the longer path (more slant) path through the ionosphere whereas, in the high latitude case, the length of the slant path through the irregularities would be shorter. Thus it can be seen that the worst geometrical situation is at low rather than high latitudes although this would be the case for both phase and amplitude scintillation. Secondly it has been seen how much the low cut-off employed in the receiver or by the detrending or determined by the length of the data set can alter the value of the phase scintillation index [2].The effect of this could be rather different between low and high latitudes due to varying Fresnel frequencies. The amplitude scintillation will be significantly affected through generally be considerably higher at high than lower latitudes. Another factor could also be different properties of the irregularities or in the mesoscale structures in which they are embedded (e.g. plasma bubbles or polar patches) between the two regions. Then they may be some combination of these effects. In order to determine the comparative importance of all these effects, in this paper the ratio of the S4 index to the phase scintillation index will be determined for a variety of different scenarios to see which factors or parameters have the greatest effect in altering this ratio, looking in particular for an effect that will minimise this ratio at low latitudes and maximise it at high latitudes. Two different methods will be used to determine the scintillation indices σ_φ and S4; the phase screen method of Rino [3] and the Hybrid method of Gherm et al [4]. Employing two different methods will ensure that the calculation are not too scintillation determination method specific. The method of Rino [3] finds the fluctuations of both the phase and amplitude of the field below a phase screen which represents the diffracting effect of the entire ionosphere and is normally place near or a little above the height of the F region and commonly at 350 km altitude at least when this is not known. The Hybrid Scintillation Propagation Model (HPSM) method [4] uses the complex phase method in combination with the random screen technique. The parameters of the random screen (situated below the ionosphere) are determined as the result of a rigorous solution to the problem of propagation inside the ionosphere using the complex phase method. The random two-dimensional spatial spectrum at the screen is then transferred down to the Earth's surface employing the rigorous relationships of the random screen theory.

2. Theoretical formulations employed.

For the single phase screen method, the theory given by Rino [3] was employed. For determining S4 his equation (31) was used and the integral in his equation (32) was integrated using the hypergeometric function as given by his equation (34). For determining the phase scintillation (σ_ϕ), his equation (19) was used for the no bandwidth limit and, for including finite cut-off frequencies, the formula (1) for the phase psd at frequency f was integrated over the respective range of fading frequencies.

$$S_\phi = \frac{T}{(f_0^2 + f^2)^{p/2}} \qquad (1)$$

T (the spectral strength of the phase psd at 1 Hz) in equation (1) was obtained from equation (18) of [3] and twice the resultant integral of (1) was then equated to σ_φ^2. Here $f_o = 2\pi/L_o$ where L_o is the outer scale of the turbulence formed by the irregularities. Whereas in WBMOD [5], the product CkL is determined using empirical data, here it can be fully calculated using the formulae in [3] as all the properties of the ionosphere and electron density profile are modelled. In order to determine the scintillation indices for this method with respect to the same irregularity parameter σ_N^2 (normalised variance of their electron density) as the HPSM method, the product CsL (linked to CkL by a multiplying factor) was obtained as:-

$$CsL = 8\pi^{3/2} \frac{\Gamma(\nu+0.5) q_o^{(2\nu-2)} \sigma_N^2}{\Gamma(\nu-1)\left(q_o^2+q^2\right)^{\nu+0.5}} \int_{h_b}^{ht} N(h)^2 dh \qquad (2)$$

where ν=0.5p and N(h) is the electron density vertical profile which exists over the altitude range from h_b to h_t. q is given by equation (8) of [3] and q_o is the outer scale cut-off wave number. The formulae employed to obtain phase and amplitude scintillation index variations for the Hybrid method are those given by [4].

2. Determination of the scintillation index ratios

The ratio of the amplitude (S4) to phase scintillation indices for the 2 methods was determined for a number of different parameters. When fixed, parameters took the following values: variance of the irregularity electron density: σ_N^2 : 0.001 (which is assumed here to be height independent); the irregularities cross-field aspect ratio: 1 and longitudinal aspect ratio: (the length of the irregularities compared with their diameter) 10; their outer scale (Lo): 10 km, their velocity East-West: 500m/s; the power law of the irregularity anisotropic spatial spectrum: 3.7 (then the slope of the received phase psd on log log axes for weak/moderate scintillation is 2.7); transmission frequency: 1575 MHz; azimuth of path :180 °; path elevation: 45° and geomagnetic field dip: 60°. The vertical electron density profile used for these calculations had a maximum of 4.63×10^{11} el m^{-3} at an altitude of 285 km and a TEC of 14.78. The signal was considered to be transmitted from an altitude of 20,000 km as from a GPS satellite.

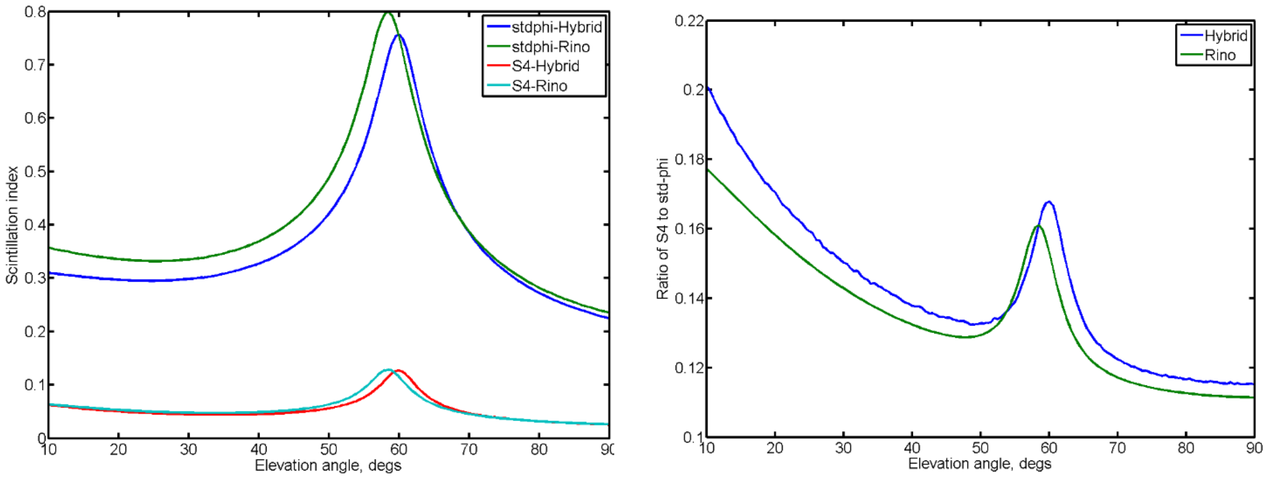

Figure 1(a) (left) shows the scintillation indices for the two methods and 1(b) (right) the corresponding scintillation ratio of S4 to σ_φ plotted against elevation angle in the geomagnetic meridian.

It is clear that both determination methods give fairly similar results and that the scintillation ratio varies significantly between high and low elevation angles (by a factor of about 0.6) and also increases by about 20% when the path elevation and dip angle are aligned. This would mean that where field aligned paths occur, the S4 index would be larger with respect to the phase index than for non-aligned paths. This might explain why the strongest scintillation occurs at low latitudes as here the scintillation index can be increased both by the low elevation of the path and a propagation direction aligned with the irregularities since these can both occur together whereas at high latitudes, only higer elevation paths could be field-aligned. The scintillation is is also be increased when the aspect ratio of the irregularities is larger. For example, increasing the aspect ratio from 10 to 20 increases the scintillation indices by a factor of about 1.5. This

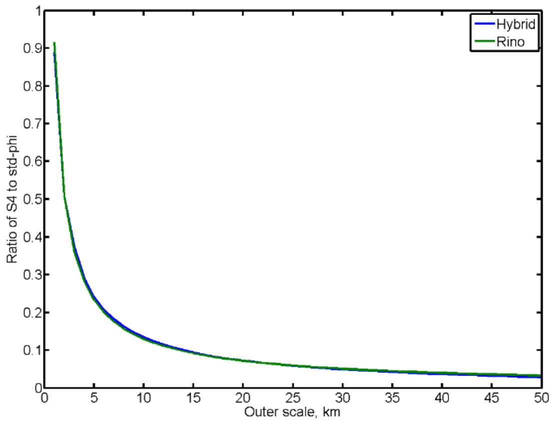

Figure 2 Scintillation index ratio versus outer scale

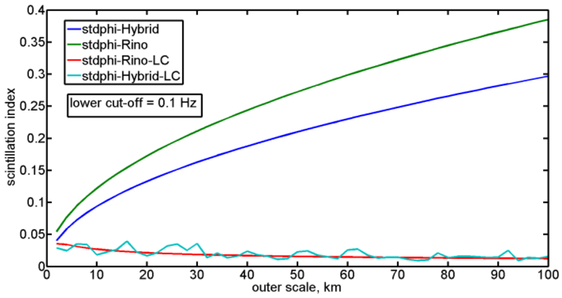

Figure 3 Variation of S4 and σ_ϕ with outer scale for no cut-off and a lower cut-off frequency of 0.1 Hz.

then could also introduce differences in scintillation between the two regions if the irregularity aspect ratios in one are rather larger than in the other. The variation of the scintillation index with outer scale is shown in Figure 2 for 1 to 50 km. In this figure, it is assumed that the low frequency cut-off of the phase psd is determined by the ratio v_{eff}/Lo where v_{eff} is the vector sum of the satellite and irregularity velocities perpendicular to the path for isotropic irregularities but for anisotropic irregularities is reduced by the aspect ratio according to the relative direction of the path to geomagnetic field direction for irregularities elongated in this direction. The ratio of the scintillation indices in the figure would be increased for large outer scales when the lower cut-of frequency corresponding to the detrending, the receiver or the length of the data set is larger than v_{eff}/Lo. It is clear that there is a very appreciable dependence on the ratio with outer scale varying from about 0.9 at 1 km to about 0.03 at 50 km. This is because the increasing outer scale continually decreases the lower limit of the fading frequency range for which the phase scintillation is seen at the receiver. It does not have a corresponding effect on S4 because the lower frequency limit for this is generally already determined by the Fresnel frequency due to Fresnel filtering. However the lowest frequency contributions to the phase scintillation index may well also be lost by the lower cut-of frequency introduced by the receiver, the detrending or the length of the data set. This is illustrated by figure 3 where a low cut-off frequency of 0.1 Hz is modelled and the variation of the scintillation indices with outer scale for both the Hybrid method and the Rino phase screen method is determined together with the corresponding determinations with no cut-off for comparison. In the case of the HPSM method, the results for the lower cut-off were obtained by measuring the area under the amplitude psd between 0.1 Hz and the upper cut-off frequency which explains the random variations on the curve produced by errors in measuring the area. It is clear that the phase scintillation index is significantly reduced by a lower cut-off and actually slowly decreases with increasing outer scale rather than increasing with it, fairly similar in fact to the S4 variation with outer scale. S4 would be much less affected because Fresnel filtering removes the lower frequencies below the Fresnel frequency. This frequency would typically be less than 0.1 Hz at low latitudes but could reach as high as 5 Hz for high latitudes due to the typically much faster irregularity drift velocity there.

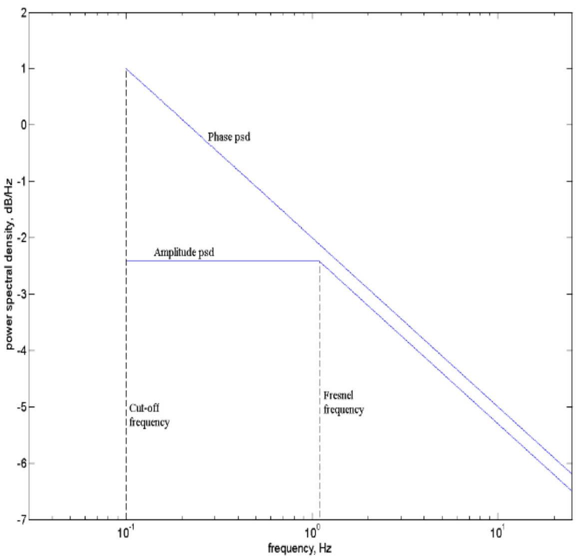

Figure 4 Idealised PSDs for amplitude (σ_χ) and phase (σ_ϕ) scintillation indices.

3 Consideration of phase and amplitude psds

The difference between the two scintillation indices can best be understood with reference to the idealised psds for phase and amplitude shown in figure 4 (as justified in [6]). The phase spectrum is modelled to have a single slope spectrum between its upper and lower limits on log-log axes. The amplitude spectrum is modelled similarly to the phase spectrum above the Fresnel frequency (the lines showing this part of the spectrum would then be coincident but are shown a little apart for clarity in figure (5). Such a separation could, in fact occur for strong scintillation conditions. Below the Fresnel frequency, the amplitude psd is taken to be constant. σ_ϕ^2 is then equal to twice the area under the psd of the phase variation and σ_χ^2 (the amplitude scintillation index/S2) is equal to twice the area under the psd of the amplitude variation [6]. S4 would be approximately twice the amplitude psd, this depending a little on the amplitude distribution since it is strictly an intensity index. Then we can see that the difference between these two scintillation indices (σ_ϕ and σ_χ) corresponds to the triangular area bounded by the psds and the lower cut-off frequency (although it should actually be measured when drawn on linear axes). It is clear that this difference will be reduced as the cut-off frequency is increased to approach the Fresnel frequency or the Fresnel frequency is reduced approaching the lower cut-off frequency. The Fresnel frequency is given by $f_F = \frac{v_{rel}}{\sqrt{2\lambda z}}$ where v_{rel} is the relative drift velocity and z is the distance from the equivalent diffraction screen to the receiver. The relative drift velocity is the vector sum of the irregularity drift velocity and the equivalent satellite velocity at the height of the irregularities where the latter is generally rather smaller than the former particularly for high latitude conditions. Thus we can see the conditions that S4 will be larger than σ_ϕ will be those when the Fresnel frequency is low which corresponds to the lower irregularity drift velocities seen at low latitudes. It is also clear from the equation that the Fresnel frequency will be reduced by a larger value of z which corresponds to a higher altitude for the irregularities (and thus also a lower satellite elevation). It is thought that most of the scattering of L band signals at low latitudes occurs within a thin layer surrounding the peak of the F layer in the equatorial region [7]. Depending on the solar activity, daytime and season, the peak density height may range from 350 to 500 km at equatorial latitudes and from 250 to 350 km at mid-latitudes [8]. The height of irregularities at auroral latitudes may well be lower depending on the auroral phenomenon. For example, in [9], scintillation is shown to be well correlated spatially with aurora seen at both 120 and 200 km altitude. Thus we can see that both the lower height and the higher velocity of the irregularity producing scintillations are likely at higher latitudes and that both will increase the Fresnel frequency and thus decrease the S4 index compared with low latitudes.

4 Conclusions

Three factors have been found which will tend to increase the ratio of S4 to σ_ϕ; lower values of the irregularity drift velocity and higher values of the altitude of the irregularities which both increase the Fresnel frequency and also lower values of the outer scale of the turbulence. The former conditions serve to increase S4 at low latitudes and correlate with lower latitude conditions while the latter condition reduces σ_ϕ. For this to serve as one possible explanation for lower σ_ϕ at low/equatorial latitudes it would necessarily need to be shown that the scale of the turbulence is typically much smaller in this region than at auroral or polar latitude. However, it should be noted that the important ratio of v_{eff}/Lo will be smaller at low latitudes even for similar values of the outer scale because v_{eff} depends on the irregularity drift velocity which will generally be considerable smaller at low/equatorial than high latitudes. Further investigation is also required of the possibility of refractive scattering which occurs off irregularity structures with larger scales than the Fresnel scale so that the long wave cut-off in the spectrum of amplitude fluctuations then occurs, not at the Fresnel scale but at a larger scale that increases as the percentage fluctuation of ionization density increases [10].

5 References

1. B. Forte (2007), On the relationship between the geometrical control of scintillation indices and the data detrending problems observed at high latitudes, Annals of Geophysics, vol. 50(6), pp.699-706

2. T.L. Beach (2006), Perils of the GPS phase scintillation index (σ_ϕ). Radio Sci., 41(5), 2006 doi: 10.1029/2005RS003356

3. C. L. Rino,(1979) "A Power Law Phase Screen Model for Ionospheric Scintillation: 1. Weak Scatter," *Radio Sci.*, **14**, 1979, pp. 1135–1145

4. V.E Gherm, N. N. Zernov and H. J. Strangeways (2005), Propagation model for transionospheric fluctuational paths of propagation: Simulator of the transionospheric channel, *Radio Sci*., 40, RS1003, 2005, doi:10.1029/2004RS003097

5. J.A. Secan, R. M. Bussey, E. J. Fremouw and S. Basu, (1997) High-latitude upgrade to the Wideband ionospheric scintillation model, Radio Sci, 32, pp. 1567-1574, doi:10.1029/97RS00453

6. H.J. Strangeways, (2009), Determining scintillation effects on GPS receivers, *Radio Sci*., 44, RS0A36, doi:10.1029/2008RS004076, 2009

7. S. Basu, S., Su. Basu, J.P. McClure, W.B. Hanson and H.E. Whitney,(1983) High resolution topside in situ data of electron densities and VHF/GHz scintillations in the equatorial region, J. Geophys. Res., 88, 403–415, doi:10.1029/JA088iA01p00403.

8. M. M. Hoque and N. Jakowski (2012) A new global model for the ionospheric F2 peak height for radio wave propagation, Ann. Geophys., 30, 797–809, doi:10.5194/angeo-30-797-2012

9. J. Kinrade, C. N. Mitchell, N.D. Smith, Y. Ebihara, A.T. Weatherwax and G.S. Bust (2003), GPS phase scintillation associated with optical auroral emissions: First statistical results from the geographic South Pole, J. geophys. Res.,118(5), pp.1825–2764, doi: 10.1002/jgra.502

10. C.M. Crain, H.G. Booker and J.A. Ferguson (1979), Use of refractive scattering to explain SHF scintillation, Radio Sci., 14(1), pp. 125-134