Calculating the Physical Bounds for Antennas Above a Ground Plane

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Abstract

The physical bounds for antennas above a ground plane is analyzed in this paper. Optimal antenna G/Q ratio and Q factor are calculated using convex optimization from stored electric and magnetic energy matrices, obtained from a method of moments solver. Physical bounds for a patch antenna with different heights above a ground plane are investigated. Results are verified using commercially available electromagnetic solver FEKO.

1. Introduction

Small antennas are found in the most commonly used devices in our daily lives [1]. The most widely known application area for small antennas today is mobile phones. The antennas in mobile phones are expected to perform near-optimal in any condition, to conform to the ever increasing standards and services demanded by the user. As the size of the antenna is restricted to fulfill consumer demands, antenna designers are faced with the tough problem of achieving the expected specifications. Due to the fact that antenna performance decreases with the antennas physical size, antenna designers have to make the most out the volume allocated to the antenna.

The physical bounds are the optimal performance limits for an antenna of specific geometry. Physical bounds for small antennas have been originally determined by Wheeler [2] using equivalent circuits, and was succeeded by Chu [3]. The results obtained in these first investigations provide valuable insight to the physics of small antennas. The paper presented by Chu has been the classical example for deriving the physical bounds analytically with spherical mode expansions. One major drawback of these results is the limitation of antenna geometries to spherical structures.

Physical bounds for antennas of arbitrary shape have been investigated in [4]. With expressions from Vandenbosch [5] to calculate the stored energies from surface current densities, and the formulations in [6], the stored energies can be computed for small antennas of any geometry by manipulating a Method of Moments (MoM) program. These energies are later used in current optimization to obtain the physical bounds.

Convex optimization is a valuable tool for calculating the physical bounds for antennas as illustrated in [7]. The optimal G/Q ratio and minimum antenna Q for a predefined radiation field are formulated as convex problems where the stored energy matrices are used to determine the constraints on the current; this is also called current optimization.

This paper extends the previous results on current optimization of free-space antennas to antennas above a ground plane. The initial results of structures such as; strip dipole and folded dipole above a ground plane, and patch antennas are presented. The physical bounds are also compared to simulation results from the commercial electromagnetic solver FEKO [8].

2. Theoretical Background

The energies for a radiating antenna situated in free-space, can be divided into two parts; the radiated energy and the stored energy. Although, the radiated energy is estimated in terms of the far-field, determining the stored energies is a demanding task. Here we present an overview of the results in [6] for the computation of stored energies.

The stored energies are found by subtracting the far-field energy from the energy density. The stored electric energy is expressed as,

$$W_{\mathrm{e}} = \frac{\epsilon_0}{4}\int_{\mathbb{R}^3} |\boldsymbol{E}(\boldsymbol{r})| - \frac{|\boldsymbol{F}(\hat{\boldsymbol{r}})|^2}{r^2}\mathrm{dV}, \tag{1}$$

where ϵ_0 is the permittivity of free-space, $\boldsymbol{F}(\hat{\boldsymbol{r}})$ is the electric far-field, $r = |\boldsymbol{r}|$, and $\hat{\boldsymbol{r}} = \boldsymbol{r}/r$. The integration in (1) is over an infinite sphere. The magnetic energy is found similarly by subtracting the far-field from the

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magnetic energy density. From (1) the stored electric energy is, [5]

$$W_{\mathrm{e}} = \frac{\eta_0}{4\omega}\int_V\int_V \nabla_1 \boldsymbol{J}_1 \cdot \nabla_2 \boldsymbol{J}_2^* \frac{\cos(kR_{12})}{4\pi k R_{12}} - \left(k^2 \boldsymbol{J}_1 \cdot \boldsymbol{J}_2^* - \nabla_1 \boldsymbol{J}_1 \cdot \nabla_2 \boldsymbol{J}_2^*\right) \frac{\sin(kR_{12})}{8\pi} \,\mathrm{dV}_1 \,\mathrm{dV}_2, \tag{2}$$

with the currents defined as; $\boldsymbol{J}_1 = \boldsymbol{J}(\boldsymbol{r}_1)$, $\boldsymbol{J}_2 = \boldsymbol{J}(\boldsymbol{r}_2)$, and the distance $R_{12} = |\boldsymbol{r}_1 - \boldsymbol{r}_2|$, and η_0 is the free-space impedance. Similarly the stored magnetic energy is,

$$W_{\mathrm{m}} = \frac{\eta_0}{4\omega}\int_V\int_V k^2 \boldsymbol{J}_1 \cdot \boldsymbol{J}_2^* \frac{\cos(kR_{12})}{4\pi k R_{12}} - \left(k^2 \boldsymbol{J}_1 \cdot \boldsymbol{J}_2^* - \nabla_1 \boldsymbol{J}_1 \cdot \nabla_2 \boldsymbol{J}_2^*\right) \frac{\sin(kR_{12})}{8\pi} \,\mathrm{dV}_1 \,\mathrm{dV}_2, \tag{3}$$

the Q factor, an essential parameter for small antennas, is calculated from the energies,

$$Q = \frac{2\omega \max\{W_{\mathrm{e}}, W_{\mathrm{m}}\}}{P_{\mathrm{r}}}, \tag{4}$$

with ω the radial frequency and P_{r} the total radiated power for a lossless antenna.

Another significant parameter for small antennas is the G/Q ratio. The G/Q ratio is an indicator of the available bandwidth for a certain antenna gain, it is expressed as

$$\frac{G(\hat{\boldsymbol{r}}, \hat{\boldsymbol{e}})}{Q} = \frac{\pi |\hat{\boldsymbol{e}}^* \cdot \boldsymbol{F}(\hat{\boldsymbol{r}})|^2}{\omega \eta_0 \max\{W_{\mathrm{e}}, W_{\mathrm{m}}\}}, \tag{5}$$

where η_0 is impedance of free space and the vectors $\hat{\boldsymbol{r}}$ and $\hat{\boldsymbol{e}}$ are unit vectors describing the direction of propagation and polarization, respectively. The far-field radiation vector is,

$$\hat{\boldsymbol{e}}^* \cdot \boldsymbol{F}(\hat{\boldsymbol{r}}) = \frac{-\mathrm{j}k\eta_0}{4\pi}\int_S \hat{\boldsymbol{e}}^* \cdot \boldsymbol{J}(\boldsymbol{r}) \mathrm{e}^{\mathrm{j}k\hat{\boldsymbol{r}}\cdot\boldsymbol{r}} \,\mathrm{dS} \tag{6}$$

3. Current Optimization Using Convex Optimization

The electric stored energy in (3), is similar to the electric field integral equation (EFIE) in the method of moments (MoM). Subsequently, the stored energies can be calculated numerically using a modified MoM code. In MoM implementations, the antenna structure is discretized into triangular or quadrilateral elements and the test functions are selected to be of Galerkin type. The currents on the structure are then written as,

$$\boldsymbol{J}(\boldsymbol{r}) = \sum_n J_n \boldsymbol{\psi}_n(\boldsymbol{r}), \tag{7}$$

where ψ, the basis functions are selected to be rooftop type for rectangular elements. The MoM implementation produces the matrix elements,

$$Z_{\mathrm{e},ij} = \frac{1}{\mathrm{j}k}\int_V\int_V \nabla_1 \cdot \boldsymbol{\psi}_{i1} \nabla_2 \cdot \boldsymbol{\psi}_{j2} \frac{\mathrm{e}^{-\mathrm{j}kR_{12}}}{4\pi R_{12}} \,\mathrm{dV}_1 \,\mathrm{dV}_2, \tag{8}$$

$$Z_{\mathrm{m},ij} = -\mathrm{j}k\int_V\int_V \boldsymbol{\psi}_{j1} \cdot \boldsymbol{\psi}_{j2} \frac{\mathrm{e}^{-\mathrm{j}kR_{12}}}{4\pi R_{12}} \,\mathrm{dV}_1 \,\mathrm{dV}_2, \tag{9}$$

and

$$X_{\mathrm{em},ij} = \frac{-1}{8\pi}\int_V\int_V \left(k^2 \boldsymbol{\psi}_{i1} \cdot \boldsymbol{\psi}_{j2} - \nabla_1 \cdot \boldsymbol{\psi}_{i1} \nabla_2 \cdot \boldsymbol{\psi}_{j2}\right) \sin(kR_{12}) \,\mathrm{dV}_1 \,\mathrm{dV}_2, \tag{10}$$

the EFIE impedance matrix is calculated by the addition of the first two matrices $\mathbf{Z} = \mathbf{Z}_{\mathrm{m}} - \mathbf{Z}_{\mathrm{e}}$. While the stored energy matrices are,

$$\mathbf{X}_{\mathrm{e}} = \mathrm{Im}\{\mathbf{Z}_{\mathrm{e}}\} + \mathbf{X}_{\mathrm{em}}, \tag{11}$$

$$\mathbf{X}_{\mathrm{m}} = \mathrm{Im}\{\mathbf{Z}_{\mathrm{m}}\} + \mathbf{X}_{\mathrm{em}}, \tag{12}$$

for antenna dimensions less than approximately $\lambda/2$, the energy matrices are positive semi-definite [7]. To compute antennas above a PEC ground plane the image currents are added to the Green's function in (8),(9), and (10). The Green's function for a antenna above an infinite PEC plane is,

$$\mathcal{G}_{\mathrm{m}} = \frac{\mathrm{e}^{-\mathrm{j}kR_{12}}}{4\pi R_{12}} \mp \frac{\mathrm{e}^{-\mathrm{j}k\tilde{R}_{12}}}{4\pi \tilde{R}_{12}}, \tag{13}$$

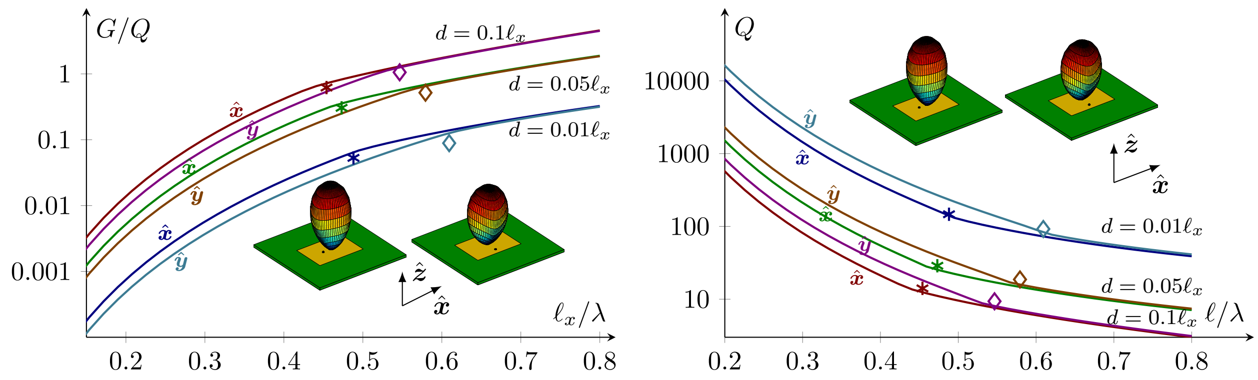

Figure 1: Optimal G/Q (left) and Q factor (right) using convex optimization (14) **for a rectangular patch above an infinite PEC ground plane using $\ell_y = 0.8\ell_x$, $d = \{0.01, 0.05, 0.1\}\ell_x$, $\ell_x/\lambda \leq 0.8$, $\hat{\boldsymbol{r}} = \hat{\boldsymbol{z}}$, and $\hat{\boldsymbol{e}} = \{\hat{\boldsymbol{x}}, \hat{\boldsymbol{y}}\}$. The corresponding patch antennas are simulated using FEKO and the results are indicated for the $\hat{\boldsymbol{x}}$ polarization (asterisk) and $\hat{\boldsymbol{y}}$ polarization (diamond).**

for an infinite ground plane at $z = 0$, the distance between current $\mathbf{J}_1$ and the image current $\mathbf{J}_2$ is $\tilde{R}_{12}$, and the $\mp$ signs are for horizontal and vertical currents, respectively.

Current optimization for maximum G/Q ratio is a quadratic form, that can be written as a convex optimization problem [9]. The maximum of the stored energies are minimized over the currents for a specific radiation pattern, the convex problem is then,

$$\begin{aligned} &\text{minimize} \quad \max\{\mathbf{J}^{\mathrm{H}}\mathbf{X}_{\mathrm{e}}\mathbf{J}, \mathbf{J}^{\mathrm{H}}\mathbf{X}_{\mathrm{m}}\mathbf{J}\} \\ &\text{subject to} \quad \mathrm{Re}\{\mathbf{F}^{\mathrm{H}}\mathbf{J}\} = 1 \end{aligned} \tag{14}$$

where $W_{\mathrm{e}} \sim \mathbf{J}^{\mathrm{H}}\mathbf{X}_{\mathrm{e}}\mathbf{J}$, $W_{\mathrm{m}} \sim \mathbf{J}^{\mathrm{H}}\mathbf{X}_{\mathrm{m}}\mathbf{J}$, and $\mathrm{Re}\{\hat{\boldsymbol{e}}^* \cdot \boldsymbol{F}(\hat{\boldsymbol{k}})\} \sim \mathrm{Re}\{\mathbf{F}^{\mathrm{H}}\mathbf{J}\}$. The minimum Q factor for the radiation pattern is also obtained from (14). The convex optimization problem is carried out in MATLAB using the CVX toolbox [10].

4. Antennas Above a Ground Plane

The theoretical results presented in the previous section are used to derive the physical bounds for a patch antenna above an infinite PEC ground. The current distributions of the aforementioned antenna is optimized with the convex optimization problem (14) to obtain the maximum G/Q quotient and the minimum Q factor.

The results from the convex optimization are plotted versus different patch sizes, ℓ/λ. Moreover, the antenna is assumed to radiate in the normal direction to the ground plane $\hat{\boldsymbol{z}}$, in both $\hat{\boldsymbol{x}}$ and $\hat{\boldsymbol{y}}$ polarization. The antenna to ground plane distance is varied to illustrate the change in both the G/Q ratio and minimum Q value.

For verification the antennas are additionally simulated in FEKO and added for comparison. The antenna Q factor from simulations is calculated using the definition in [11],

$$Q \approx Q_{Z'} = \frac{\omega_0}{2R_0}\left|\frac{\mathrm{d}Z_{\mathrm{m}}}{\mathrm{d}\omega_0}\right|, \tag{15}$$

where ω_0 is the angular frequency, R_0 is the real part of the input impedance at the specified angular frequency. The antenna is matched at the specified angular frequency with a lumped circuit element, the matched impedance is Z_{m}.

A patch antenna with dimensions $\ell_y = 0.8\ell_x$ is used to illustrate the current optimization. The patch is placed above a ground plane with different heights $d = \{0.1, 0.05, 0.01\}\ell_x$. Figure 1 shows the G/Q ratio and Q factor versus antenna size, it is clear that the antenna performance improves significantly as the antenna is placed further from the ground plane. When the antenna is located in close proximity to the PEC ground, it is basically short circuited to the ground. As, the antenna is placed further away the radiated power increases, lowering the Q-factor.

5. Conclusions

Initial physical bounds have been presented for antennas above a ground plane. The approach uses a MoM solver to compute the stored energies that are then optimized with convex optimization. The initial results of patch antennas have been verified with simulations in FEKO.

References

1. J. Volakis, C. C. Chen, and K. Fujimoto. *Small Antennas: Miniaturization Techniques & Applications*. New York: McGraw-Hill, 2010.

2. H. A. Wheeler. "Fundamental limitations of small antennas". *Proc. IRE*, vol. 35(12), pp. 1479–1484, 1947.

3. L. J. Chu. "Physical limitations of omni-directional antennas". *J. Appl. Phys.*, vol. 19, pp. 1163–1175, 1948.

4. M. Gustafsson, C. Sohl, and G. Kristensson. "Physical limitations on antennas of arbitrary shape". *Proc. R. Soc. A*, vol. 463, pp. 2589–2607, 2007.

5. G. A. E. Vandenbosch. "Reactive energies, impedance, and Q factor of radiating structures". *IEEE Trans. Antennas Propagat.*, vol. 58(4), pp. 1112–1127, 2010.

6. M. Gustafsson and B. L. G. Jonsson. "Stored Electromagnetic Energy and Antenna Q". Tech. Rep. LUTEDX/(TEAT-7222)/1-25/(2012), Lund University, 2012.

7. M. Gustafsson and S. Nordebo. "Optimal antenna currents for Q, superdirectivity, and radiation patterns using convex optimization". *IEEE Trans. Antennas Propagat.*, vol. 61(3), pp. 1109–1118, 2013.

8. FEKO. "EM Simulation Software". https://www.feko.info/.

9. S. P. Boyd and L. Vandenberghe. *Convex optimization*. Cambridge Univ Pr, 2004.

10. M. Grant and S. Boyd. "CVX: Matlab software for disciplined convex programming, version 1.21". cvxr.com/cvx, Apr. 2011.

11. A. D. Yaghjian and S. R. Best. "Impedance, bandwidth, and Q of antennas". *IEEE Trans. Antennas Propagat.*, vol. 53(4), pp. 1298–1324, 2005.