System and Formulation Options for Biomedical Microwave Imaging

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Abstract

The various options in the design of microwave imaging systems for biomedical applications, as well as the associated options in the mathematical formulation of the corresponding inverse scattering problem, are delineated and discussed. These options impact the imaging performance of the overall system because they affect 1) the amount of scattered field information obtained, 2) the signal-to-noise ratio of the measurements, 3) the modelling error incurred in the inverse scattering problem, and 4) the nonlinearity and ill-posedness of the inverse problem. Here we describe how some of these options can be used to design and enhance the imaging performance of microwave imaging systems.

1. Introduction

Microwave imaging (MWI) for biomedical applications has come a long way since its early beginnings in the late 70's [1]. This imaging modality has expanded to many industrial applications and many sophisticated algorithms and measurement systems have been developed [2, 3]. Although some work on reconstructing magnetic material properties has been reported, the principle goal of MWI is to reconstruct an "image" of the complex permittivity of the object or region being imaged. For biomedical applications much progress has been made on imaging for the detection and identification of breast cancer [4, 5], and many other medical imaging applications are on the horizon [6]. In this paper we review some of the measurement system and formulation options available which can be utilized to enhance the imaging results. Some of these options have been investigated in the past whereas others have not yet been fully analyzed. This paper emphasizes options available for biomedical applications, focuses on full nonlinear quantitative inversions using time-harmonic sources of microwave energy, and is written from the perspective which we've gained during the past several years through our experience in developing systems and algorithms in the Electromagnetic Imaging Laboratory at the University of Manitoba.

2. Biomedical Microwave Imaging Systems

2.1 Chambers and Matching Media

The basic MWI system, schematically represented in Figure 1, consists of a chamber within which an object-of-interest, or target, that is to be imaged is placed. The important considerations here are the shape and material composition of the chamber as well as the inclusion of any matching medium (fluid or otherwise) within which the target is surrounded inside the chamber. The purpose of the matching medium is two-fold: 1) to increase the amount of "interrogation energy" coupled from the microwave energy source into the target, and 2) to add propagation loss in the region between the target and the chamber walls so that a simplified inversion model can be used without incurring too much modelling error. That is, if too little interrogation energy couples into the target because of large reflections at the target's surface then the amount of information gained about the interior of the target will be too small for successful inversion of the data. Due to the large dielectric value of many biological tissues it is not uncommon to use matching fluids with the dielectric constant ranging in value from $\varepsilon_r = 3$ to 80. With such large values the boundary of the chamber will introduce reflections which

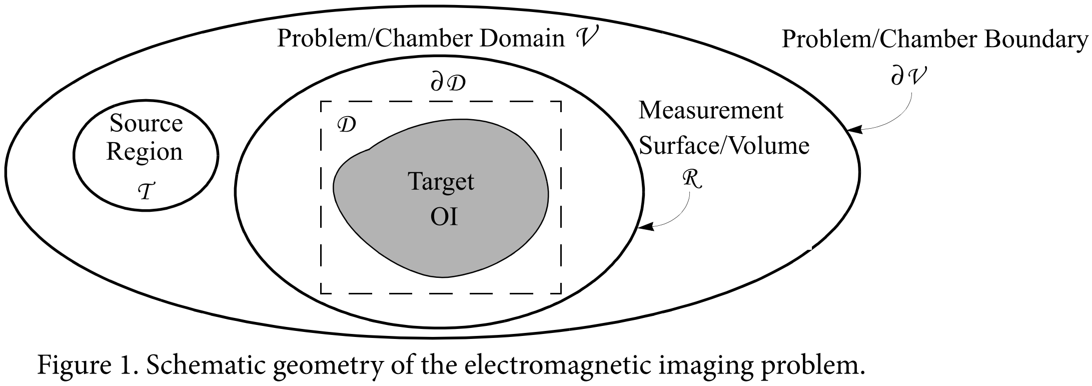

Figure 1. Schematic geometry of the electromagnetic imaging problem.

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must be taken into account in the inversion model irrespective of what material is used for the chamber walls. Whether a co-resident antenna system or movable antenna is used to perform the measurements the required mechanical infrastructure will also introduce reflections. Thus, one way to simplify the inversion model (by not having to model these features of the system) is to introduce path loss into the matching fluid. The introduction of path loss decreases the dynamic range of the measurement system resulting in a reduction in the S/N of the measurement data. One could attempt to use a simplified inversion model without the introduction of loss but this results in an increase in the modelling error. This has been analyzed quantitatively in our recent study [7]. One additional reason to introduce loss when performing 2D inversions is to reduce the 3D to 2D modelling error.

2.2 Antennas, Field Probes, and Polarization

The interrogation energy is usually introduced via antennas in the source region $\mathcal{T}$ within the chamber and the resulting total field information is measured using antennas or field probes on the measurement surface or volume $\mathcal{R}$. In the so called direct approach antennas are used to measure the field whereas an indirect approach, such as the Modulated Scatterer Technique (MST), can be implemented which uses small field probes in addition to collector antennas [8, 9]. One advantage of MST is that fewer collector antennas are required than receiving antennas in the direct approach, while the probes can be made small so as to not significantly perturb the field (thus allowing simpler inversion models). Also, as was shown in [9], multiple polarizations can be accommodated using MST simply by orienting the field probes in the direction of the polarization which is to be measured. When using a direct approach with co-resident antennas, unless a sufficient amount of loss is introduced into the matching fluid, one must consider whether or not to include the transmitting and receiving antennas within the inversion model. Including the antennas into the model will incur large computational demands of the inversion algorithm whereas omitting them from the model will increase modelling error and deteriorate the image quality which can be obtained. An interesting technique described in [10] images at specially selected frequencies where the co-resident antenna array becomes almost transparent to the microwave energy that is radiated by the transmitting antenna. Ultimately, for enhanced imaging, one must introduce a sufficient number of incident field distributions (including different polarizations) which interrogate the target while measuring the scattered field at a large number of receiver locations for each incident field. An interesting way to accomplish a diversity of incident fields without using a different transmitting antenna location is to make use of the chamber's boundary. That is, by reconfiguring or moving the conducting boundary of the chamber one can vary the incident interrogating field while keeping the transmitting antenna fixed with respect to the target. Such a technique has been demonstrated in [11] using a rotatable conductive enclosure having a triangular shape. The importance of varying the polarization has been emphasized in [12].

3. Mathematical Formulations

To delineate the options available in the mathematical formulation of the problem it is sufficient to focus on the scalar formulation represented by Helmholtz's equation for some total scalar field $u(x|\omega)$:

$$\Delta u(x|\omega) + k^2(x|\omega)u(x|\omega) = -s(x|\omega) . \tag{1}$$

Here $\Delta(\cdot)$ is the Laplacian operator, x is the position vector in whatever dimensional space is being considered, ω is radial frequency $(2\pi f)$, $s(x|\omega)$ is the source term and $k(x|\omega) = \omega\sqrt{\varepsilon_0\varepsilon(x|\omega)\mu_0}$ is the complex wave number. Depending on the boundary condition that must be accommodated at the chamber walls, one may have to impose either

$$u(x) = 0 \qquad x \in \partial\mathcal{V} \tag{2}$$

for a conducting wall, or a Sommerfeld radiation boundary condition,

$$\lim_{r \to \infty} r^{(N-1)/2}(\partial_r u + ju) = 0 , \tag{3}$$

where N denotes the dimension of space being considered (2 or 3) and $r = \|x\|$. Other boundary conditions are possible but these suffice to demonstrate the options. It is usual to first split the total field into "incident" and "scattered" fields as

$$u(x) \triangleq u^i(x) + u^s(x) \tag{4}$$

where the PDEs satisfied by these new fields are defined by choosing a "numerical" wavenumber $k_n(x) = \omega\sqrt{\varepsilon_n(x)\mu_0}$ which may be a function of position and complex valued if a lossy numerical background is chosen. That is,

$$\Delta u^i(x) + k_n^2(x)u^i(x) = -s(x) , \tag{5}$$

$$\Delta u^s(x) + k_n^2(x)u^s(x) = -(k^2(x) - k_n^2(x))u(x) . \tag{6}$$

Note that the incident field corresponds to the field inside the empty chamber if $k_n = k_b$, the wavenumber of the background matching medium, and in this case may be measured at the receiver locations on the measurement surface. Otherwise, this "numerical" incident field must be modelled. The total field is measured with the target in place and then the scattered field is calculated using $u^s = u - u^i$.

The *contrast source* is defined as

$$w(x) \triangleq (k^2(x) - k_n^2(x))u(x) = k_n^2(x)(k^2(x)/k_n^2(x) - 1)u(x) = k_n^2(x)\chi(x)u(x) \tag{7}$$

whereas the *contrast* is defined as

$$\chi(\boldsymbol{r}) = \chi' + i\chi'' \triangleq k^2(x)/k_n^2 - 1 = (\varepsilon(x) - \varepsilon_n(x))/\varepsilon_n(x) . \tag{8}$$

Note that the contrast is independent of the transmitted field whereas the contrast sources will vary with this field.

For a system with enough loss in the matching medium such that a homogeneous open-region inversion model with Sommerfeld radiation conditions can be applied for all fields, and if one chooses $k_n = k_b$, then the so-called data equation arises using the Green's function solution of the PDEs:

$$u^s(x) = k_b^2 \int_{\mathcal{D}} g(x|x')\chi(x')u(x')dx' = \int_{\mathcal{D}} g(x|x')w(x')dx' . \tag{9}$$

Here, for example, in 2D the scalar Green's function for a homogeneous open-region with Sommerfeld radiation condition is $g(x|x') = (-j/4)H_0^{(2)}(k_b\|x - x'\|)$. Given the measured scattered field at several discrete measurement locations, $x^m \in \mathcal{R}$, this represents a discrete data nonlinear first-kind integral equation for the contrast or contrast source.

For an MWI system with conductive chamber walls, although the total field will satisfy (2) one could define the incident field as in (5) but with a numerical boundary condition $u^i(x) = -\beta(x)$, $x \in \partial\mathcal{V}$. The scattered field is then governed by

$$\begin{cases} \Delta u^s(x) + k_n^2(x)u^s(x) = -k_n^2(x)w(x) & x \in \mathcal{V} \\ u^s(x) = \beta(x) & x \in \partial\mathcal{V} \end{cases} \tag{10}$$

Note again that $k_n^2(x)$ and $u^i(x \in \partial\mathcal{V})$ don't have to be the physical background and BCs. If we now define the Green's function for these PDEs as

$$\begin{cases} \Delta G(x|x') + k_n^2(x)G(x|x') = -\delta(x - x') & x, x' \in \mathcal{V} \\ G(x|x') = \gamma(x) & x \in \partial\mathcal{V} \end{cases} \tag{11}$$

where we're allowing the inhomogeneous boundary function $\gamma(x)$ to be an arbitrary function on $\partial\mathcal{V}$. Then the new data equation becomes

$$u^s(x) = \int_{\mathcal{V}} k_n^2(x)G(x|x')\chi(x')u(x')dx' + \int_{\partial\mathcal{V}} \gamma(x')\partial_n u^s + u^i(x')\partial_n G dx' . \tag{12}$$

where ∂_n denotes the normal derivative at the boundary. If we choose $\beta(x) = \gamma(x) = 0$ and $k_n = k_b$ then the surface integral goes away and we arrive back at (9) but now with the Green's function satisfying (11) as opposed to the Green's function for a homogeneous open-region [13, 14]. Of course many other options are available depending on the choice of $\beta(x)$, $\gamma(x)$, and k_n. What should be noticed is that formulating the data equation in this way changes the kernel of the volumetric integral equation as well as the surface integral arising because of the inhomogeneous boundary term $\gamma(x)$. The kernel $G(x|x')$ as well as the measurement locations $x^m \in \mathcal{R}$ determine the singular values and vectors of the operator [15]. The surface integral effectively changes the data $u^s(x^m)$ as follows. If we define the data as

$$d(x^m) = u^s(x^m) - \int_{\partial\mathcal{V}} \gamma(x')\partial_n u^s + u^i(x')\partial_n G(x^m|x')dx' \tag{13}$$

then the measured scattered field data $u^s(x^m)$ is modified by subtracting a weighted integral of the normal derivative of the scattered field and incident field at the boundary. The weighting coefficients are the chosen $\gamma(x')$ and the normal derivative of the resulting Green's function at the boundary with source point at the measurement point x^m. This sort of "transformation" from the Green's function used in the integral equation to the effective data provides considerable flexibility in handling the ill-posedness of the nonlinear inversion problem based on minimizing the data error

$$\|d - \mathcal{G}_d(\chi u)\|_{\mathcal{R}} \tag{14}$$

where $\mathcal{G}_d(\cdot)$ represents one of the integral data operators and d represents the measured data appropriately modified for the data operator being used. Although these options have been described in terms of modifying the Green's function kernel, they can be effectively implemented using Finite-Element inverse operators and can correspond to introducing prior information into the problem [16].

4. Conclusions

Some of the options available in the design of MWI systems and in the mathematical formulation of the associated inverse problem have been reviewed. These design options impact the imaging performance of the overall system and can be used to enhance the resulting images. The successful implementation of some of these options by our research group and by others have been referred to in the references. Many of these possible options have not been attempted yet but provide options for considerations in future MWI system designs.

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