Ray Tracing of Spherical Waves in Magnetoplasma

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Abstract

In general, it is very difficult to find analytical solutions of the electromagnetic field in an inhomogeneous magnetoplasma such as the ionosphere and magnetosphere, even under the approximation of geometrical optics [Budden, 1988]. As a powerful technique, numerical ray tracing can give solutions in these cases, and it is widely used to reveal the physical property of wave propagation in magnetoplasma and to interpret observations. As it is well known, when the electromagnetic wave excited from a source is propagating in a magnetoplasma, in the classical ray-tracing, the wave is treated as one of two plane waves, either L mode or R mode, and the error introduced is generally believed to be negligibly small. The task of ray-tracing is to numerically solve the ray equation system including six partial differential equations for a given or an assumed three dimensional structure of the electron density and the imposed geomagnetic field [Haselgrove, 1955]. Because of the anisotropic property of the magnetoplasma, the group and phase velocity vectors for any mode plane wave are in general differing in both magnitude and direction. This results in a very complicated coupling situation for the three components of the refractive index vector in Haselgrove's ray equation system, and numerical computations are time-consuming. It is time consuming in computation to find the numerical solutions, a disadvantage for real time applications in a time varying. We are proposing a spherical wave solution for which the directions of the phase and group vectors are equal.

In an isotropic medium, waves originated from a small source have spherical wave fronts at distances large compared to the source dimension. Huang and Reinisch [2012] have shown that electromagnetic waves originating from an arbitrary current source confined in a limited region in a uniform cold magnetoplasma can also be presented by spherical waves. When the z-axis is set along the direction of the imposed magnetic field and using the complex time factor $e^{j\omega t}$, the relative dielectric tensor, $\boldsymbol{\varepsilon}$, can be written in matrix form with the standard notation:

$$\boldsymbol{\varepsilon}=\begin{bmatrix}\varepsilon_1 & -j\varepsilon_2 & 0\\ j\varepsilon_2 & \varepsilon_1 & 0\\ 0 & 0 & \varepsilon_3\end{bmatrix},\qquad \begin{cases}\varepsilon_1=1-\dfrac{XU}{U^2-Y^2},\ \ \varepsilon_2=\dfrac{XY}{U^2-Y^2},\ \ \varepsilon_3=1-\dfrac{X}{U}\\ X=\dfrac{{\omega_{pe}}^2}{\omega^2},\ \ Y=\dfrac{\omega_{ce}}{\omega},\ \ U=1-jZ,\ \ Z=\dfrac{\nu}{\omega}\end{cases} \tag{1}$$

In order to describe the phase variation with distance, a refractive index of spherical waves, n_s, is introduced:

$$\begin{aligned} n_s &= n_{zs}\left|\cos\alpha\right| + n_\rho \sin\alpha, \qquad \left(-\frac{\pi}{2}\le \arg(n_s)\le 0\right)\\ n_{zs}(n_\rho) &= \left(\frac{2\varepsilon_1\varepsilon_3-(\varepsilon_1+\varepsilon_3){n_\rho}^2+q_{s\pm}(n_\rho)}{2\varepsilon_3}\right)^{1/2}\\ q_{s\pm}(n_\rho) &= \pm\left((\varepsilon_1-\varepsilon_3)^2{n_\rho}^4-4{\varepsilon_2}^2\varepsilon_3{n_\rho}^2+4{\varepsilon_2}^2{\varepsilon_3}^2\right)^{1/2}\end{aligned} \tag{2}$$

Here α is the angle between the propagation direction and the imposed magnetic field, $q_{s\pm}$ serves as the discriminator of the two modes designated as "+" and "-" mode, respectively, and the values of n_ρ are the roots of the following equation:

$$n_\rho\left[-(\varepsilon_1+\varepsilon_3)q_s+(\varepsilon_1-\varepsilon_3)^2 n_\rho{}^2-2\varepsilon_2{}^2\varepsilon_3\right]\ \left|\cos\alpha\right|+2\varepsilon_3 n_{zs} q_s \sin\alpha=0 \qquad (3)$$

As pointed out in our previous paper, this nonlinear equation can be transformed to a sextic (sixth-order polynomial) equation, and the roots can be found with numerical methods. For a given direction, the refractive indices of spherical waves and plane waves are generally different except for the directions parallel and perpendicular to the imposed magnetic field. In this paper we show that, for a given direction α, the refractive index of spherical waves can be calculated with the refractive index of plane waves in another direction using the formula:

$$\begin{aligned} n_s(\alpha) &= \cos\delta\ n(\alpha-\delta) \\ \tan\delta &= -\frac{1}{n(\theta)}\frac{dn(\theta)}{d\theta}\bigg|_{\theta=\alpha-\delta} \end{aligned} \qquad (4)$$

Here $n(\theta)$ and $n(\alpha-\delta)$ are the values of the refractive index of plane waves in the direction θ and $(\alpha-\delta)$, respectively, δ is the deviation angle included by the phase and group velocities for plane waves. Since the refractive index of plane waves can be calculated analytically, equation (4) makes it possible to calculate the refractive index of spherical waves analytically. Meanwhile, the group refractive index surfaces for spherical and plane waves are identical and this means that in any direction the electromagnetic energy is flowing with the same velocity either for a progressive spherical or a plane wave.

It is important to note that for spherical waves the time-averaged energy flow is always in the radial direction, parallel to the wave normal. The directional agreement of the phase and group velocities of the spherical waves makes the anisotropic plasma look like an isotropic medium, and ray tracing in a magnetoplasma is reduced to the isotropic task of finding the ray position in the spherical coordinate system $\mathbf{r}=\mathbf{r}(\tau)$ from the ray equation valid for isotropic media [Born and Wolf, 1999]:

$$\frac{d}{d\tau}\left(n_s(\mathbf{r})\frac{d\mathbf{r}}{d\tau}\right)=\nabla n_s(\mathbf{r}) \qquad (5)$$

Here $n_s(\mathbf{r})$ is the refractive index of the spherical wave at a point **r** in space, and τ is the length along the ray. At a cut-off point where the index value is zero, and/or at a boundary where the index value is discontinuously changing, the ray equation is invalid and the classical Snell's law should be applied to find the ray direction after reflection.

Numerical ray tracing based on equation (5) has been tested for rays transmitted from the ground using the IRI model for the 3D ionospheric electron density distribution and the IGRF geomagnetic field. The results agree within the computational error range with the rays calculated from the Haselgrove's equation system. A few examples are shown in the paper. Equation (5) is much simpler and several program codes are available to achieve optimization and efficiency, for example, Software Ray Trace 3.0, written by Sam Buss, Department of Mathematics, University of California San Diego.

References

1. Born, M. and E. Wolf (1999), Principles of Optics: Electromagnetic Theory of Propagation, Interference and Diffraction of Lights, Cambridge University Press.

2. Budden, K.G. (1988), The Propagation of Radio Waves: The Theory of Radio Waves of Low Power in the Ionosphere and Magnetosphere, Cambridge University Press.

3. Haselgrove, J. (1955), Ray Theory and a New Method for Ray Tracing, London Physical Society Report of Conference on the Physics of the Ionosphere. pp. 355–364.

4. Huang, X. and B. Reinisch (2012), Excited spherical waves in unbounded cold magnetoplasma and applications in radio sounding, Radio Sci., 47, RS0L08, doi: 10.1029/2011RS004940.