Focusing properties of hollow Gaussian beams

Guohua Wu and ***Wen Dai***
School of Electronic Engineering, Beijing University of Posts and Telecommunications, Beijing 100876, China

Abstract

On the basis of Fresnel diffraction integral formulation, an analytical intensity distribution of hollow Gaussian beam (HGB) passing through an unapertureless lens is obtained.The three-dimensional intensity in the focal region is studied in detail. The irradiance distribution in the focus plane approaches Gauss shape with some lobes at two sides, which differs from its initial doughnut shape. We also investigate the focal shift of HGB depending on various parameters.

1. Introduction

In recent years, dark hollow beams (DHBs) have been investigated with growing interest for their unique physical properties, such as heating-free effect, diffraction-free propagation and may carry spin and orbital angular momentum [1], [2]. They have been a very useful tools for manipulating and controlling neutral atoms. Also, they have potential applications in laser optics, computer-generated holography and materials science. There are many methods for generating dark hollow beams. Making use of a small hollow fiber, DHBs was generated by Jianping Yin *et al* [3]. R. M. Herman *et al* [4] obtained DHBs by using the conical lens. Xiao Wang *et al* [5] produced DHBs with a variable-radius ring by selecting the transverse-mode. Through axicon-type computer-generated holograms, the generation of DHBs were demonstrated by Carl Paterson *et al* [6].

In theory, several models have been put forward to describe DHBs, such as the TEM_{01} beam [7], Laguerre-Gaussian beam [8] and the higher-order Bessel beam [6], [9], and so on. More recently, a new theoretical model called the hollow Gaussian beam (HGB) has been put forward to describe DHBs [10], [11].The propagation and transformation of HGB through an unaperture [10] and aperture [11] optical system have been well studied. Atomic trapping and guiding by using quasi-dark hollow beams was theoretically investigated [12]. However, the focusing properties of HGB is not studied. The analysis of the three-dimensional light distribution in the vicinities of the focus is of particular importance. As known, under certain conditions the point of maximum intensity in the diffracted field for focused beams does not coincide with the geometric focus but is shifted along the axis toward the focusing lens[13], [14]. This effect, referred to as focal shift, has been verified experimentally [15]. It has been recognized that focal shift occurs only if the Fresnel number is of the order of unity or less [13], [14]. When the Fresnel number is close to unity, the intensity distribution does not possess this symmetry and focal shift occurs. In other words, evident focal shift only when the focusing lens is placed too close to the focal plane so that the lens is within the near field of the focal plane [16].

In present paper, the focusing properties of HGB is examined based on the encircled-energy criterion. With the help of Fresnel diffraction integral formulation, an analytical intensity expression for focused HGB is obtained in section 2. The structure of the light distribution near the focus through an unapertured aplanatic lens is well studied in section 3. Because the focal shift is also embodied in the focused HGB, the focal shift of HGB depends on various parameters is investigated in section 3. Finally, conclusions is drawn in section 4.

2. Theoretical model

As shown in Fig. 1, a hollow Gaussian beam is focused by an aplanatic lens of focal length f, which may be expressed in the forms [10], [11]

$$E(r,0) = G_0 \left[\frac{r^2}{w_0^2}\right]^n \exp\left[-\frac{r^2}{w_0^2}\right] \tag{1}$$

where n is the order of HGB, w_0 is the beam waist width and G_0 is a constant. Within the paraxial approximation, the propagation of electrical field $E(r, z)$ is governed by the following formula [17]

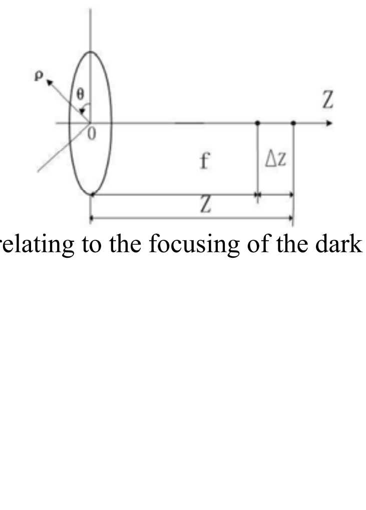

Fig 1Notation relating to the focusing of the dark hollow beams.

$$E(r,z)=\int_0^{2\pi}\int_0^{\infty}E(\rho,0)K(r,\rho,z)\rho d\rho d\theta \qquad (2)$$

where the propagation kernel $K(r,\rho,z)$ in our case can be expressed as [17]

$$K(r,\rho,z)=\frac{k\exp(ikz)}{2\pi iz}\exp\left[\frac{ik}{2z}(r-\rho)^2\right]\exp\left[-i\frac{k\rho^2}{2f}\right] \qquad (3)$$

The term k=2π/ λ is wave number, ρ and θ are the radial and the azimuth angle coordinates, respectively. Recalling the following integral formula

$$\int_0^{2\pi}\cos(n\theta)\exp\left[ix\cos\theta\right]d\theta=2\pi i^n J_n(x) \qquad (4)$$

the double integral can be reduced into a single integral owing to the cylindrical symmetry of the problem, and we obtain

$$E(r,z)=\frac{k}{iz}\exp\left[ikz+\frac{ikr^2}{2z}\right]\int_0^{\infty}E(\rho,0)\exp\left[-\left(\frac{ik}{2f}-\frac{ik}{2z}\right)\rho^2\right]J_0\left[\frac{kr\rho}{z}\right]\rho d\rho \qquad (5)$$

where $J_0(x)$ denotes a Bessel function of the first kind, of order 0. After integration, the final result can be arranged as

$$E(r,z)=\frac{G_0Z_R}{iz}\exp\left[ikz+\frac{ikr^2}{2z}\right]n!\,p^{-(n+1)}\exp\left[-\frac{Z_R^2r^2}{z^2pw_0^2}\right]L_n\left[\frac{Z_R^2r^2}{z^2pw_0^2}\right] \qquad (6)$$

where

$$p=1+iZ_F-i\frac{Z_R}{z} \qquad (7)$$

The terms $Z_R=kw_0^2/2$ is the Rayleigh distance and $Z_F=kw_0^2/2f$ coincides with _ times the Fresnel number [19], respectively. The irradiance distribution of HGB through an apertureless aplanatic lens at the point (r,z) is given by

$$I(r,z)=\left[\frac{G_0Z_Rn!}{z|p|^{n+1}}\right]^2\exp\left[-\frac{2Z_R^2r^2}{z^2|p|^2w_0^2}\right]\left|\mathrm{L}_n\left[\frac{2Z_R^2r^2}{z^2|p|^2w_0^2}\right]\right|^2 \qquad (8)$$

where $|x|$ gives the absolute value of the real or complex number x.

For more general beams, it is more suitable to use the encircled-energy criterion [20], [21], [22]. The beam width r_0 is defined as that radius within which 80% of the beam's power is enclosed [20]. The actual focus is located at the point z_0 where the smallest value of r_0 satisfies

$$\frac{\int_0^{2\pi}\int_0^{r_o}I(r,\theta,z_0)rdrd\theta}{\int_0^{2\pi}\int_0^{\infty}I(r,\theta,z_0)rdrd\theta}=0.8 \qquad (9)$$

With the help of Eq. (11), the actual focal plane z_{max} can be found numerically. Therefore, the relative focal shift $\frac{\Delta z}{f}=\frac{z_{max}-f}{f}$ may be obtained.

3. Numerical calculations and analysis

To characterize focusing properties and focal shift of the DHB, we give some numerical results by using of Eqs. (9) and (12). The irradiance distribution of the focused dark hollow beams in the region of the geometric focus is complicated, as shown in Fig. 2. The calculation parameter is f = 50mm. As can seen from Fig. 2 (a) and (c), when the Fresnel number is not very large compared to unity, the maximum intensity is located before the geometric focus (z = 50mm), i.e., focal shift occurs. And a distinct asymmetry of intensity distribution with respect to the focal plane exhibits. The asymmetry properties become less distinct with the increasing of the Fresnel number, as shown in Fig. 2 (b) and (d). In other words, the focal shift $\triangle z = z_0 - f$ decreases with the increasing of the Fresnel number. The exact focal shift of the focused dark hollow beam may be found in Fig. 4 and Fig. 5. Further more, the energy is more concentrated in the region of the geometric focus with the increasing of the Fresnel number. The structure of the beams is shown further in Fig. 3, which plots the normalized irradiance distribution at the focal plane for different beam order

n and z_F. The calculation parameters are the same as in Fig. 2. As can seen from Fig. 3, the irradiance distribution at the focal plane approaches Gauss shape with some lobes at two sides, which differs from its initial ring-shape, i. e., a null intensity center on the beam axis in the propagation direction [2]. This is because HGB is not a pure-mode but a combination of Laguerre-Gaussian beams with different parameters. Different modes evolve differently with the propagation distances. Different modes overlap and interfere in propagation resulting in the shape in Fig. 3.

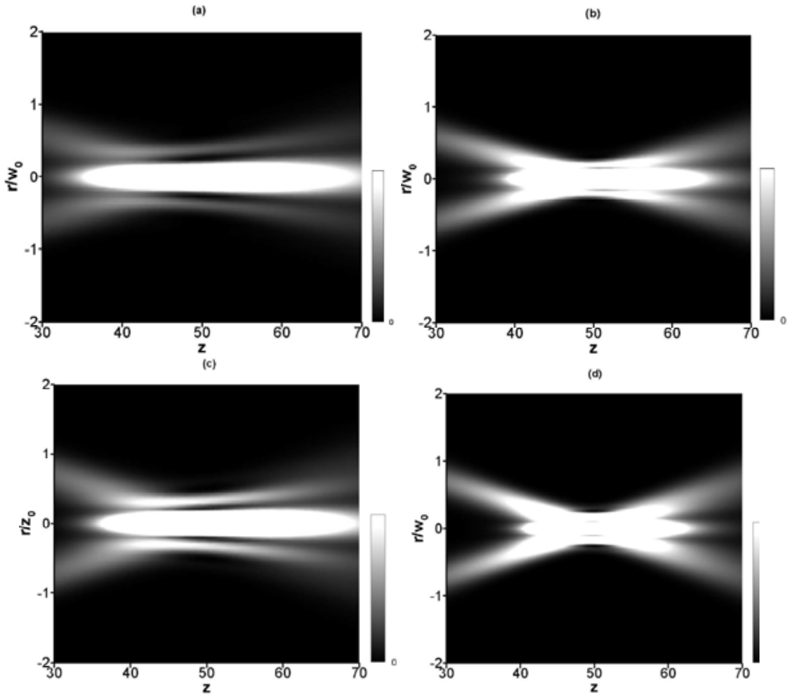

Fig 2 Irradiance distribution of the focused dark hollow beams in the region of the geometric focus for different mode numbers and Fresnel number: (a) n=2, $z_F = 3$, (b) n=2, $z_F = 5$, (c) n=3, $z_F = 3$, (d) n=3, $z_F = 5$.

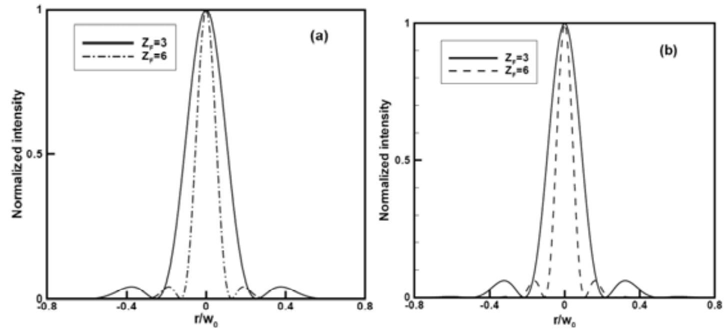

Fig Normalized irradiance distribution at the focal plane for different beam order n and z_F. (a) $n = 2$, (b) $n = 3$.

Beam width based on the encircled-energy criterion against propagation distance z for different beam order n and z_F is plotted in Fig. 4. The focal length is still $f = 50$mm. The power is more concentrated at a point just before the focal plane $z = f$, i. e., focal shift, which is further confirmed in Fig. 4. The asymmetry about focal plane $z = f$ plane is apparent here. As indicted by Fig. 4, the beam width of each case decrease as propagation distance z increases and reaches a minimum beam width just before the focal length. The effect of focal shift become more evident for smaller z_F. The beam width increases as the propagation distance z increasing after that. For the same beam order n, the beam width is always bigger for smaller z_F. That is to say the energy is more concentrated for bigger z_F. Just as shown by Fig. 4, the beam width decreases more quickly before the actual focus for bigger beam order n and increases more quickly after the actual focus when other parameters are fixed. This indicates that HGB with bigger beam order n has better focusing properties. However, the focal shift Δz for $n = 2$ is nearly identical with that of $n = 3$ when the Fresnel number is fixed.

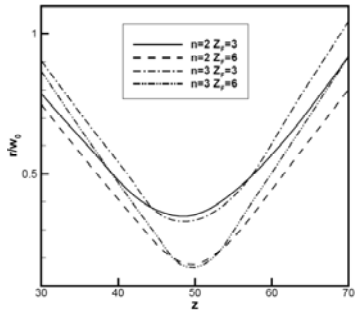

Fig 4 Beam width based on the encircled-energy criterion varies with propagation distance z for different beam order n and Z_F .

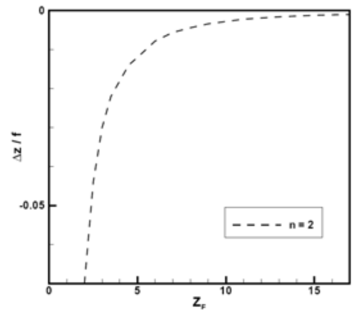

Fig 5 Relative focal shift $\triangle z/f$ plotted against Z_F for the beam order $n = 2$.

In Fig. 5, the relative focal shift $\triangle z/f$ for the beam order $n = 2$ is plotted as a function of Z_F . The calculation parameters are the same with that of Fig. 2. As Fig. 5 shows, the relative focal shift $\triangle z/f$ is always negative. This indicates the best focusing plane is closer to the focusing lens than the geometrical focal plane. As expected, the relative focal shift tends to vanish for high value of Z_F . This confirms that the effect of focal shift is only evident when the Fresnel number is of the order of unity or smaller [13], [14]. Although the focal shift of HGB is well studied above, further explanation of it is needed. As known, HGB can be expressed as a finite series of Laguerre-Gaussian modes [10], [11], i. e., the HGB can be seen as multi-Laguerre-Gaussian modes with different parameters. Different modes experience different focal shifts which leads to the whole focal shift as shown in Fig. 5.

4. Conclusion

We have derived an analytical formulation for evaluating the three-dimension irradiance distribution of the focused dark hollow beams in the region of the geometric focus. It has shown that the irradiance distribution at the focus plane approaches Gauss shape with some lobes at two sides, which differs from its initial doughnut shape. Investigation indicates the focal shift is also embodied in the focusing of HGB. The focal shift strongly depend on the Fresnel number, which is investigated by the encircled-energy criterion. The relative focal shift is evident when the Fresnel number is of order of unity, and decreases as the increasing of the Fresnel number.

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6. References

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