Improved Calibration Technique for the Transmit Beamforming by a coherent MIMO Radar with Collocated Antennas

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Abstract

In this paper we present a calibration method for mitigating the effects of mutual coupling affecting the signal radiated by MIMO radar with collocated antennas. The paper contains the measurement results that verify the proposed method. The improved scan-dependent calibration technique is proposed for elimination of the coupling effect. The experiments with a real X-band antenna array validate the calibration procedure presented in this paper.

1. Introduction

In recent years there has been a significant increase in researches related to phased array antennas. Many systems, such as broadcast radio stations, space probes, radars of different applications employ phased array systems for beam-forming and steering [1]. Normally, phased array systems employ a set of radiating elements geometrically configured and excited in order to obtain a specific radiation pattern. By employing all the antenna elements (or sub-arrays) and transmitting different waveforms in multiple-input multiple-output (MIMO) system, the MIMO radar performance can be significantly improved. Since the separation between the antenna elements is small, the radar using an antenna array is called coherent or collocated [2].

Mutual coupling is a common problem in the applications of antenna arrays, also in collocated coherent MIMO systems. Coupling is present in all antenna arrays to some degree and can significantly affect their operation [3,4]. Obviously, the deep and quick evolution of MIMO systems could only go together with a development of the calibration strategies. The different effects of antenna mutual coupling on the performance of MIMO systems has been investigated [5-7]. A MIMO radar with the space-time waveform transmission operates at a new level of complexity, where the coupling effects also show up depending on the radiated space-time waveforms [8]. A commonly used measure for assessing waveform selection in radar systems is their ambiguity function (AF). The mutual coupling on transmit changes the radiated group signal consisting of different waveforms. Therefore the MIMO AF analysis might be an effective tool for estimation of the coupling effects and their compensation in actual practice. The detailed analysis of antenna coupling effects on the beam-forming on transmit implemented by means of signal processing in the MIMO radar receiver has been done in [8]. Although the existing calibration technique is robust and rather simple in implementation, it takes into account only the influence the adjacent antenna elements with regard to the transmitting one. The purpose of the here-presented technique is to perform the calibration taking into account the mutual coupling between all the array elements and keeps the same computational cost as the basic calibration technique [8].

The paper is organized into the following sections. Section II introduces the angular transmit diagram and the error induced in it due to the coupling effect between the radiating array elements. The improved calibration technique taking into account the coupling coefficients between all the elements is described in Section III. The validation of the improved calibration on transmit is presented in the experimental part, Section IV. Section V draws conclusions.

2. Angular Transmit Diagram

A MIMO radar transmits multiple different waveforms $s^n(t)$ whether correlated or uncorrelated. Without the mutual coupling between the antenna elements, the group signal radiated by a linear antenna array in the angular (e.g. elevation) direction θ_0 in the absence of noise can be written as

$$s_T^0(t,\theta_0) = \sum_{n=1}^{N} P_n(\theta_0) \cdot e^{j\cdot\bar{k}(\theta_0)\cdot\bar{x}(n)} \cdot s^n(t) \tag{1}$$

where N is a number of channels on transmit; $P_n(\theta_0)\cdot$ is the embedded pattern of the *n*th radiating element, $\bar{k}$ is a wave-vector, $\bar{x}(n)$ is the position vector of the *n*th radiating element. The upper index '0' for the transmitted signals here means that the coupling between the antenna array elements is not taken into account. We assume the phased array antenna (see Fig. 1) is to be linear, the active elements are placed in a line and equally spaced by a distance Δx.

The multi-dimensional transmit ambiguity function (AF) described in [9,10] is used for performance evaluation of the beamforming capabilities together with evaluation of the MIMO waveforms. The ambiguity function is calculated as the matched filter output in the aiming direction θ_0 in the absence of noise, assuming Doppler effect negligible within one pulse and leaving the complex coefficient due to the propagation and scattering:

$$\chi_{\theta_0}^0(\tau,\theta) = \int s_T^0(t,\theta)\cdot(s_T^0(t+\tau,\theta_0))^* dt, \tag{2}$$

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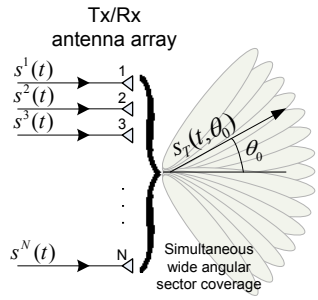

Figure 1: MIMO transmission

where θ is the observed angular direction, $\tau = 2 \cdot R/c$ is the roundtrip travel time of the scattered signal, R is the distance to the observed object, c is the light velocity.

The multi-dimensional ambiguity function $\left|\chi_{\theta_0}^0(\tau,\theta)\right|^2$ is a 3-dimensional function, giving for each aiming direction θ_0 the delay-angle ambiguity. For demonstration of the beam-forming, the angular transmit diagram, which is a cut of the ambiguity function at the exact range of the target, $\tau_0 = 0$, is used:

$$D^0(\theta,\theta_0) = \left|\chi_{\theta_0}^0(0,\sin\theta)\right|. \tag{3}$$

For interpretation of the angular transmit diagram in the logarithmic scale, $20 \cdot \log(D^0(\theta,\theta_0))$ should be used. Sometimes, the power of two is applied to the matched filer output to identify the diagram $D^0(\theta,\theta_0) = \left|\chi_{\theta_0}^0(0,\sin\theta)\right|^2$. Then $10 \cdot \log(D^0(\theta,\theta_0))$ should be used for the data interpretation, what, basically, gives the same result. It is a function of the angular aiming direction θ_0 and the observed angular direction θ. The angular transmit diagram is obtained by means of signal processing in one (in our case) or in each (in general) receiver channel.

Because of the linearity in Maxwell's equations, the coupling between the array elements is modeled as a linear system via the mutual coupling matrix [11]. A complex valued, linear, angle- and frequency-dependent coupling is introduced in the model as a complex symmetrical matrix. The ($N \times 1$) MIMO system is simulated with the $N \times N$ coupling matrix on transmit,

$$\mathbf{R}_T = \begin{bmatrix} 1 & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & 1 & \ddots & \rho_{2N} \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{N1} & \rho_{N2} & \cdots & 1 \end{bmatrix}. \tag{4}$$

Notwithstanding, the amplitude and phase errors due to the passing of the signals through the RF part in the MIMO transmitter could be included in the main diagonal elements of $\mathbf{R}_T$.

When mutual coupling is present, the signal transmitted in free space (see Eq. 1) in the direction θ_0 becomes

$$s_T(t) = \sum_{n=1}^{N} P_n(\theta_0) \sum_{m=1}^{N} \mathbf{R}_T(n,m) \cdot \left(e^{j \cdot \bar{k}(\theta_0) \cdot \bar{x}(n)} \cdot s^m(t)\right) \tag{5}$$

where n is the number of the antenna element, $n \in [1 \ldots N]$, m is the index of the transmitted waveform, $m \in [1 \ldots N]$.

In the presence of coupling between the radiating elements, the matched filter output (see Eq. 2) can be written as

$$\chi_{\theta_0}(\tau,\theta) = \int s_T^0(t,\theta_0) \cdot (s_T(t+\tau,\theta_0))^* \, dt, \tag{6}$$

or in the extended form:

$$\chi_{\theta_0}(\tau,\theta) = \sum_{n=1}^{N} \sum_{n'=1}^{N} \sum_{m \neq n'}^{N} e^{j \cdot \left(\bar{k}(\theta) \cdot \bar{x}(n) - \bar{k}(\theta_0) \cdot \bar{x}(n)\right)} \cdot P_n(\theta_0) \cdot \mathbf{R}_T^*(n,m) \cdot \int s^n(t+\tau) \cdot \left(s^m(t)\right)^* dt, \tag{7}$$

where n' stands as an index for the replica of the signal radiating through the n-th radiating element. The presence of the coupling changes the transmit beamforming properties, described by the angular transmit diagram $D(\theta,\theta_0) = \left|\chi_{\theta_0}(0,\sin\theta)\right|$. The error signal presented in the angular transmit diagram due to the coupling effects can be written as:

$$D^{Err}(\theta,\theta_0) = \left|D^0(\theta,\theta_0) - D^0(\theta,\theta_0)\right|. \tag{8}$$

3. Calibration Technique

Taking into account the fact that the normalized self-coupling ($m = n$) integral between the orthogonal waveforms is equal to *1* and the normalized cross-coupling integral ($m \neq n$) in case of (almost) orthogonal waveforms with large BT-products is equal to *0* for $\tau' = 0$ [12], that is $\tau = \tau_0$, Eq. (7) can be simplified:

$$\chi_{\theta_0,\tau_0}(\theta') = N \cdot \sum_{n=1}^{N} \sum_{n' \neq n}^{N} P_{n'}(\theta_0) \cdot e^{j \cdot \left(\bar{k}(\theta') \cdot \bar{x}(n) - \bar{k}(\theta_0) \cdot \bar{x}(n')\right)} \cdot \mathbf{R}_T^*\left(n', n\right) \cdot \tag{9}$$

As previously stated, the radar waveforms $s^n(t)$ occupying the same time interval and the same bandwidth can not be completely orthogonal, meaning that Eq (7) is an approximation of the signal for the case of the ideally orthogonal waveforms. Therefore, the error signal is expected to exhibit some differences for different types of radiated signals.

To compensate the mutual coupling effects, some known solutions propose to introduce the transformation matrix, which is equal to $\mathbf{R}_T^{-1}$ [13,14], into the transmitting signal. However, the amplitude and phase change can suppress the coupling effects on transmit from one side, but, from another side, the compound signal radiated by the antenna array can be affected by the angular- and frequency dependency of the coupling effects, which is absolutely not desirable. So, the proposed calibration technique will be performed in the receiver by means of the signal processing.

The calibration technique proposed by the authors of this work [8] mitigates the coupling effects only of the neighboring antenna elements. The improved calibration technique, proposed in this paper, takes into account the whole coupling matrix. The correcting signals are applied on the useful signals' replicas for matched filtering in a receiver channel. In fact, these correcting signals A_n describe the amplitude change in the replicas of the useful signals $s^n(t)$; and they can be recalculated as the following addition to the main diagonal of the coupling matrix on transmit (4):

$$\mathbf{R}_T = \begin{bmatrix} \left(1 - A_1\right) & \rho_{12} & \cdots & \rho_{1N} \\ \rho_{21} & \left(1 - A_2\right) & \ddots & \rho_{2N} \\ \vdots & \ddots & \ddots & \vdots \\ \rho_{N1} & \rho_{N2} & \cdots & \left(1 - A_N\right) \end{bmatrix}. \tag{10}$$

Assume that the orthogonal signals are perfectly orthogonal. In this case, the error signal can be written as a function of the index n, which is representing the number of the transmitted waveform.

$$\chi_{\theta_0,\tau_0}^{er}(\theta') = N \cdot \sum_{n=1}^{N} e^{j \cdot \bar{x}(n) \cdot \left(\bar{k}(\theta') - \bar{k}(\theta_0)\right)} \cdot \sum_{n' \neq n}^{N} P_{n'}(\theta_0) \cdot e^{-j \cdot \bar{k}(\theta_0) \cdot x_{n',n}} \cdot \mathbf{R}_T^*\left(n', n\right), \tag{11}$$

where $\bar{x}_{n',n}$ is the distance between the n' and n antenna elements. If the antenna array is equidistant with a spacing d between the elements, then $\bar{x}_{n',n} = (n' - n) \cdot d$. From whence the amplitudes of the angular-dependent correcting signals taking into account the coupling between all the antenna elements are derived as

$$A_{n,\theta_0} = -\sum_{n' \neq n}^{N} P_{n'}(\theta_0) \cdot e^{-j \cdot \bar{k}(\theta_0) \cdot \bar{x}_{n',n}} \cdot \mathbf{R}_T^*\left(n', n\right). \tag{12}$$

4. Experimental Results

The following set-up is used for the beamforming demonstration in the MISO test-bed. The pulse duration is equal to 100µs. The carrier frequency is 8.5 GHz. The BT product of all the modelled signals was chosen relatively small, 255, for better visibility of the results. The antenna array is made of $N = 8$ patch antennas spaced $\lambda/2$ from each other. Two sets of measurements have been done using 8 maximum length sequences (M-sequences) and 8 quadratic Alltop signals, correspondingly. The coupling between the eight elements of a suitable X-band antenna array was measured using a vector network analyzer. The antenna is an array of 32 by 4 X-band patches. The radiation patterns and the coupling of 8 elements have been measured in the operating configuration. In this way, the measured patterns are taking into account the coupling effects. The selected line of 8 elements is performing as a linear equidistant array. The remaining elements are terminated by 50 Ω loads, since the antenna used in the experiment has been designed for 50 Ω impedance matching. Due to the calibration, the measured coupling matrix has been normalized. The error presented in the angular transmit diagram due to the coupling effects is used for analysis of the efficiency of the developed calibration technique and its predecessor.

Fig. 2 presents the error of the angular transmit diagram obtained for 8 orthogonal M-sequences and 8 orthogonal Alltop signals (upper and lower rows, correspondingly). The embedded antenna patterns and the cross-correlation between the signals are taken into account. Each vertical cut of the errors presented in Figs. 2.a-f are the errors of the transmit beamformed patterns obtained on receive. Figs. 2.g-h demonstrate such cuts in $\theta_0 = 0$ angular direction for the considered cases: without any calibration, with the calibration [8] and the improved calibration presented in this paper. It can be seen that without calibration (Fig. 2.a-b) the beamforming on transmit is affected by the angular-dependent error, which values were equal to -14dB in $\theta_0 = 0$ direction and less in other directions for both considered types of signals. Application of the calibration technique (Fig. 2.c-d) decreases the error significantly. For example, in $\theta_0 = 0$ direction it becomes equal to -22dB and -26dB

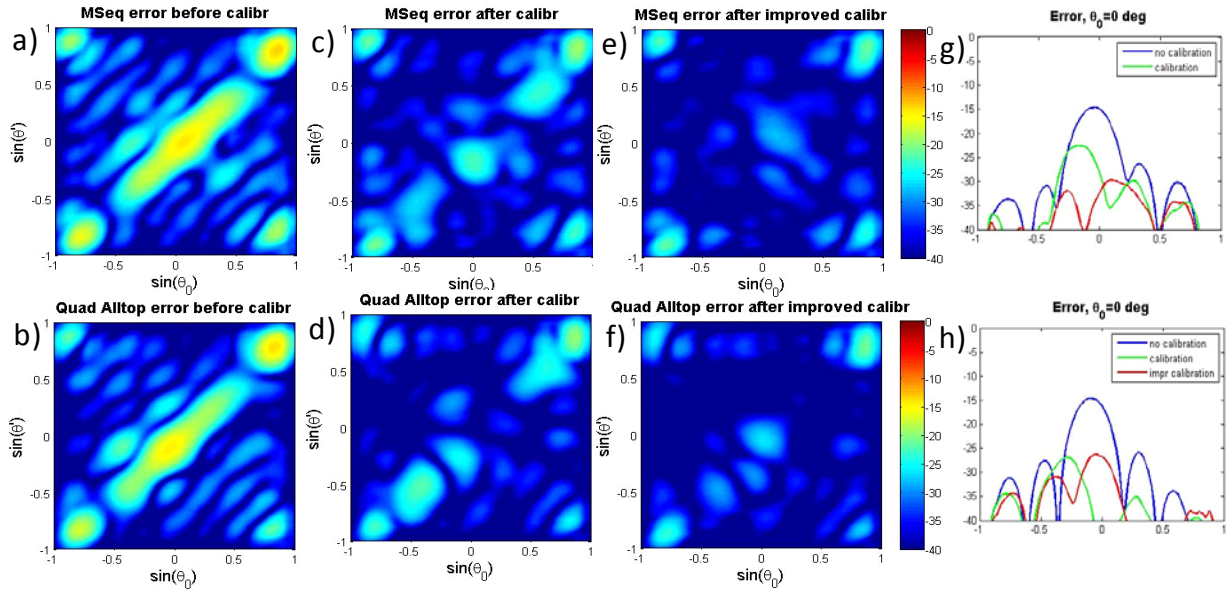

Figure 2: Error of the transmit diagram $D^{Err}(\theta,\theta_0)$, in dB

for M-sequences and Alltop signals, correspondingly. In spite of the fact, that the used array has very small coupling coefficients between the non-adjacent elements, the improved calibration technique still decreases the error. The improvement is can be clearly seen for most of the angular directions in Figs. 2.e-f. As for the specific beamformed direction $\theta_0 = 0$ (see Figs. 2.g-h), the maximal error value decreases to – 29.5dB for M-sequences and remains the same for Alltop signals.

5. Conclusions

In this paper we presented an improved calibration method for MIMO radar with the space-time waveform transmission that can compensate or decrease the mutual coupling effects. The main advantage of the proposed method over the published one is taking into account not only the adjacent elements, but all the elements. The method was successively demonstrated by simulation. In addition, the experiments with the real codes and the real X-band antenna array have validated the efficiency of the calibration technique presented in the paper.

6. References

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