Efficient Characterization of Stochastic Electromagnetic Fields using Eigenvalue Decomposition and Principal Component Analysis Methods

Tatjana Asenov[1,2], ***Johannes A. Russer***[1], and ***Peter Russer***[1]

[1] Institute for Nanoelectronics, Technische Universität München, Germany, e-mail: `russer@tum.de`
[2] Faculty of Electronic Engineering, University of Niš, Serbia, e-mail: `tatjana.asenov@elfak.ni.ac.rs`

Abstract

Stochastic electromagnetic fields can be described by the correlation function of the field amplitudes in all pairs of space points. We show that the description of stochastic electromagnetic fields by correlation matrices can be simplified using the principal component analysis (PCA) for eigenvalue decomposition. In this paper, the principal component analysis and the eigenvalue decomposition approach are applied for decomposing and reducing the correlation matrix describing the correlations of the sampled field amplitudes. Subsequently conventional eigenvalue decomposition and the PCA approaches are compared.

1 Introduction

A stochastic electromagnetic field is the statistic counterpart to a deterministic field. Its time evolution is random and we cannot assign an amplitude or phase to stochastic fields [1, 2]. Stochastic electromagnetic fields occur as noise or electromagnetic interference, i.e. as disturbing quantities or information carrying signals as for example in radiometry. Under the assumption that a stochastic electromagnetic field is characterized by a Gaussian process it can be described by the auto- and cross correlation spectra of the electric and magnetic field values at pairs of points in space [2]. In [3] we have shown that the description of stochastic electromagnetic fields by correlation matrices can be simplified considerably by eigenvalue decomposition of the correlation matrix representing the field correlations in the sampling points. Since the number of statistically independent excitation sources in many cases is far below the number of sampling points the dimension of the correlation matrix describing the filed can be reduced considerably. The computation of the stochastic field can be performed by computing the field distributions for deterministic unit vector excitation and then superimposing the field dyadics representing the unit vector excitations weighted by the eigenvalues of the correlation matrix of the scanned electric field samples describing the excitation. In this work we compare a standard method of eigenvalue decomposition as implemented in MATLAB with the Principal Component Analysis (PCA) which is a computationally efficient method for reducing the data set of correlations of measured field samples [4].

2 The Principal Component Analysis

Principal Component Analysis (PCA) is a statistical procedure allowing for reduction of the dimensionality of a multivariate data set without loss of relevant information as well as for the identification of the principal directions in which the data are varying. The PCA technique is described in detail in [4]. In general, the reduction is achieved by linear transformation to a new hierarchical set of uncorrelated data, called the principal components (PC). Each PC represents a linear combination of data at specific coordinates (variables) at different values of a chosen parameter (called observations) [5]. Computation of PCs is based on calculating the eigenvectors and eigenvalues (λ) of the data covariance matrix. The determination of eigenvectors is an iterative process. In the first step, the eigenvector with the largest eigenvalue is calculated. The subsequent eigenvector is the orthogonal vector with the second largest eigenvalue etc. In the following step, the eigenvectors are ranked by the eigenvalue, highest to lowest. Therefore, the PCs in order of significance are obtained.

Let $\mathbf{x}$ be the input vector of random variables, ψ the covariance matrix and z the vector whose kth

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element is the kth PC for $k = 1, 2, .., p$. Then $\mathbf{z}$ is given by

$$\mathbf{z} = \mathbf{A}^{'}\mathbf{x}, \tag{1}$$

where $\mathbf{A}$ is the orthogonal matrix, whose kth column α_k is the kth eigenvector of ψ. The eigenvectors can be obtained by solving the characteristic equation or from the singular value decomposition of ψ. This yields

$$\psi = \mathbf{A}\mathbf{\Lambda}\mathbf{A}^{'}, \tag{2}$$

where $\mathbf{\Lambda}$ is the diagonal matrix, whose kth element λ_k is the kth eigenvalue of ψ, and

$$\lambda_k = var(\alpha_k^{'}\mathbf{x}) = var(\mathbf{z_k}). \tag{3}$$

Several criteria have been proposed for determining a suitable number of PCs to account for most of the variation in a data set [4]. According to a (cumulative) percentage of total variation rule, the selected PCs should account for more than 80% or even 90% of the total variation. The most frequently used visual technique for determining the appropriate number of retainable PCs is a scree graph. The scree graph plots the eigenvalues of the covariance matrix in decreasing order of magnitude illustrating the fraction of total variance in the data represented by each PC and the cumulative proportion of the variance accounted for the significant PCs. Cattell's rule discards all PCs beyond the first point on the scree graph of high-to-low ranked eigenvalues of the covariance matrix for which the graph forms a straight line, not necessarily horizontal. The number of the last PC retained is defined by the first point on the more or less straight line [4].

3 Numerical Example

In the following we evaluated two approaches for reducing the computation of stochastic electromagnetic fields. The eigenvalue decomposition of the measured electromagnetic noise is given in detail in [3]. In this paper results obtained by performing eigenvalue decomposition [6] and PCA [7] on the correlation matrix are compared. In both cases we have used the respective MATLAB routines. We investigated the case of stochastic noise sources at different positions. The stochastic electromagnetic field has been characterized by sampling the field values in pairs of five sampling points. A single source is located at one position or two sources are located at two distinct positions in space. The experimental setup is sketched in Fig. 1.

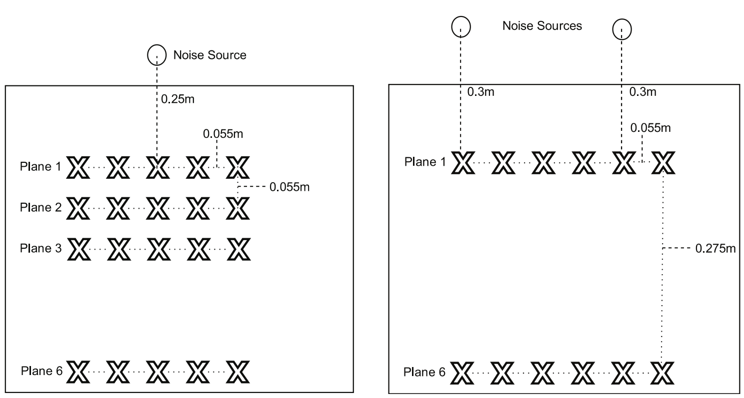
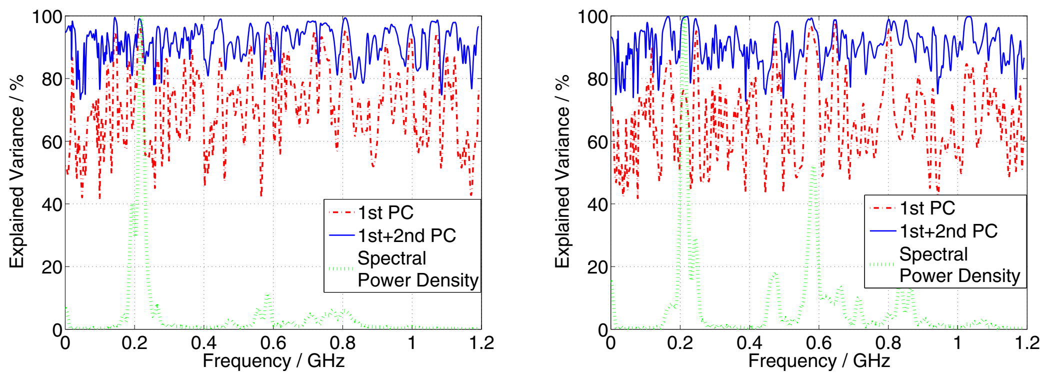


Figure 1: Schematic of measurement setup showing location of noise sources and spatial smple points.

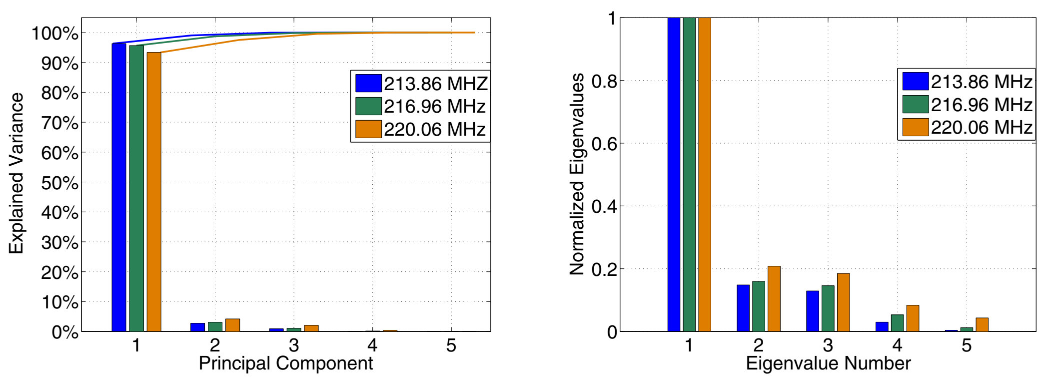
To keep the example simple we assume a linearly polarized field and therefore scalar tangential field values. The field has been scanned using small electrical dipoles. As test object an arrangement of two micro-motors has been used. For the scanning of the electric near field two dipoles were positioned at sampling points spaced at 5.5 cm distance along a line. The signals were recorded by a two-channel digital sampling oscilloscope with a sample time length of 1 ns. From this, the autocorrelation functions for every sampling point and the cross correlation functions for every pair of sampling points have been evaluated and from this, by numerical Fourier transformation the correlation matrix has been computed. Subsequently, PCA and eigenvalue decomposition have been applied to the correlation matrix. Plots of spectral power density obtained in the observation plane and explained variance of the PCs over a frequency range for a single and two noise sources are shown in Fig. 2.



(a) (b)

Figure 2: Spectral power density and explained variance over a frequency range for (a) a single noise source and (b) two noise sources.

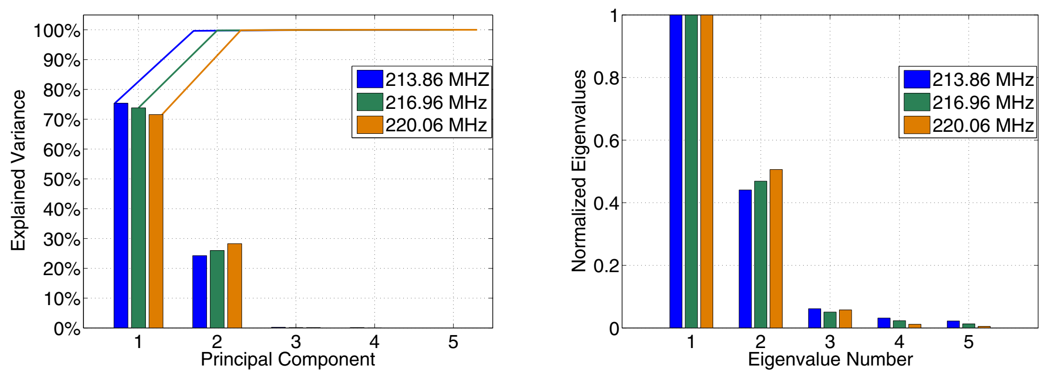
First, the case of a single noise source has been considered. The scree graph for the correlation matrix in a frequency range with the highest spectral power density is given in Fig. 3(a). In this case we can see that there is one dominant PC at three frequencies when using the (cumulative) percentage of total variation rule.



(a) (b)

Figure 3: (a) Scree graph for the correlation matrix for a single noise source case and (b) eigenvalues.

According to Fig. 3(b) the eigenvalue decomposition of correlation matrix has generated the same conclusion. Fig. 3(b) shows the five eigenvalues of the correlation matrix of the field samples at three different frequencies. The eigenvalues have been normalized to the maximum eigenvalue. Fig. 2(b) shows the spectral power distribution and explained variance of the PCs over a frequency range in the case of two noise sources. The scree graph obtained in Fig. 4(a) confirms the dominance of the first two PCs in absolute terms for the chosen frequency. The five eigenvalues of the correlation matrix are illustrated in Fig. 4(b). As we can see at least two eigenvalues should be taken into consideration. Hence, both approaches allow to simplify the computation of environmental field on the basis of measure field data. However, due to various rules regarding the number of retainable PCs the PCA approach leads to clear evaluation criteria and therefore, it has advantage over eigenvalue decomposition. This advantage is more pronounced when more intricate problems of stochastic EM fields are modelled involving many sources or complex geometries.



(a) (b)

Figure 4: (a) Scree graph for the correlation matrix for two noise sources case and (b) eigenvalues.

An enhanced analysis along with further numerical and experimental results will be presented at the conference.

4 Conclusion

We have shown that the description of stochastic electromagnetic fields by correlation matrices can be simplified using the principal component analysis (PCA) or eigenvalue decomposition. In this paper, the principal component analysis and the eigenvalue decomposition approach have been applied for decomposing and reducing the correlation matrix describing the correlations of the sampled field. Subsequently, both approaches have been compared. The propagation of the stochastic electromagnetic field can be computed by transformation of the correlation matrices with the help of analytic or numerical Green's function. The PCA method has the potential to considerably reduce the computational effort for computing the environmental field from the field sampled by identifying and retaining only the relevant variables without loss of information. Thus, PCA can be used to represent the sources of the stochastic electromagnetic fields by their principal components. The principal components are a set of linearly uncorrelated field variables. This facilitates to compute the propagation of the PCs of the EM field independently.

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