A Compact Fractional-order Model for Terahertz Composite Right/Left Handed Transmission Line

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Abstract

A compact fractional-order model is developed for CMOS on-chip Composite Right/Left Handed Transmission Line (CRLH-T-Line) at Terahertz. With consideration of loss from frequency-dependent dispersion, the compact model can provide higher accuracy of both characteristic impedance and prorogation constant when compared to the traditional integer-order model within the wide frequency range of 220GHz to 325GHz.

-1. Introduction-

The terahertz (THz) frequency region (0.1~10 THz) bridges the gap between electronics and photonics. THz can be potentially applied in short-range communication and imaging with large bandwidth and high detection sensitivity [1]. For example, there is recent trend in using THz-based imaging system due to its high sensitivity and penetrability to various human tissues without harmful radiation, which have successfully implemented in diagnosing for skin and breast cancer [2]. Meanwhile, thanks to the scale-down of CMOS process that has made it possible to realize whole THz system on chip-based platform including antenna. According to the International Technology Roadmap for Semiconductors (ITRS), the prospective cut-off frequency of CMOS transistors will be 0.9 THz in 2021.

CRLH T-Line is a well-know metamaterial structure [3][4][5] that can realize positive, negative or zero phase propagation. The modeling of on-chip CRLH transmission line (CRLH T-line) is under significant interest in assisting various designs [6]. The unit cell of one typical on-chip CRLH T-line is shown in Fig.1, and it is fabricated by 65nm CMOS process with 9 metal layers. Top aluminum layer (LB) is employed as signal layer to maximum distance to the ground layer (M1) to obtain the highest efficiency. An equivalent circuit of CRLH T-Line is shown in Fig.2(a), The L_p of the CRLH T-line is contributed by the microstrip line connected to the ground and C_s is implemented with inter-digital capacitor. L_s is the inductor derived from microstrip line, C_p is the parasitic capacitor between arm and ground layer as shown in Fig.1. Both shunt and series intrinsic losses are synthesized by G and R, respectively. However, such an integer-order model is inaccurate to describe the T-line performance at THz band because the loss expression is difficult to describe models of the dispersion loss and non-quasi-static effects [7]. The concept of fractional-order model has been examined by modeling capacitor (C) and inductor (L) more precisely at high frequency region. The I-V function of a capacitor is found to follow a fractional-order, so does the eddy current and hysteresis effect in inductors [8-9]. It inspires us to re-examine the CRLH T-line device characterization model at THz with measurement verifications at THz.

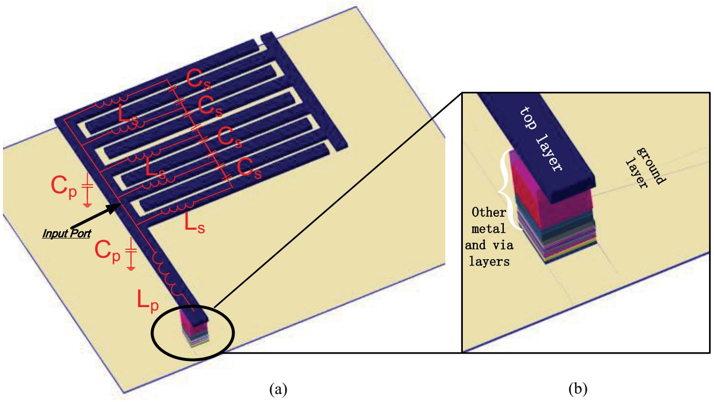

Fig.1. Structure of on-chip CRLH unit-cell.

In this paper, we have developed a fractional-order model for the periodic composite right/left handed (CRLH) transmission-line (T-line) structure at THz with the following advantages. Firstly, the fractional-order model can describe dispersion and non-quasi-static effect in THz. Moreover, the fractional-order model has a compact form that can be extracted from measurement results. The proposed factional-ordered CRLH T-line model is verified with S-parameter measurement results of CRLH T-line from 220 GHz to 325 GHz. Compared to the conventional integer-order model, the proposed fractional-order model demonstrates improved accuracy for both characteristic impedance and propagation constant. The rest of this paper is organized as follows. Section 2 briefly reviews the theory of fractional model, and the application to the fractional-order CRLH T-line model. The measurement results of fractional-order CRLH T-line are presented in Section 3. The paper is concluded in Section 4.

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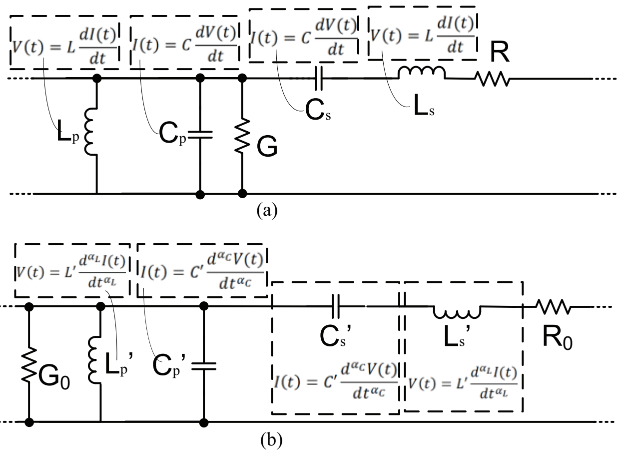


Fig.2. CRLH unit-cell equivalent circuits: (a) integer-order model; and (b) fractional-order model.

-2. FRACTIONAL-ORDER CRLH TRANSMISSION LINE MODEL-

2.1. Fractional-order Capacitance and Inductance Models

In this paper, we show that fractional-order model for T-line can be built by introducing fractional-order terms in the conventional RLGC model as shown in Fig. 1(b). The admittance and impedance of fractional-order capacitor and inductor can be obtained from the [10] I-V relation by

$$Y'(\omega) = \omega^{\alpha_C} C' e^{\frac{j\alpha_C \pi}{2}} \tag{1}$$

$$Z'(\omega) = \omega^{\alpha_L} L' e^{\frac{j\alpha_L \pi}{2}} \tag{2}$$

where C' is the fractional capacitance with order α_C, and $\alpha_C \in (0,1]$ is the fractional-order relating to the loss of capacitor and L' is the fractional inductance with order α_L, and $\alpha_L \in (0,1]$ is the fractional-order relating to the loss of inductor.

When α_L *or* $\alpha_C \neq 1$, we can expect the existence of real-parts at the right-hand sides of (1) and (2), which represent the frequency-dependent loss. Physically in a particular device, the fractional-order operator indicates the transfer of the energy storage to energy loss. As such, the distributed frequency-dependent terms are considered by L' and C' elements in fractional-order terms.

2.2. Fractional-order CRLH T-line Model

Note that the fractional-order T-line can be analyzed in a similar fashion as to the traditional T-line. The characteristic impedance (Z_0) of T-line can be found by $\sqrt{Z/Y}$, where Z and Y are the series impedance and shunt admittance, respectively. Based on (1) and (2) with consideration of resistance R_0 and conductance G_0, one can have

$$Z_0 = \sqrt{(R_0 + \omega^{\alpha_{Ls}} L_s' e^{\frac{j\alpha_{Ls}\pi}{2}} + 1/\omega^{\alpha_{Cs}} C_s' e^{\frac{j\alpha_{Cs}\pi}{2}}) / (G_0 + \omega^{\alpha_{Cp}} C_p' e^{\frac{j\alpha_{Cp}\pi}{2}} + 1/\omega^{\alpha_{Lp}} L_p' e^{\frac{j\alpha_{Lp}\pi}{2}})} \tag{3}$$

From the equivalent circuit of CRLH T-line, we find that each cell consist parallel and serial LC resonators, and there is a gap between parallel resonate frequency (ω_p)and serial resonate frequency (ω_s). When $\omega = \omega_p$, Z_0 reach maximum value, when $\omega = \omega_s$, Z_0 reach its minimum value, thus Z_0 shows a peek-valley or valley-peek curve as frequency grows within the whole range. This is verified by simulation and measurement results shown in Fig.4(b). According to (3), real terms of serial resonator and parallel conductance are expressed as (4) and (5), respectively.

$$\Re(Z) = R_0 + \omega^{\alpha_{Ls}} \bullet L_s' \bullet cos\frac{\alpha_{Ls}\pi}{2} + \frac{1}{\omega^{\alpha_{Cs}} \bullet C_s'} \bullet cos\frac{\alpha_{Cs}\pi}{2} \tag{4}$$

$$\Re(Y) = G_0 + \omega^{\alpha_{Cp}} \bullet C_p' \bullet cos\frac{\alpha_{Cp}\pi}{2} + \frac{1}{\omega^{\alpha_{Cs}} \bullet C_s'} \bullet cos\frac{\alpha_{Lp}\pi}{2} \tag{5}$$

From (4) and (5), we observe that, different from conventional integer order model, fractional model introduces frequency-depended terms to loss and conductance equations that affect a lot in terms of Z_0's peak and valley magnitude.

In THz frequency region, ω is in the order of 10^{11}~10^{13}. We have $R_0 \ll \omega^{\alpha_{Ls}} L' e^{\frac{j\alpha_{Ls}\pi}{2}}$, also $G_0 \ll \omega^{\alpha_{Cp}} C' e^{\frac{j\alpha_{Cp}\pi}{2}}$. At a frequency that much lower than zero-phase-shift frequency (around 280GHz), terms $1/\omega^{\alpha_{Cs}} C_1' e^{\frac{j\alpha_{Cs}\pi}{2}}$ and $1/\omega^{\alpha_{Lp}} L_2' e^{\frac{j\alpha_{Lp}\pi}{2}}$ are dominant, much larger than $\omega^{\alpha_{Ls}} L' e^{\frac{j\alpha_{Ls}\pi}{2}}$ and $\omega^{\alpha_{Cp}} C' e^{\frac{j\alpha_{Cp}\pi}{2}}$ as well as R_0 and G_0. Thus Z_0 can be approximated as

$$Z_0 = \sqrt{L_p'/C_s'} \bullet \omega^{\frac{\alpha_{Lp}-\alpha_{Cs}}{2}} \bullet \left[cos\frac{(\alpha_{Lp}-\alpha_{Cs})\pi}{4} + jsin\frac{(\alpha_{Lp}-\alpha_{Cs})\pi}{4} \right]. \tag{6}$$

In the high frequency region, which is far above 280GHz, $1/\omega^{\alpha_{Cs}} C_1' e^{\frac{j\alpha_{Cs}\pi}{2}}$ and $1/\omega^{\alpha_{Lp}} L_2' e^{\frac{j\alpha_{Lp}\pi}{2}}$ are small. In the contrast, $\omega^{\alpha_{Ls}} L' e^{\frac{j\alpha_{Ls}\pi}{2}}$ and $\omega^{\alpha_{Cp}} C' e^{\frac{j\alpha_{Cp}\pi}{2}}$ contribute most part of Z_0. So (3) can be approximated as

$$Z_0 = \sqrt{L_s'/C_p'} \bullet \omega^{\frac{\alpha_{Ls}-\alpha_{Cp}}{2}} \bullet \left[cos\frac{(\alpha_{Ls}-\alpha_{Cp})\pi}{4} + jsin\frac{(\alpha_{Ls}-\alpha_{Cp})\pi}{4} \right] \tag{7}$$

Thus, we can see that Z_0 is a frequency-depended parameter, determined by $\omega^{\frac{\alpha_{Lp}-\alpha_{Cs}}{2}}$ and $\omega^{\frac{\alpha_{Ls}-\alpha_{Cp}}{2}}$ respectively. Moreover, for this CRLH T-line design, α_L is regularly smaller than α_C, due to loss of inductor in microstrip lines is larger than capacitor leakage. Thus at lower frequency and higher frequency ends, Z_0 goes down when ω grows.

In terms of propagation constant, $\gamma = \alpha + j\beta$, where α is the attenuation constant and β is the phase constant. Since $R_0 \ll \omega^{\alpha_{Ls}} L' e^{\frac{j\alpha_{Ls}\pi}{2}}$ and $G_0 \ll \omega^{\alpha_{Cp}} C' e^{\frac{j\alpha_{Cp}\pi}{2}}$ in THz frequency region, with (1) and (2), one can have $\gamma = \sqrt{ZY}$ as

$$\gamma = \sqrt{\left(\omega^{\alpha_{Ls}} L_s' e^{\frac{j\alpha_{Ls}\pi}{2}} + 1/\omega^{\alpha_{Cs}} C_s' e^{\frac{j\alpha_{Cs}\pi}{2}}\right)\left(\omega^{\alpha_{Cp}} C_p' e^{\frac{j\alpha_{Cp}\pi}{2}} + 1/\omega^{\alpha_{Lp}} L_p' e^{\frac{j\alpha_{Lp}\pi}{2}}\right)} \tag{8}$$

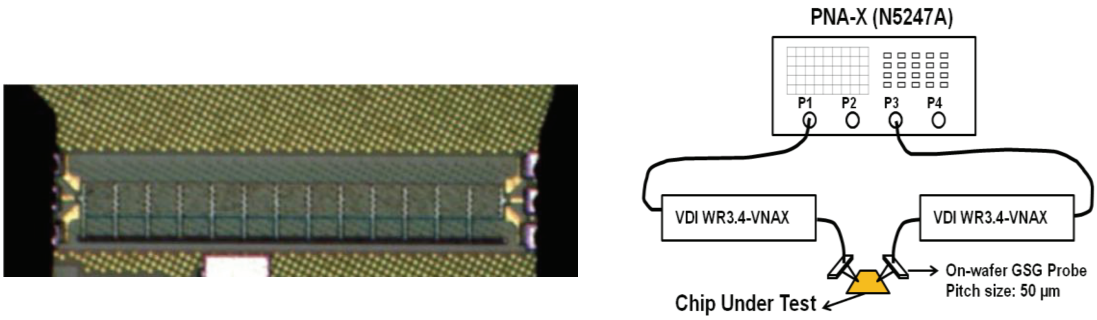
Eq. (8) can be simplified as following if the fractionality for both inductance and capacitance are all constants that $\alpha_{Cp} - \alpha_{Cs} = 0$ and $\alpha_{Lp} - \alpha_{Lp} = 0$,

$$\gamma = \sqrt{\omega^{\alpha_{Ls}+\alpha_{Cp}} \bullet L_s' C_p' \bullet e^{\frac{j\pi(\alpha_{Ls}+\alpha_{Cp})}{2}} + \frac{1}{\omega^{\alpha_{Cs}+\alpha_{Lp}} \bullet C_s' L_p'} \bullet e^{-\frac{j\pi(\alpha_{Cs}+\alpha_{Lp})}{2}}} \tag{9}$$

The zero-phase-shift frequency (ω_0) can be obtained from (9) when $\beta = 0$ that Eq. (9) can be simplified when where $\alpha_{Cs} + \alpha_{Lp}$ and $\alpha_{Ls} + \alpha_{Cp}$ are approaching 2.

$$\omega_0^{\alpha_{Ls}+\alpha_{Cp}+\alpha_{Cs}+\alpha_{Lp}} \bullet C_S' L_P' L_S' C_P' = \frac{\alpha_{Cs}+\alpha_{Lp}}{\alpha_{Ls}+\alpha_{Cp}}. \tag{10}$$

Eq. (10) reveals an exponential relationship between all fractional parameters. This equation can be used in fitting process to reduce parameter number to make the work easier.



(a) **(b)**

Fig.3. (a) Chip micrograph of fabricated CRLH T-line; (b) Measurement setup of on-wafer S-parameter testing up to 325GHz.

-3. MEASUREMENTS-

3.1. Circuit and Layout Implementation

In order to minimize the characterization error and to improve radiation efficiency for each unit cell, one 13-cell CRLH T-line is fulfilled in Global Foundry with 65nm CMOS process. As shown in Fig.3(a), it has a chip size of 145μm x 660μm excluding the RF Pads. The L_p of the CRLH T-line is synthesized by the microstrip line connected to the ground and C_s is implemented with inter-digital capacitor. Both right-handed elements L_s and C_p are contributed by the intrinsic parasitic.

3.2. Circuit Simulation and measurement results

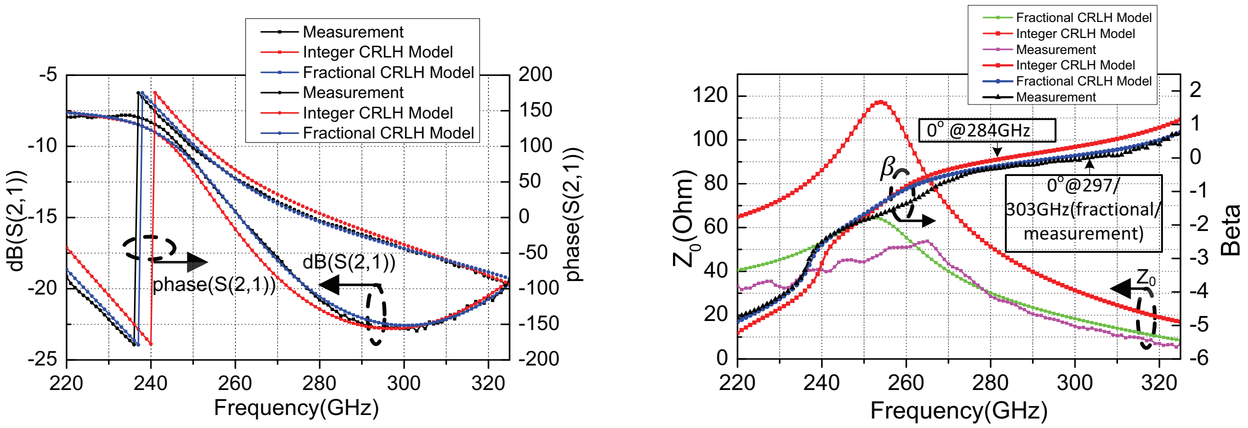
The 13-cell CRLH T-line design is verified by circuit simulation in ADS from 220 to 325GHz. From this we get the integer-order and fractional-order simulation results. The fabricated 13-cell CRLH T-line structure is measured on probe station (CASCADE Microtech Elite-300) with VNA extender (VDI WR3.4-VNAX). Two waveguide GSG probes with 50μm pitch are used for the S-parameter measurement from 220 to 325GHz, as shown in Fig.3(b). Note that the testing pads and traces are deembedded (open, short) from both sides with recursive modeling technique [11]. We also compare the measurement results with integer-order circuit simulation and with fractional-order circuit simulation for CRLH T-line. The circuit simulation is conducted with the equivalent circuit of unit cell shown in Fig.1, and the value of each circuit elements are listed in TABLE I, obtained by curve fitting technique.

TABLE I

MODELING PARAMETERS OF INTEGER-ORDER and FRACTIONAL-ORDER MODEL FOR CRLH T-LINE

INTEGER-ORDER MODEL			FRACTIONAL-ORDER MODEL		
PARAMETER	VALUE	UNIT	PARAMETER	VALUE	UNIT
L_S	15.6	pH	L'_S	14	pH
C_S	14.7	fF	C'_S	1407.8	fF
G	770	Ω	$\alpha_{Ls}/\alpha_{Lp}/\alpha_{Cs}/\alpha_{Cp}$	0.9847/0.9766/0.9939/0.9973	-
R	2.8	Ω	L'_p	39.41	pH
C_p	13.8	fF	C'_p	1732.1	fF
L_p	28.3	pH	R_0/G_0	0.3396/902	Ω

As shown in Fig.4, the phase and magnitude of S21are almost identical for both fractional-order and integer-order models in the measured frequency range of 220-325GHz. But the extracted phase constant (β) of fractional-order model is closer to measurement than that of integer-order one while considering the dispersion effects. More importantly, fractional-order model accurately fits the measurement results at the frequency with β=0, which is the boundary between left-handed and right-handed regions, while that from integer-order model is 13GHz less. Moreover, Fig.4(b) shows a remarkable difference between integer-order and fractional-order results in terms of characteristic impedance Z_0. The measurement Z_0 fit very well to fractional-order model at zero-phase-shift region from 260GHz, also a smaller error of Z_0 at low frequency region compared to integer-order fitting result. The average accuracy improvement of 78.8% is obtained by fractional-order model compared to the integer-order counterpart with correlated measurement and simulation results of Z_0.



(a) (b)

Fig.4. Verification of fractional-order T-line model with measurement and integer-order results: (a) magnitude in dB and phase delay of S21 (b) the phase constant β results Z_0

-4. Conclusion-

For the first time, a compact fractional-order model of composite right/left handed transmission line (CRLH T-line) is developed in this paper. As verified by the measurement (correlated with the circuit simulation) from 220 to 325GHz, the accuracy of the characteristic impedance Z_0 by the fractional-order model is 78.8% improved on average when compared to the conventional integer-order model.

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