Safety assessment of a transmission line with EMC requirements

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Abstract

This paper addresses the safety assessment of an ElectroMagnetic Compatibility (EMC) device, namely a transmission line. We assume that both the physical parameters of this transmission line and those of the illumination wave are uncertain. The objective is to assess the probability of occurrence of an extreme event defined in terms of requirements imposed on the current circulating in the line. The work is based on methods of the structural reliability research community. The efficiency and accuracy of the applied methods are compared with reference results obtained by simulations.

1. Problem statement

The ElectroMagnetic Compatibility (EMC) of complex systems within critical devices cannot rely on the sole use of deterministic approaches. Quantifying the effects of uncertainties on the overall system behavior becomes nowadays of paramount importance in regards to EMC standards and this field of research has gained a growing interest over the past few years [1,2]. The present work focuses on safety assessment in an EMC context. For the illustration purpose, we consider a Transmission Line (TL) of length L, diameter d and attenuation constant α, located at height h above an infinite ground plane and illuminated by a linearly polarized (direction of the electric field θ_e) plane wave at frequency f with a given incidence characterized by angles ϕ_p and θ_p and an electric field magnitude E_0 (see Fig.1). The current I measured at the output load impedance Z_L (assuming that Z_0 is the second load impedance) can be analytically approximated as a function of the defined parameters as follows (please refer to [2] for the function description):

$$I = R\big(L, h, d, Z_L, E_0, Z_0, \theta_e, \theta_p, \phi_p, f, \alpha\big), \tag{1}$$

where R is a known function from $\mathbb{R}^n$ into $\mathbb{C}$ with $n = 11$. We focus here on the magnitude $Y = |I|$ of I assuming that all the parameters in Eq. (1) are uncertain and modeled as mutually independent $\mathbb{R}$-valued continuous random variables (r.v.) denoted by $X_i,\ i = 1, \dots, n$. As a result, Y is a $(\mathbb{R}_+)$-valued r.v. such that $Y = r(\mathbf{X})$, where r is the function from $\mathbb{R}$ into $\mathbb{R}_+$ such that, $\forall\, \mathbf{x} \in \mathbb{R}^n,\ r(\mathbf{x}) = |R(\mathbf{x})|$ and $\mathbf{X} = (X_1, \dots, X_n)$ is the n-dimensional random vector which gathers the n X_i's r.v.. It is assumed that the probability density functions (pdf) f_{X_i} of the r.v. X_i are known for $i = 1, \dots, n$. Therefore, due to the independence of the X_i's, the joint pdf $f_{\mathbf{X}}$ of $\mathbf{X}$ is also known and is given by: $\forall\, \mathbf{x} = (x_1, \dots, x_n) \in \mathbb{R}^n,\ f_{\mathbf{X}}(\mathbf{x}) = f_{X_1}(x_1) \times \cdots \times f_{X_n}(x_n)$.

Let $\bar{y} \in \mathbb{R}_+^*$ be a threshold value of the magnitude of the current and E_f be the so-called *failure event* such that $E_f = \{\bar{y} - Y \le 0\} = \{\bar{y} - r(\mathbf{X}) \le 0\} = \{g(\mathbf{X}) \le 0\}$, where the introduced g function $\mathbf{x} \to g(\mathbf{x})$ from $\mathbb{R}^n$ into $\mathbb{R}$ is known as the *limit-state function*. The aim of this work is then to calculate, for a given value of $\bar{y}$, the probability that the random magnitude Y takes values greater than this threshold value, that is the probability $P_f = \mathbb{P}\big(E_f\big)$, known as the *failure probability* and given by the n-dimensional integral:

$$P_f = \int_{D_{fx}} f_{\mathbf{X}}(\mathbf{x})\, \mathrm{d}\mathbf{x} \ = \ \int_{\mathbb{R}^n} \mathbb{I}_{D_{fx}}(\mathbf{x})\, f_{\mathbf{X}}(\mathbf{x})\, \mathrm{d}\mathbf{x} \tag{2}$$

where $\mathbf{x} = (x_1, \dots, x_n),\ \mathrm{d}\mathbf{x} = \mathrm{d}x_1 \dots \mathrm{d}x_n$ and $\mathbb{I}_{D_f}$ is the indicator function of the set $D_{fx} = \{\mathbf{x} \in \mathbb{R}^n : g(\mathbf{x}) \le 0\}$, known as the *failure domain*. A natural strategy for estimating this type of integral is to resort to a crude Monte Carlo procedure. However, this solution is not efficient for low failure probabilities and the use of more efficient and advanced Monte Carlo methods is preferred. Another solution is to make simplifying assumptions about the failure domain (*i.e.* the integral domain), either by linearizing the limit-state function G or using a quadratic approximation. These three alternatives are briefly introduced in the next section.

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2. Problem solving

Formulation of the problem in the standard space. The problem defined in Eq. (2) is reformulated in terms of mutually independent standard Gaussian r.v.'s. Let $\mathbf{u} \to T(\mathbf{u})$ be the function from $\mathbb{R}^n$ into $\mathbb{R}^n$ defined as follows: $\mathbf{x} = T(\mathbf{u}) \Leftrightarrow \{x_i = T_i(u_i) = (F_{X_i}^{-1} \circ \Phi)(u_i),\ i = 1, \dots, n\}$, where F_{X_i} represents the cumulative density function (cdf) of X_i and Φ is the standard Gaussian cdf. Let $\mathbf{U} = (U_1, \dots, U_n)$ be a n-dimensional standard Gaussian random vector. Then, $\mathbf{X}$ and $\mathbf{U}$ are related by the equality in distribution: $\mathbf{X} = T(\mathbf{U})$ [3]. Using this mapping, the failure event takes the form $E_f = \{G(\mathbf{U}) \leq 0\}$ and the expression of P_f rewrites:

$$P_f = \int_{\mathbb{R}^n} \mathbb{I}_{D_{fu}}(\mathbf{u})\, \varphi_n(\mathbf{u})\, \mathrm{d}\mathbf{u} \tag{3}$$

where $\mathbf{u} = (u_1, \dots, u_n)$, $\mathrm{d}\mathbf{u} = \mathrm{d}u_1 \dots \mathrm{d}u_n$, φ_n is the n-dimensional standard Gaussian pdf (*i.e.* the joint pdf of the random vector $\mathbf{U}$), $D_{fu} = T^{-1}(D_{fx}) = \{\mathbf{u} \in \mathbb{R}^n : G(\mathbf{u}) \leq 0\}$ is the failure domain expressed in the $\mathbf{u}$-space (known as the *standard space*) and G is the function from $\mathbb{R}^n$ into $\mathbb{R}$ such that: $G = (G(\mathbf{u}) = (g \circ T)(\mathbf{u}),\ \mathbf{u} \in \mathbb{R}^n)$. Note that the expression (3) of P_f can be directly obtained by performing the change of variable $\mathbf{x} = T(\mathbf{u})$ in Eq. (2). It is worth mentioning that the calculation of P_f is carried out from Eq. (3) in most of the methods developed for structural reliability analysis. As mentioned above, three approaches are mainly used in the present context for estimating P_f: the first one is based on an advanced Monte Carlo method and the two others make use of simplifying assumptions applied to the failure domain D_{fu}, either a linearization of the limit-state function G or a quadratic approximation of G. The first approximation method is known as the *First-Order Reliability Method* (FORM) and the second one as the *Second-Order Reliability Method* (SORM). A basic description of these two methods is given below.

Resolution via a Monte Carlo method. Several Monte Carlo methods were developed for estimating the probability of Eq. (3): directional simulation, importance sampling techniques, see *eg.* [3-5]. The present work is based on a relatively new one due to Au & Beck [6] which has been proved to be computationally efficient in the context of rare failure events. This method known as *subset simulation* by its authors relies on the following idea. Let $m \geq 2$ be an integer and let $\{E_1, \dots, E_m\}$ be a m-family of events such that $E_m \subset E_{m-1} \subset \cdots \subset E_2 \subset E_1$, where $E_m = E_f$. It is easy to show that $P_f = \mathbb{P}(E_1) \prod_{s=1}^{m-1} \mathbb{P}(E_{s+1}|E_s)$ starting from $E_f = \bigcap_{s=1}^{m} E_s$ and using a classical conditioning technique. Thus, P_f appears as a product of m probabilities, each of them being necessarily greater than P_f and therefore easier to estimate. The method for judiciously constructing the E_s events, $s = 1, \dots, m$, which satisfy the above defined inclusion property can be found in [6]. This factorization of P_f is the first originality of the method. The second originality lies in the calculation of the conditional probabilities $\mathbb{P}(E_{s+1}|E_s), s = 1, \dots, m-1$, $\mathbb{P}(E_1)$ being estimated using a classical Monte Carlo method. Indeed, estimating these probabilities via a Monte Carlo procedure requires the simulation of a r.v. distributed according to the conditional distribution of $\mathbf{U}$ given E_s, for any $s \in \{1, \dots, m-1\}$, and this is performed by means of Markov Chain Monte Carlo (MCMC) in the subset simulation method. The major interest of this type of method is that it enables to bring the problem of simulating a r.v. with a given distribution ν to that of the simulation of an ergodic Markov chain whose stationary distribution is ν [4,5]. Such a method leads thus to recursive simulation algorithms (due to the Markovian character of the chain) which further have the advantage of being practically insensitive to the dimension of the target random vector (and therefore of the integral (3) in our case). The MCMC algorithm used in this work is the Metropolis-Hastings Algorithm modified by Au & Beck [6].

Resolution via the FORM approach. Let β be the positive coefficient defined as the Euclidean distance between the failure domain D_{fu} and the origin O in the standard space: $\beta = d(O, D_{fu})$. This coefficient known as the *(Hasofer-Lind) reliability index*, is such that $\beta = \|\mathbf{u}^*\|$, where $\mathbf{u}^*$ is called the *Most Probable failure Point*, (MPFP) and $\|\cdot\|$ denotes the Euclidean norm on $\mathbb{R}^n$. This point represents the element of the limit-state surface $S_{fu} = \{\mathbf{u} \in \mathbb{R}^n : G(\mathbf{u}) = 0\}$ given by the solution of the following optimization problem: $\mathbf{u}^* = \arg\min\{\|\mathbf{u}\| : \mathbf{u} \in \mathbb{R}^n, G(\mathbf{u}) = 0\}$. Let $G_1 = (G_1(\mathbf{u}) = < \nabla_{\mathbf{u}} G(\mathbf{u}^*)\,,\ \mathbf{u} - \mathbf{u}^* >,\ \mathbf{u} \in \mathbb{R}^n)$ be the linear approximation of G at the MPFP $\mathbf{u}^*$, obtained by approximating G by its first-order Taylor polynomial at $\mathbf{u}^*$, where $\nabla_{\mathbf{u}} G(.)$ denotes the gradient of G w.r.t. $\mathbf{u}$ and $<.,.>$ the canonical Euclidean inner product on $\mathbb{R}^n$. Let D_{fu1} be the subset of $\mathbb{R}^n$ such that $D_{fu1} = \{\mathbf{u} \in \mathbb{R}^n : G_1(\mathbf{u}) \leq 0\}$. The FORM approach then consists in approximating the unknown probability $P_f = \mathbb{P}(E_f) = \mathbb{P}(\mathbf{U} \in D_{fu})$ by $P_{f1} = \mathbb{P}(\mathbf{U} \in D_{fu1}) = \Phi(-\beta)$ where $\beta = < \boldsymbol{\alpha}\,,\ \mathbf{u}^* >$ and $\boldsymbol{\alpha} = -\nabla_{\mathbf{u}} G(\mathbf{u}^*)/\|\nabla_{\mathbf{u}} G(\mathbf{u}^*)\|$ [3,7].

Resolution via the SORM approach. Let $G_2 = (G_2(\mathbf{u}) = < \nabla_{\mathbf{u}} G(\mathbf{u}^*), \mathbf{u} - \mathbf{u}^* > + 1/2 < \nabla_{\mathbf{u}}^2 G(\mathbf{u}^*)(\mathbf{u} - \mathbf{u}^*)\,,\ \mathbf{u} - \mathbf{u}^* >\,,\ \mathbf{u} \in \mathbb{R}^n)$ be the quadratic approximation of G at the MPFP $\mathbf{u}^*$, obtained by approximating G by its second-order

Taylor polynomial at $\mathbf{u}^*$, where $\nabla_{\mathbf{u}}^2 G(.)$ denotes the Hessian of G w.r.t. $\mathbf{u}$. Let D_{fu2} be the subset of $\mathbb{R}^n$ such that $D_{fu2} = \{\mathbf{u} \in \mathbb{R}^n : G_2(\mathbf{u}) \leq 0\}$. The SORM approach then consists in approximating the unknown probability P_f by $P_{f2} = \mathbb{P}(\mathbf{U} \in D_{fu2})$, whose an asymptotic approximation is given by the Hohenbichler formula [3,7]: $P_{f2} \approx \Phi(-\beta) \prod_{i=1}^{n-1} \{1 + \kappa_i\, \varphi(\beta)/\Phi(-\beta)\}^{-1/2}$ $(\beta \to \infty)$, in which φ denotes the one-dimensional standard Gaussian pdf, β is the FORM reliability index and the κ_i's are the principal curvatures of the limit-state surface S_{fu} at the MPFP $\mathbf{u}^*$, assuming that $\kappa_i < \Phi(-\beta)/\varphi(\beta)$ for $i = 1, \dots, n-1$.

3. Numerical application

The types of distribution of the input variables and the distribution parameters used in the present application example are given in Tab. 1. The threshold value $\bar{y}$ of the magnitude Y is set to 10^{-4} A for ensuring a sufficiently low failure probability. The calculations were all performed using the open source code FERUM [8]. N = 500000 samples are used in the subset simulation analysis for assessing $\mathbb{P}(E_1)$ and each $\mathbb{P}(E_{s+1}|E_s)$ for $s \geq 1$. The estimated failure probability is $P_{f\mathrm{ref}} = 5.60 \times 10^{-5}$ with a coefficient of variation of 1.6%. This probability obtained by this advanced Monte Carlo method and with a high accuracy is considered as a reference result. The probability obtained by FORM is $P_{f1} = 2.33 \times 10^{-3}$, which is about two order of magnitude greater than the reference solution. This therefore clearly indicates that the FORM approach is inapplicable here. The SORM result is $P_{f2} = 2.93 \times 10^{-5}$. Despite a solution closer to the reference result $P_{f\mathrm{ref}}$, the SORM analysis is not able to capture the specific geometry of the limit-state surface S_{fu}. A detailed investigation of the problem geometry has been conducted. The objective has been to analyze the geometry of the limit-state surface S_{fu} in all possible (u_i, u_j)-cuts passing through the MPFP $\mathbf{u}^*$ obtained with the FORM analysis, for $i, j \in \{1, \dots, n\}$ and $i < j$. Some of these cuts are presented in Fig. 2. This analysis shows that S_{fu} is rather smooth for all pairs of random variables except those involving the r.v. representing the angle ϕ_p (9th r.v.). The limit-state surface appears very sharp for such cases as illustrated in the 3 bottom plots of Fig. 2. Such an intricate geometry of the limit-state surface in the **u**-space clearly explains why the FORM and SORM analyses fail to give an accurate estimate of the failure probability.

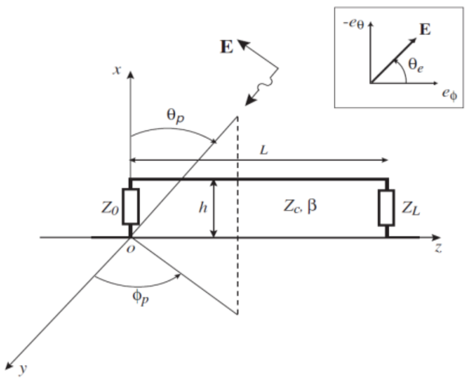

Fig. 1. Scheme of the Transmission Line [2]

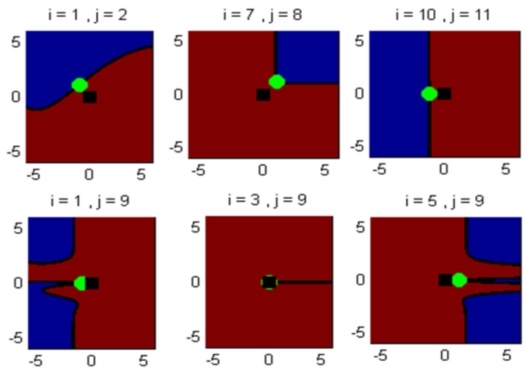

Fig. 2. Cutting planes at MPFP **u***

Tab. 1. Random variables and distribution parameters

Random variable	Type	Mean / Coef. of variation
L (m)	Lognormal	$\mu = 4.2$ / c.o.v. = 5%
h (m)	Lognormal	$\mu = 0.02$ / c.o.v. = 5%
d (m)	Lognormal	$\mu = 0.001$ / c.o.v. = 5%
Z_L (Ω)	Lognormal	$\mu = 1000$ / c.o.v = 5%
E_0 (V/m)	Lognormal	$\mu = 1$ / c.o.v. = 5%
Z_0 (Ω)	Lognormal	$\mu = 50$ / c.o.v. = 5%

Random variable	Type	Support
θ_p (rad)	Uniform	[0 ; $\pi/2$]
θ_e (rad)	Uniform	[0 ; $\pi/2$]
ϕ_p (rad)	Uniform	[0 ; 2π[
f (MHz)	Uniform	[25 ; 35]
α (−)	Uniform	[0.0005 ; 0.0015]

An alternative approach has been developed for solving the problem defined in Eq. (3). The idea is to apply a FORM then a SORM analysis conditionally on deterministic values of ϕ_p. The ϕ_p-values are set equally distributed over [0,2π[(see Fig. 3) and the failure probability is simply obtained by averaging out the conditional probabilities obtained by FORM or SORM. This procedure gave the following approximate values of P_f: $P_{f1;\,\mathrm{avg}\,\phi_p} = 3.2 \times 10^{-4}$ using FORM and $P_{f2;\,\mathrm{avg}\,\phi_p} = 5.0 \times 10^{-5}$ using SORM, which brought significant improvements in terms of accuracy in comparison with results initially obtained. The SORM result is now in close agreement with the reference value of 5.60×10^{-5}.

Importance factors are also interesting by-products in a FORM analysis. These factors represent the weight each r.v. has on the failure event. In the case of mutually independent r.v., these factors are simply obtained by squaring each component of the $\boldsymbol{\alpha}$ -vector defined in the FORM analysis and each α_i^2 -factor represents the contribution of the i^{th} variable in the unit variance of $G_1(\mathbf{U})/\|\nabla_{\mathbf{u}}G(\mathbf{u}^*)\|$. Fig.4 represents the evolution of these factors with ϕ_p values. Over the considered ϕ_p bandwidth, these factors allow us to rank the parameters from the most influential ones to those having almost no effects. A specific attention needs to be paid to $\phi_p = 0$ and $\phi_p = \pi/2$ cases according to Fig.4. The case $\phi_p = \pi/2$ is physically characterized by very low levels of coupling between the external plane wave and the TL. This implies weak effects of parameter θ_p (physically expected), Z_0 (mostly due to the gap existing with Z_L where the current I is measured) and the attenuation constant α (very low levels involving almost no losses over the TL). In comparison, the importance of other parameters is similar over ϕ_p bandwidth, except for the line length L. This may also be physically interpreted: since weak couplings with the TL are expected for $\phi_p = \pi/2$, the line length straightforward involves increase or decrease of coupled current at load impedance.

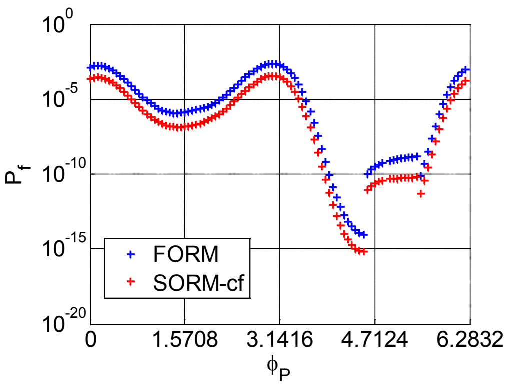

Fig. 3. Failure probability vs. ϕ_p (rad).

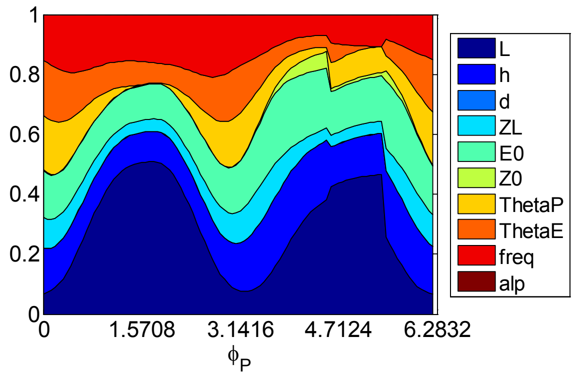

Fig. 4. Importance factors vs. ϕ_p (rad)

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