EFIE with a Novel Perturbed ILUT Preconditioner for Electromagnetic Scattering by Conducting Objects in Half Space

Lan-Wei Guo[1], Joshua Le-Wei Li[1,2], Yongpin P. Chen[1], and Jun Hu[1]

[1]School of Electronic Engineering and Institute of Electromagnetics,
University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China;
[2]Advanced Engineering Platform and School of Engineering, Monash University, Sunway, Selangor 46150, Malaysia
lanwei407@gmail.com;lwli@ieee.org;yongpinchen@gmail.com;hujun@uestc.edu.cn

Abstract

In this paper, a highly efficient and robust scheme is proposed to analyzing electromagnetic (EM) scattering of arbitrary shaped perfect electrically conducting (PEC) objects in half space. The electric field integral equation (EFIE) with the half-space Green's function is chosen as the basis of this method due to its great versatility and accuracy. A perturbed dual threshold incomplete LU factorization (ILUT) preconditioner based on the near-field interactions in the improved EFIE (IEFIE) operator is proposed as an effective preconditioner. It will be further shown that such preconditioner enhances the stability of the traditional ILUT preconditioner based on EFIE, which may occasionally encounter problems of instability for real-life problems. Numerical examples are presented to demonstrate the high efficiency and robustness of this method over other existing alternative solutions.

1. Introduction

Surface integral equation (SIE) is one of the most powerful full wave method in the simulation of electromagnetic radiation and scattering problem. One popular approach is to formulate the surface integral equation (SIE) with the kernel of half-space Green's function to make the unknowns involved only relate to the boundary of the object [1]. The integral equation thus formulated is then converted further into a matrix equation by the method of moments (MoM) [2].

Generally speaking, there are two kind of Fredholm integral operator in Surface integral equation method, one is the first-kind Fredholm integral operator, and the other is second-kind Fredholm integral operator. The electric field integral equation (EFIE), which is a first-kind Fredholm integral equation, has the spectrum clustered around zero and at infinity; the magnetic-field integral equation (MFIE) on the other hand, is a second-kind Fredholm integral operator, which has the spectrum clustered around zero [3].

The electric field integral equation (EFIE) is commonly adopted for analyzing electromagnetic scattering of arbitrary shaped objects due to its great accuracy and versatility. However, the conditioning of the matrix resulted from the discretization of the EFIE is usually poor, especially for electrically large problems. If an iterative solver is applied to solve the system, it may be very inefficient (though the matrix-vector product in each iteration is accelerated by p-FFT [4]), since the iteration may converge very slowly or even diverge in real-life problems. To gain some convergence speed-up, the magnetic field integral equation (MFIE) is also adopted to form a well-conditioned combined field integral equation (CFIE) in free space, at the expense of losing some versatility (not easy applicable for open surfaces or sharp tips for instance). In half space, however, this expense could be even higher due to the complexity of the half-space Green's function---much more components of the dyadic Green's function are required in the magnetic-type integral operators [5]. To overcome this difficulty, a well-conditioned improved EFIE (referred to as IEFIE) developed in [6-7] can be extended to our cases. In this method, the principle value term of the MFIE operator is added into the EFIE operator in an iterative way, thus avoiding the calculation of the magnetic-type Green's functions. However, we found that in some real-life problems, the performance of the IEFIE is not stable and the accuracy cannot be guaranteed.

In this paper, we will construct a perturbed incomplete ILU perconditioner [8] based on the idea of IEFIE for an efficient solution of EFIE with a half-space kernel. A near-field matrix $\mathbf{Z}_{\text{near}}$ in the IEFIE operator, which is implemented as a diagonal perturbation, is used as the building blocks for constructing a dual threshold incomplete LU factorization preconditioner for the EFIE operator. This method makes the LU factors more diagonally dominant, which increases the robustness of ILU system [9]. It should be noted that a similar idea was proposed in [10] for free space

978-1-4673-5225-3/14/$31.00 ©2014 IEEE

problem, however, we would like to emphasize that the method benefits more when extended to a half space problem. By avoiding the use of magnetic-type Green's functions in half space, the method not only reduces the number of *Sommerfeld* integrals in the near field part, but also reduces the number of FFTs in the far field computation. Exhaustive numerical tests will be presented to validate our theoretical and numerical results.

2. Formulation

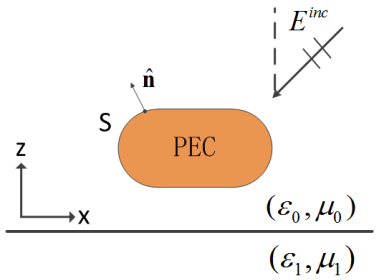


Fig. 1 The computational domain is divided into two sub-domains

Consider a 3-D PEC object of arbitrary shape located above a half space, as is shown in Fig.1. The background media is lossy half space with complex permittivity, and permeability denoted with ε, μ respectively .Throughout the following derivations the time-harmonic field variation $e^{j\omega t}$ is assumed and suppressed.

By imposing the PEC boundary condition for electric field and magnetic field, the EFIE and MFIE can be derived as

$$\bar{\mathrm{L}}(\mathrm{J}) = -\hat{n}\times E^{inc}, \quad \text{and} \quad \left[-\frac{\bar{\mathrm{I}}}{2}+\bar{\mathrm{K}}\right](\mathrm{J}) = -\hat{n}\times H^{inc} \tag{1}$$

$\bar{\mathrm{I}}$ denotes the unit tensor or identity matrix, and the operators $\bar{\mathrm{L}}$ and $\bar{\mathrm{K}}$, are defined as

$$\bar{\mathrm{L}}(\bullet) = -jw\mu_0\hat{n}\times\int dr'\bar{G}^{AJ}(r,r')\cdot(\bullet) + \frac{\nabla}{jw\varepsilon_0}\hat{n}\times\int dr' G^{VJ}(r,r')\nabla'\cdot(\bullet) \tag{2}$$

$$\bar{\mathrm{K}}(\bullet) = \hat{n}\times P.V.\int dr'\bar{G}^{HJ}(r,r')\cdot(\bullet) \tag{3}$$

Where $\bar{G}^{AJ}$, G^{VJ} and $\bar{G}^{HJ}$ are the half-space dyadic (scalar) Green's functions [11,12]. *P.V.* stands for the Cauchy principal value integration. In analogy with the CFIE, the IEFIE is constructed by adding the identity operator in EFIE [12]:

$$\hat{n}\times\bar{\mathrm{L}}(\mathrm{J}) + \alpha\frac{\eta_0}{2}\bar{\mathrm{I}}(\mathrm{J}) = -\hat{n}\times\hat{n}\times E^{inc} + \alpha\frac{\eta_0}{2}\bar{\mathrm{I}}(\mathrm{J}) \tag{4}$$

By discretizing and testing of EFIE whit the RWG basis functions in the Galerkin method, a $N\times N$ dense matrix equation is formulated in this form

$$\mathbf{Z}^{\mathrm{EF}}\cdot\mathbf{I}^{\mathrm{EF}} = \mathbf{V}^{\mathrm{EF}}, \quad \text{and} \quad Z_{mn}^{\mathrm{IF}} = \frac{1}{2}\int(\boldsymbol{t}_m\cdot\boldsymbol{f}_n)dS_m \tag{5}$$

Where $\boldsymbol{t}_m$, and $\boldsymbol{f}_n$ is the testing functions and the basis functions, respectively. $\mathbf{Z}^{\mathrm{IF}}$ is matrix for principal value integration. The linear algebraic equation of IEFIE is then given in the following iterative way:

$$\mathbf{Z}^{\mathrm{IEF}}\cdot\mathbf{I}_{i+1} = (\mathbf{Z}^{\mathrm{EF}} + \alpha\eta_0\mathbf{Z}^{\mathrm{IF}})\cdot\mathbf{I}_{i+1} \approx \mathbf{V}^{\mathrm{EF}} + \alpha\frac{\eta_0}{2}\mathbf{Z}^{\mathrm{IF}}\cdot\mathbf{I}_i \tag{6}$$

The ILU-class preconditioners are an effective and simple preconditioning technique for solving integral equation systems. Define the matrix $\mathbf{M}$ as the preconditioner matrix, which is a nonsingular approximation to $\mathbf{Z}$. When it is a right preconditioner, the MVM of MoM can be written in the form

$$\mathbf{Z}^{\mathrm{EF}}\cdot\mathbf{M}^{-1}\mathbf{u} = \mathbf{V}^{\mathrm{EF}}, \quad \text{with} \quad \mathbf{I}^{\mathrm{EF}} = \mathbf{M}^{-1}\mathbf{u} \tag{7}$$

The preconditioning matrix for EFIE matrix in this paper is constructed as

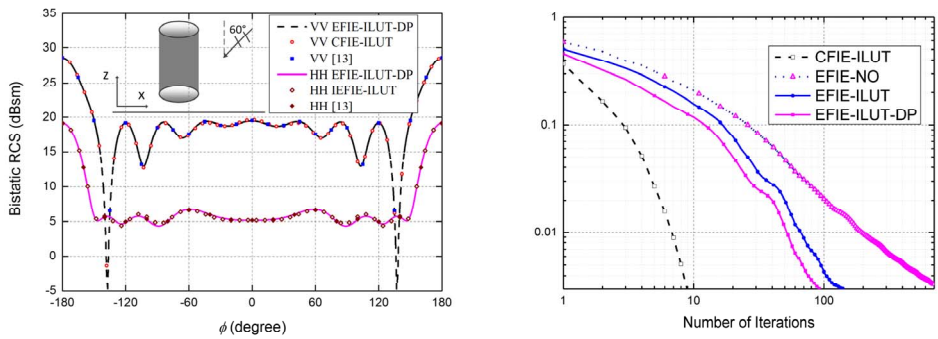
$$\mathbf{M} \approx \mathrm{ILUT}(\mathbf{Z}_{\mathrm{NFI}}^{\mathrm{IEF}}) \tag{8}$$

Where $\mathbf{Z}_{\mathrm{NFI}}$ is near-field interactions (NFIs) matrix in p-FFT, which we used as integral-equation based fast algorithms.

3. Results and Discussions

The numerical experiments are performed on an desktop computer configured by Intel I7 2600 quad core 3.4 GHz and 16 GB RAM. Numerical results are presented to validate the effectiveness of our proposed integral equation domain decomposition method. The five cases studied for the proposed 3-D p-FFT are: 1) EFIE-NO, no preconditioner is employed; 2) EFIE-ILUT, conventional ILUT (τ , p) is employed for EFIE; 3) EFIE-ILUT-DP, the diagonal perturbations ILUT (τ , p) is employed for EFIE; 4) CFIE-ILUT, conventional ILUT (τ , p) is employed for CFIE; 5) IEFIE-ILUT, conventional ILUT (τ , p) is employed for IEFIE.

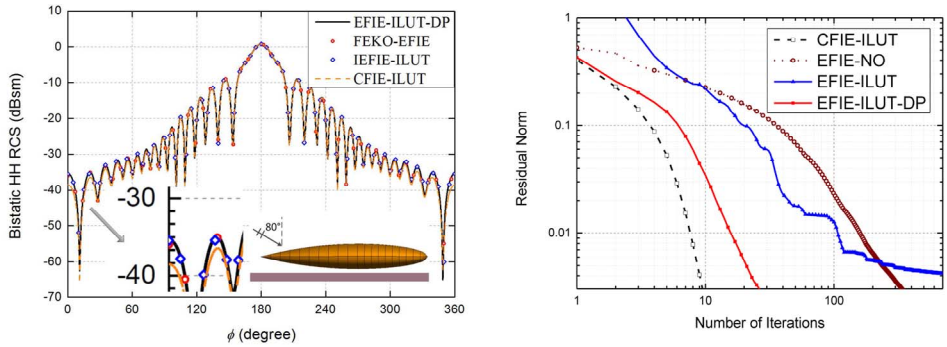
To demonstrate the demonstrate the accuracy and efficiency of the proposed perconditioner, the biostatics RCS of two benchmark targets were calculated. the biostatics RCS of three benchmark targets was calculated. They consist of a cylinder with 20,448 unknowns at 600 MHz, an almond with 43,743 unknowns at 9 GHz. All of them above the half space with the bottom layer of relative permittivity ε_r =6.38-j0.663. The cylinder and the almond geometries come from [13] and [14].



a *b*

Fig. 2 A PEC cylinder scattering problem. (a) co-popularized biostatic RCS patterns of the cylinder above the half space in the θ-cut at 600 MHz; (b) The convergence history of GMRES algorithms with different integral operators and preconditioners on the cylinder scattering problem.

In the first example, the cylinder is 3m long, has a radius of 0.5m, and is located 0.2m above the interface. A 600 MHz plane wave is incident from $\theta_{inc} = 60^\circ$ and $\phi_{inc} = 0^\circ$. The Co-polarized biostatics RCS patterns calculated with the proposed 3-D p-FFT based on IEFIE and EFIE are compared to a reference data [13] in Fig. 2. The patterns are shown to be identical. From Fig. 3, it is seen that the proposed EFIE-ILUT-DP converges more rapidly than other methods except for CFIE-LUT-DP. In this example, CFIE-LUT-DP is the best choice, however, as will be shown, it may deteriorates in accuracy or even not applicable for irregular structures.



a *b*

Fig. 3 A PEC NASA almond scattering problem, (a) co-popularized biostatic RCS patterns of the cylinder above the half space in the θ-cut at 9 GHz; (b) The convergence history of GMRES algorithms with different integral operators and preconditioners on the almond shape example..

The second example, an 9-inch almond is above the half space with the bottom layer which has relative permittivity $\varepsilon_r = 6.38 - j0.663$. It is located 0.02m above the interface. A 9 GHz plane wave is incident from $\theta_{inc} = 80^\circ$ and $\phi_{inc} = 0^\circ$. Fig. 3 shows the biostatics RCS patterns of the almond by different methods. The results are further compared with the one calculated by FEKO. All of them agree well, except for CFIE, which is less accurate. This is because EFIE can produce numerical results with much better accuracy than CFIE for irregular structures. Fig. 6 shows the convergence history of the GMRES algorithm. It can be observed that EFIE-ILUT deteriorates in this example due to the ill-conditioned factors of the ILUT preconditioning matrix, while the proposed EFIE-ILUT-DP is still robust.

5. Acknowledgments

This work was supported in part by National Science Foundation of China (No. 61171046, 61201002), and in part by Program for *Changjiang Scholars and Innovation Team in University* by Ministry of Education, China (No: IRT1113). The authors acknowledge helpful discussions with Ran Zhao form the University of Electronic Science and Technology of China and Mr. Kai Yang from the University of Texas at Austin, USA.

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