Analysis on the Structure of Distributed Space-Time Code for TWRNs with Relay Having Multiple Antennas

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Abstract

The lower bound of pairwise error probability (PEP) is derived for two-way relaying networks (TWRNs), in which the relay node has M antennas. PEP analysis shows that the error performance can decay with $\ln(\mathrm{SNR})/\mathrm{SNR}^M$ by linearly combining signals from M antennas at the same relay. Compared with the $\ln^M\mathrm{SNR}/\mathrm{SNR}^M$ reported by Jing et. al., the optimum diversity gain can be greatly improved. In addition, we also present one possible design for the case of M=4, simulations illustrate that the selected design has a preferable diversity gain function.

1. Introduction

Relaying networks have becoming more and more interesting in recent ten years due to the performance gain brought by relaying communication. Focusing on attaining the larger diversity gain, various distributed space-time codes (DSTC) are proposed for relaying networks. If channel statistical information is known at the relays, for the single-way relaying network having M relays with all nodes equipped with one antenna, error performance decaying with $\ln^M(\mathrm{SNR})/\mathrm{SNR}^M$ has been reported in [1-2]. Furthermore, under the assumption of perfect channel information at relays, a DSTC is proposed in [3] for more general networks. Pairwise error probability (PEP) analysis shows that the ln(SNR) part in the PEP upper bound expression can be reduced or eliminated. The improvement is mainly obtained due to both the combining method and the perfect known fading coefficient at the relay, which may not be easily attainable.

Inspired by the performance improvement achieved in the one-way relaying networks, a bi-directional relaying protocol was also proposed in [4] to enhance bandwidth efficiency. Since then, various two-way relaying protocols have been reported, including DSTC-based schemes [5-12], which do not require modem at the relays. Since multiple antenna deployment is limited at the terminals in many scenarios, such as mobile ad hoc, we consider only one antenna can be employed to every source node and each relay node can be equipped with multiple antennas in this paper. For the specific TWRN with one dual-antenna relay, we have proposed a distributed concatenated Alamouti code and analyzed the asymptotic symbol error probability [10], which shows its superiority to conventional DSTC [1-2]. We extend our work to relaying networks with two relays [12] and more general 2^N relays [11], where dual-antenna relays are considered. The new results show that the lower bound of PEP can be improved by using more relays, whereas the relay number, which determines the exponential term of the numerator of the PEP, also limits the diversity performance, i.e., $\ln^N\mathrm{SNR}/\mathrm{SNR}^{2N}$.

From those results in [10-12], we reasonably assume that the lower bound of PEP can be further improved if all relay antennas are assigned to one relay node. The main contribution of our work is to give a lower bound expression on the PEP of the TWRNs with the relay having M antennas and give out a possible DSTC design to achieve this bound. The theoretical and simulation result verified our assumption, which is superior to that of networks where M antennas are allocated to more than one relay node.

Notation: Column vectors and matrices are boldface lowercase and uppercase letters, respectively. Notation$\|\mathbf{r}\|$ denotes a 2-norm of $\mathbf{r}$; $(.)^{\mathrm{T}}$, (.)*and $(.)^{\mathrm{H}}$ denote the matrix transpose, conjugate, and conjugate transpose, respectively; $E[.]$ denotes the expected value of the expression in brackets; $\mathbf{I}_{\mathrm{N}}$ denotes the $\mathrm{N}\times\mathrm{N}$ identity matrix. We use $\otimes$ denote the Kronecker product, and vec(A) denotes the vectorization of matrix A. $\Re$ and $\Im$ denote the real and imaginary part.

2. TWRN Model with Relay Having Multiple Antennas and Lower Bound on PEP

In this section, we consider a half-duplex amplify-and-forward TWRN consisting of two sources with each having a single antenna and one relay having M antennas. As shown in Fig. 1, the transmission can be described as follows. In the L time slots/symbol duration of the first phase, both T_1 and T_2 transmit their

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messages $\mathbf{x}_1=[x_{1,1}, x_{1,2},\ldots,x_{1,L}]^{\mathrm{T}}$ and $\mathbf{x}_2=[x_{2,1}, x_{2,2},\ldots,x_{2,L}]^{\mathrm{T}}$ to the relay node with the transmission power being P_1 and P_2, respectively. The received signal vector by the relay at the *l*-th time slot can be written as

$$\mathbf{r}_l = \sqrt{P_1}\mathbf{f}_j x_{1,l} + \sqrt{P_2}\mathbf{g}_j x_{2,l} + \mathbf{n}_l\,, \tag{1}$$

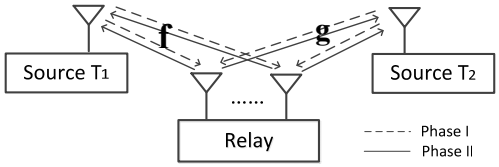

Fig.1. TWRN model with one relay having multiple antennas

where $E[\mathbf{x}_1\mathbf{x}_1^{\mathrm{H}}]=E[\mathbf{x}_2\mathbf{x}_2^{\mathrm{H}}]=\mathbf{I}_L$, $\mathbf{r}_l=[r_{1,l}, r_{2,l},\ldots, r_{M,l}]^{\mathrm{T}}$. Furthermore, $\mathbf{f}=[f_1,f_2,\ldots, f_M]^{\mathrm{T}}$ denotes the channel vector between T_1 and R, $\mathbf{g}=[g_1,g_2,\ldots,g_M]^{\mathrm{T}}$ denotes the channel vector between T_2 and R. We assume f_{j} and g_{j} are independent non-identically distributed (*i.n.i.d*) complex Gaussian random variables with zeros-mean and variances Ω_1 and Ω_2 respectively. We further assume f_{j} and g_{j} remain constant in one transmission block. The noise vectors $\mathrm{n}_l=[\mathrm{n}_{1,l},\mathrm{n}_{2,l},\ldots\mathrm{n}_{M,l}]^{\mathrm{T}}$ are assumed to be independent identically distributed (*i.i.d*) zero-mean complex Gaussian random variables with $E[n_{j,l}]=\sigma^2$. Let $\mathbf{r}=[\mathbf{r}_1^{\mathrm{T}},\mathbf{r}_2^{\mathrm{T}},\ldots\mathbf{r}_L^{\mathrm{T}}]^{\mathrm{T}}$, we have (2) in the 1st phase.

$$\mathbf{r} = \sqrt{P_1}\mathbf{F}_L\mathbf{x}_1 + \sqrt{P_2}\mathbf{G}_L\mathbf{x}_2 + \mathbf{n}\,, \tag{2}$$

where $\mathbf{F}_L = \mathbf{I}_L \otimes \mathbf{f}$, $\mathbf{G}_L = \mathbf{I}_L \otimes \mathbf{g}$ and $\mathbf{n} = [\mathbf{n}_1^T, \mathbf{n}_2^T, \cdots, \mathbf{n}_L^T]^T$.

In the 2nd phase, the relay R combines the *ML* received signals into new symbols and transmits them with power P_r in consecutive *N* time slots simultaneously ($N >= L$). By using $\mathbf{z}_k=[\mathrm{z}_{k,1},\mathrm{z}_{k,2},\ldots\mathrm{z}_{k,N}]^{\mathrm{T}}$, k=1,2, denote the received signals of $\mathrm{T_k}$ during the *N* time slots in the second broadcasting phase, we have

$$z_{1,n} = \mathbf{f}^T\mathbf{t}_n + \eta_{1,n}, \quad z_{2,n} = \mathbf{g}^T\mathbf{t}_n + \eta_{2,n}\,, \tag{3}$$

where $\mathbf{t}_j=[\mathrm{t}_{j,1},\mathrm{t}_{j,2},\ldots\mathrm{t}_{j,N}]^{\mathrm{T}}$ and $\mathrm{t}_{j,\mathrm{n}}$ denotes the *n*-th transmitted signal from the *j*-th antenna at the R node. $\mathbf{t}_n$ is generated by linearly combining $\mathbf{r}$ and its conjugate $\mathbf{r}^*$ as

$$\mathbf{t}_n = \sqrt{\beta}(\mathbf{A}_n\mathbf{r} + \mathbf{B}_n\mathbf{r}^*)\,, \tag{4}$$

where $\mathbf{A}_n$ and $\mathbf{B}_n$ are coefficient matrices adopted for the *n*-th antenna at relay. Eq.(4) is normalized to make the average transmitted power be P_r. Here we also assume that the total power is fixed as 1, i.e., $MPr+P_1+P_2=1$. Since the symmetry of the TWNR, we just consider the received signals by T_1.

$$z_{1,n} = \sqrt{\beta}(\mathbf{f}^T\mathbf{A}_n\mathbf{r} + \mathbf{f}^T\mathbf{B}_n\mathbf{r}^*) + \eta_{1,n} \tag{5}$$

For diversity analysis, $\mathbf{r}$ in (2) can also be written as

$$\mathbf{r} = \sqrt{P_1}\mathbf{X}_1\mathbf{f} + \sqrt{P_2}\mathbf{X}_2\mathbf{g} + \mathbf{n}\,, \tag{6}$$

where $\mathbf{X}_k=[x_{k,1}\mathbf{I}_{\mathrm{M}}, x_{k,2}\mathbf{I}_{\mathrm{M}}, \ldots x_{k,L}\mathbf{I}_{\mathrm{M}}]^{\mathrm{T}}$, k=1,2. By replacing $\mathbf{r}$ in (5) with (6) and neglecting the term on $\mathbf{X}_1$, we have a vector form as

$$\tilde{\mathbf{z}}_1 = \sqrt{P_2\beta}\mathbf{U}\mathbf{F}_M\mathbf{g} + \sqrt{P_2\beta}\mathbf{V}\mathbf{F}_M\mathbf{g}^* + \boldsymbol{\xi}_1\,, \tag{7}$$

where $\mathbf{F}_M = \mathbf{I}_M \otimes \mathbf{f}$ and

$$\mathbf{U}^T = \left[\mathrm{vec}(\mathbf{U}_1), \mathrm{vec}(\mathbf{U}_2), \cdots, \mathrm{vec}(\mathbf{U}_N)\right]^T, \quad \mathbf{V}^T = \left[\mathrm{vec}(\mathbf{V}_1), \mathrm{vec}(\mathbf{V}_2), \cdots, \mathrm{vec}(\mathbf{V}_N)\right]^T$$

in which $\mathbf{U}_n = \mathbf{A}_n\mathbf{X}_2$, $\mathbf{V}_n = \mathbf{B}_n\mathbf{X}_2^*$. Further, (7) can also be equivalently written as

$$\overline{\mathbf{z}}_1 = \sqrt{P_2\beta}\mathbf{K}_N^H\mathcal{F}\mathbf{K}_M\overline{\mathbf{g}} + \overline{\xi}_1\,, \tag{8}$$

where the noise vector $\overline{\xi}_1 = \sqrt{\beta}\mathbf{K}_N^H\mathcal{M}\mathbf{K}_{ML}\overline{\mathbf{n}} + \overline{\eta}_1$ and operation $\bar{(\cdot)} = \left[\Re^T(\cdot) \quad \Im^T(\cdot)\right]^T$,

$$\mathbf{K}_N = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_N & j\mathbf{I}_N\\ \mathbf{I}_N & -j\mathbf{I}_N\end{bmatrix}, \mathbf{K}_M = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_M & j\mathbf{I}_M\\ \mathbf{I}_M & -j\mathbf{I}_M\end{bmatrix}, \mathbf{K}_M = \frac{1}{\sqrt{2}}\begin{bmatrix}\mathbf{I}_{ML} & j\mathbf{I}_{ML}\\ \mathbf{I}_{ML} & -j\mathbf{I}_{ML}\end{bmatrix}, \mathcal{F} = \begin{bmatrix}\mathbf{U}\mathbf{F}_M & \mathbf{V}\mathbf{F}_M\\ \mathbf{V}^*\mathbf{F}_M^* & \mathbf{V}^*\mathbf{F}_M^*\end{bmatrix}, \mathcal{M} = \begin{bmatrix}P & Q\\ Q^* & P^*\end{bmatrix}.$$

For general linear combining space-time codes based on (8), we can obtain *Theorem 1* by using the property of positive definite matrices, the Fisher's inequality and Hadamard's inequality [12-13]. We omit the detailed proof for space limitation.

Theorem 1. The lower bounds of PEP at T_1 for the considered two-way network with one relay which has *M* antennas is given by

$$P_{LB}\left(\mathbf{x}_2 \to \mathbf{x}_2'\right) = C\rho^{-M}\ln\rho + \mathrm{O}\left(\rho^{-M}\right), \tag{9}$$

where $C = \dfrac{2^{M-1}(2M-1)!!}{\left(\lambda_{\max} P_2 \Omega_1 \Omega_2 \beta\right)^M (M-1)!!(2M)!!}$ and $\rho = \dfrac{1}{\sigma^2}$ is defined as the signal-to-noise ratio (SNR), $\lambda_{\max}$ is the maximum eigenvalue of the transmission signal matrix related with $\mathbf{x}_2$.

For the network with M single-antenna relays, it is known that error performance can decay with $\ln^M(\text{SNR})/\text{SNR}^M$, whereas *Theorem 1* shows that the lower bound PEP of general designs is proportional to $\ln(\text{SNR})/\text{SNR}^M$. Although the ln(SNR) factor can be reduced to one, it cannot be eliminated if channel state information is unknown at the relay. If multiple relays are used, a reasonable assumption is that the signals cannot be exchanged between different relays since it requires extra channel resources. It has been reported that the index of ln(SNR) can be decreased in half with dual-antenna relays by combining the signals from two antennas on the same relay. We believe that the increased improvement of PEP is due to the fact that the larger processing freedom provided by the received signals from the multiple antennas on one relay. The result can be extended to the networks consisting of N relays with each having M antennas. The lower bound of PEP in that case should be proportional to $\ln^N(\text{SNR})/\text{SNR}^{MN}$.

3. A Possible DSTC Design for TWRN with Relay having Four Antennas

To achieve the lower bound given by *Theorem 1*, the transmitted symbol length L and the combination matrices at relays should be carefully designed. In case of M=4, we denote our proposed DSTC transmission matrix and the new combined vector from all received signals at relay as **R** and **y**. In the following, inspired by the quasi-orthogonal space-time block code (QO-STBC) in [14], we give a possible distributed design, and we will show that this design has better diversity gain function by simulations in Section 4.

Let $y_1 = r_{1,1} - r_{2,2}^* - r_{3,3}^* + r_{4,4}$, $y_2 = r_{2,1} + r_{1,2}^* - r_{4,3}^* - r_{3,4}$, $y_3 = r_{3,1} - r_{4,2}^* + r_{1,3}^* - r_{2,4}$, and $y_4 = r_{4,1} + r_{3,2}^* + r_{2,3}^* + r_{1,4}$, we get the new combined vector $\mathbf{y} = [y_1\ y_2\ y_3\ y_4]^{\mathrm{T}}$, then the transmission matrix can be expressed as

$$\mathbf{R} = \begin{bmatrix} y_1 & y_2 & y_3 & y_4 \\ y_2^* & -y_1^* & y_4^* & -y_3^* \\ y_3^* & y_4^* & -y_1^* & -y_2^* \\ y_4 & -y_3 & -y_2 & y_1 \end{bmatrix}. \tag{11}$$

In addition, according to (4), the transmitted signal matrix can be presented as $\mathbf{R}=[\mathbf{t}_1\ \mathbf{t}_2\ \mathbf{t}_3\ \mathbf{t}_4]$, thus the corresponding combining coefficient matrices are $\mathbf{A}_1 = [\mathbf{I}_4\ \mathbf{O}\ \mathbf{O}\ \mathbf{P}]$, $\mathbf{A}_2 = [\mathbf{O}\ \mathbf{I}_4\ \text{-}\mathbf{P}\ \mathbf{O}]$, $\mathbf{A}_3 = [\mathbf{O}\ \text{-}\mathbf{P}\ \mathbf{I}_4\ \mathbf{O}]$, $\mathbf{A}_4 = [\mathbf{P}\ \mathbf{O}\ \mathbf{O}\ \mathbf{I}_4]$, $\mathbf{B}_1 = [\mathbf{O}\ \text{-}\mathbf{Q}\ \text{-}\mathbf{R}\ \mathbf{O}]$, $\mathbf{B}_2 = [\mathbf{Q}\ \mathbf{O}\ \mathbf{O}\ \text{-}\mathbf{R}]$, $\mathbf{B}_3 = [\mathbf{R}\ \mathbf{O}\ \mathbf{O}\ \text{-}\mathbf{Q}]$, $\mathbf{B}_4 = [\mathbf{O}\ \mathbf{R}\ \mathbf{Q}\ \mathbf{O}]$, where $\mathbf{P} = \mathbf{J} \otimes \mathbf{J}$, $\mathbf{Q} = \mathbf{I}_2 \otimes \mathbf{J}$, $\mathbf{R} = \mathbf{J} \otimes \mathbf{I}_2$ and $\mathbf{J} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$.

4. Simulation Result

Fig.2 compares the BER performance of the new design in Section 4 with those of two DSTC designs proposed in [2] and [12]. The DSTC in [2] doesn't use signal combination at all, whereas the design in [12] combines signals from two antennas. The new design utilizes all the signals from four antennas. It should be

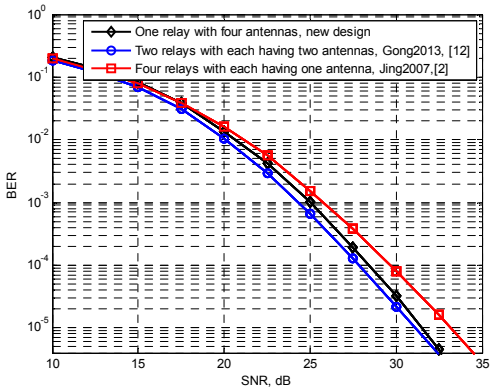

Fig.2. BER comparison of three kinds of DSTC designs

pointed out that our design is only one possible design, which probably is not optimum. However, the slope of the new design is larger than that of the optimal one given in [12]. These results help the readers approve that the PEPs of [2], [12] and the new design are in proportion to $\ln^4 SNR/SNR^4$, $\ln^2 SNR/SNR^4$ and $\ln(SNR)/SNR^4$ respectively.

5. Conclusion

In this paper, we focus on the TWRNs with one relay having multiple antennas. We derive the lower bound of PEP and give a possible example which utilizes all the signals from four antennas. The diversity gain is verified by simulations. The coding gain of this kind of designs may be further improved by carefully designing the signal combination at the relay. Our next step is to find an optimal design as that in [12].

6. Acknowledgments

The work was supported by National Natural Science Foundation of China (NSFC) (61372067).

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