

# A Novel Iterative Symbol Detection of OFDM Systems in Time and Frequency Selective Channels

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## Abstract

When orthogonal frequency division multiplexing (OFDM) technology is applied in time- and frequency-selective, or doubly selective channels, the time-variant character of the channel destroys subcarrier orthogonality, resulting in inter-carrier interference (ICI). In general, the classical OFDM detection algorithms are neither effective to mitigate the ICI due to the high error floor in BER curves, nor applicable to the practice due to the high computational complexity encountered in the large-sized matrix inversion. To resolve this problem, an iterative symbol detection scheme, combined with linear preprocessing at the receiver to restrict ICI support and band approximation of the channel matrix is proposed. The simulation results demonstrate that the proposed algorithm achieves enhanced performances with lower complexities compared to the classical methods.

## 1. Introduction

When OFDM transmission is applied in scenarios with high mobility and high carrier frequencies, the time variance of channel can destroy the subcarrier orthogonality, resulting in the inter-carrier interference (ICI) [1]. To resolve this problem, several approaches have been developed to suppress the ICI [2-6]. A linear MMSE scheme with a successive interference cancellation method were developed in [2]. Although effective but the method has a high complexity of more than  $O(N^3)$ , where the  $N$  is the number of the subcarriers. In [3], a two stage detection method was proposed, where a time domain windowing is launched in the first stage followed by a iterative MMSE estimation. Due to the band approximation, the algorithm has a complexity of  $O(N)$ . In [4], a block MMSE equalization is developed combined with band approximation and LDL<sup>H</sup> matrix inversion to achieve a complexity of  $O(N)$ . In [5], a decision feedback equalization based on MMSE was proposed. To reduce the complexity, the algorithm also resorted to band approximation. In [6], a sequential detection method based on the neighboring symbol search and band approximation was proposed. All the methods mentioned above are neither effective to mitigate the ICI, nor applicable to the practice due to the high computational complexity.

In this paper, we propose a novel iterative OFDM symbol detection algorithm based on the time domain windowing [3], band approximation [4] and neighboring search [6]. time domain windowing is to limit the ICI in a small region, so the band approximation is more accurate. Different from [3], we use exponential based window to allow a white windowed noise. And then we use the neighboring search to improve the detection performance. Therefore, the algorithm can yield further enhanced performance, while retaining a low complexity of  $O(N)$ .

## 2. System Structure and Model

An equivalent baseband model of OFDM system with  $N$  subcarriers and a  $L_c$ -length cyclic prefix (CP) is considered, as illustrated in Fig. 1. The transmitter contains  $N$ -point IFFT operation, CP insertion, and parallel to serial conversion. After distortion of the linear doubly selective channel and discarding the CP at the receiver, the inter-symbol interference (ISI) can be mitigated.

The relationship between the time domain receiving signal  $\mathbf{y}_t$  and the transmitted symbols  $\mathbf{x}$  can be expressed as  $\mathbf{y}_t = \mathbf{H}_t \mathbf{F}_N \mathbf{x} + \mathbf{z}_t$ , where  $\mathbf{H}_t$  is the “pseudo-circulant” time domain channel matrix,  $\mathbf{F}_N$  is the unitary  $N$ -point IDFT

matrix,  $\mathbf{z}_t$  is the white Gaussian noise. After the FFT operation, the frequency-domain receiving signal  $\mathbf{y}_f$  can be expressed by  $\mathbf{y}_f = \mathbf{F}_N^H \mathbf{y}_t = \mathbf{F}_N^H \mathbf{H}_t \mathbf{F}_N \mathbf{x} + \mathbf{F}_N^H \mathbf{z}_t = \mathbf{H}_f \mathbf{x} + \mathbf{z}_f$ , where  $\mathbf{H}_f$  is channel frequency response matrix.

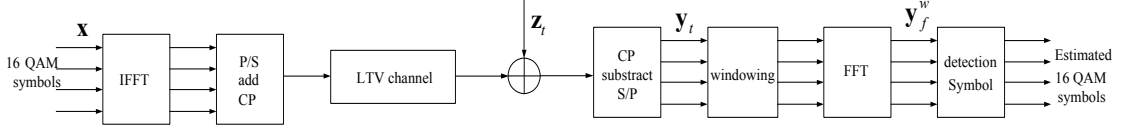


Figure 1. Block diagram of OFDM system

When the multipath channel is time-invariant,  $\mathbf{H}_f$  is a diagonal matrix, and the single-tap equalization can be simply launched with complexity of  $O(N)$ . Nevertheless, when the channel is time-variant, the off-diagonal elements of  $\mathbf{H}_f$  are not zero anymore, and ICI comes up as follows:

$$y_f(n) = H_f(n, n)x(n) + \sum_{k=0, k \neq n}^{N-1} H_f(n, k)x(k) + z_f(n) = H_f(n, n)x(n) + ICI_n + z_f(n) \quad (1)$$

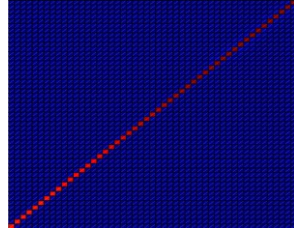
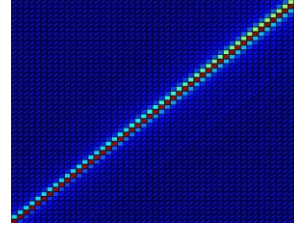


Figure 2. (a) the power of ICI for  $f_d$  0.001



(b) the power of ICI for  $f_d$  0.001

Fig. 2 represents part of  $\mathbf{H}_f$  and shows how the energy leaks out to the off-diagonal elements of  $\mathbf{H}_f$  as the Doppler frequency increases, giving rise to ICI. Here, TU6 channel has been considered, high power is represented by lighter color. Note that in Fig. 2 (b), one of the characters of the ICI power distribution is the obvious centralization in the diagonal area. So in order to reduce the complexity of the equalization, band matrix approximation can be made to the  $\mathbf{H}_f$  by assuming some terms,  $H_f(m, n)$ ,  $|m - n| > p$ , to be zero. But neglecting the elements away from the diagonal will degrade the detection performance due to the approximation error, especially in the fast time-variant channels. To improve the detection performance after the band approximation, as in the Fig. 1, time-domain windowing is utilized to squeeze the ICI power further around the diagonal area before FFT operation. The windowed of the FFT result  $\mathbf{y}_f^w$  can be expressed by  $\mathbf{y}_f^w = \mathbf{F}_N^H \text{diag}\{\mathbf{w}\} \mathbf{y}_t = \mathbf{F}_N^H \text{diag}\{\mathbf{w}\} \mathbf{H}_t \mathbf{F}_N \mathbf{x} + \mathbf{F}_N^H \text{diag}\{\mathbf{w}\} \mathbf{z}_t = \mathbf{H}_f^w \mathbf{x} + \mathbf{z}_f^w$  where the vector  $\mathbf{w}$  is the discrete window function, and  $\text{diag}\{\mathbf{w}\}$  is the corresponding diagonal matrix.  $\mathbf{H}_f^w = \mathbf{F}_N^H \text{diag}\{\mathbf{w}\} \mathbf{H}_t \mathbf{F}_N$  is the windowed channel frequency response matrix. It is expected that the band approximation of  $\mathbf{H}_f^w$  is more accurate than that of the  $\mathbf{H}_f$ .

### 3. Symbol Detection Schemes

In this section, we describe the OFDM symbol estimation schemes under the condition of the aforementioned time domain windowing operation and band approximation. In mobile circumstances, time-selective fading destroys the subcarrier orthogonality, resulting in ICI. The ICI leads to an error floor in BER, increasing with the normalized Doppler frequency, which is the product of the Doppler frequency and the symbol block duration.

Before introducing the proposed iterative detection scheme, we first define the band approximation of  $\mathbf{H}_f^w$  to be  $B_p(\mathbf{H}_f^w)$

$$[B_p(\mathbf{H}_f^w)]_{m,n} = \begin{cases} [\mathbf{H}_f^w]_{m,n}, & |m-n| \leq p \\ 0 & |m-n| > p \end{cases} \quad (1)$$

where the parameter  $2p$  denoted the number of dominant ICI terms against the  $m$ th desired subcarrier. And the error metric of the  $n$ th symbol is defined as

$$\xi_n = \left| y_f^w(n) - \sum_{k=(n-p)_{\text{mod } N}}^{(n+p)_{\text{mod } N}} [B_p(\mathbf{H}_f^w)]_{n,k} \hat{x}(k) \right|^2 \quad (2)$$

where  $\hat{x}(k)$  is the estimation of the symbol at the  $k$ th subcarrier.

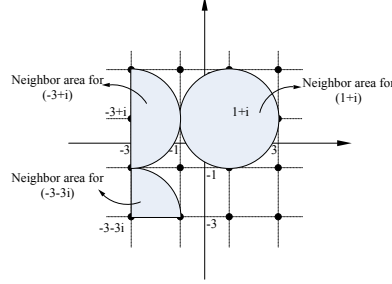


Figure 3. Neighbor areas of symbols in 16QAM constellation

Finally the neighbor area of a 16QAM symbol is defined as the set of points in constellation closest to it. As demonstrated in Fig. 3, there are three kinds of the neighbor area (the shadowed area in the figure): for the four central points, the neighbor area is a round with central on it; for the point of  $(3+3i)$ ,  $(3-3i)$ ,  $(-3+3i)$  and  $(-3-3i)$ , the corresponding neighbor area is a quarter-round; for the other points, the corresponding neighbor area is a half-round.

An efficient iterative OFDM symbol detection algorithm is proposed to minimize the cumulated error metrics,

$\xi = \sum_{n=0}^{N-1} \xi_n$ . The detailed processing of the algorithm is described as follows:

**STEP 1.** Obtain initial symbol decisions by using LMMSE estimation

$$\hat{\mathbf{X}}_{MMSE} = Q([B_p(\mathbf{H}_f^w)]^H B_p(\mathbf{H}_f^w) + \sigma_z^2 \sigma_s^2 \mathbf{I}_N)^{-1} B_p(\mathbf{H}_f^w)^H \mathbf{Y} \quad (4)$$

where  $Q(\cdot)$  is the decision function based on the minimum Euclidean distance principle.

**STEP 2.** Calculate the error metric  $\xi_n^0$ ,  $n = 0, \dots, N-1$ , based on the initial  $\hat{\mathbf{X}}_{MMSE}$

**STEP 3.** Start to search neighboring symbols in the constellation and update as follows. for  $n = 0, 1, \dots, N-1$

Search the neighboring symbols  $\{\hat{x}_n^s\}$ ,  $1 \leq s \leq S_n$ , of  $\hat{x}_{MMSE}$  in the constellation, where  $S_n$  is the number of neighboring candidate symbols,  $\{\hat{x}_n^s\}$ . Calculate the error metric  $\xi_n^s$  using the searched candidate symbols  $\{\hat{x}_n^s\}$ . If there exists  $\xi_n^s$  such that  $\min_{1 \leq s \leq S_n} \xi_n^s < \xi_n^0$ , update  $\hat{x}_{MMSE} = \hat{x}_n^s$ .

It can be seen that the computational complexity of step 2 and step 3 is  $O(N)$ . Resorting to the LDLH band matrix decomposition technique, the complexity of LMMSE equalization in step 1 is  $O(N)$ , so the overall complexity of the algorithm is  $O(N)$ , while the conventional ZF and LMMSE has a complexity of  $O(N^3)$ .

## 4. Simulation Results

In this section, we evaluate the performance of the proposed detection algorithm over doubly selective fading channels. Throughout the simulations, we assume that the receiver has perfect channel estimation and synchronization. A Gray-coded 16-QAM modulation, and 64 subcarriers with 500 kHz bandwidth is used. The OFDM symbol duration

is 134 us, with the guard interval to be 6 us, which is larger than the maximum channel delay 4 us. Jake' model is used for the 4-ray i.i.d. Rayleigh fading channel, with equal power and delays of 0, 1, 2 and 4 us. The considered normalized Doppler frequencies are 0.1 and 0.01 for the severe doubly selective channels.

The BER performance of four detection schemes are compared: the existing ZF detectors, LMMSE detectors, the block MMSE equalization based on band approximation and the proposed iterative algorithm.

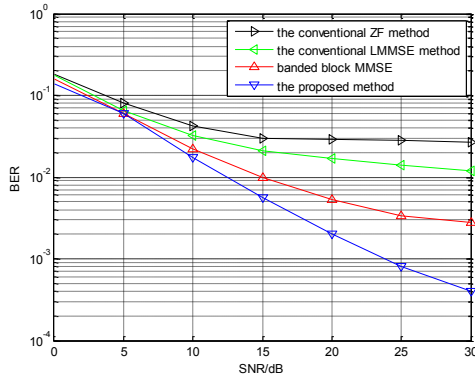


Figure 4. BER varying with SNR for  $f_d=0.01$

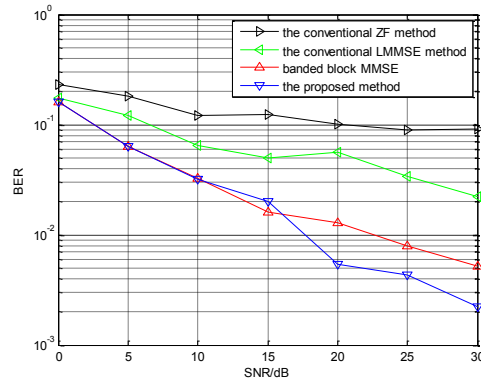


Figure 5. BER varying with SNR for  $f_d=0.1$

Figure 4 and Figure 5 depicts the performance of algorithms when normalized Doppler frequency to be 0.01 and 0.1 respectively. The conventional ZF and MMSE equalization has a high error floor when SNR larger than 15 dB. And the proposed algorithm is better than the banded block MMSE algorithm with a 3 dB gain at 10<sup>-2</sup> BER level.

## 5. Conclusion

To suppress the ICI produced by time-frequency selective fading channels and more precisely estimate received symbols, a novel iterative detection scheme has been proposed. Compared to the existing methods, the proposed algorithm exhibit performance improvement at a low complexity of  $O(N)$ , which is validated by computer simulations.

## 6. Acknowledgments

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## 7. References

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