Linear Block Physical-layer Network Coding for Multiple-User Multiple-Relay Wireless Networks

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Abstract

We propose a novel scheme for physical-layer network coding (PNC), constructed from linear block codes for multiple-user multiple-relay wireless networks. In the proposed design, each relay computes a linear combination of source symbols, namely, the network coded symbol (NCS) and forwards it to the destination. The destination collects all NCSs and the original source symbols to form a valid codeword of the linear code. The resulting codeword can be decoded to reliably extract the original source symbols. The numerical results show that our proposed design provides m-th order diversity when there are m relays. Moreover, the proposed design provides a significant sum-rate enhancement over the orthogonal multiple access scheme with network coding described in previous literature.

I. Introduction

In recent years, leveraging the linear network coding in wireless relaying networks has drawn increasing interest [1, 2]. In [1], a novel finite field network coding (FFNC) was proposed for multiple-user multiple-relay (MUMR), wireless networks where each relay transmits a linear combination of source symbols, namely, the network coded symbol (NCS), to form a codeword of the linear code. The authors confirmed that their scheme can achieve $m+1$-th order diversity assuming the number of relays is m. The authors in [2] then extended the FFNC to the frame-wise binary network coding (FBNC), constructed from the LDPC code, which can achieve the same diversity order as FFNC in [1] and improve the error performance.

However, we note that in both FFNC or FBNC, the transmissions from sources to relay or relay to destinations require several channel uses, which sacrifices spectral efficiency. The newly emerged physical-layer network coding (PNC) [3] shows a clear throughput enhancement in two-way relay channels (TWRC), over the conventional orthogonal multiple access schemes, as it only consumes two channel uses. Motivate by these, we propose a new PNC scheme which is constructed from the linear block codes. We refer to such strategy as linear block PNC (LB-PNC), where the NCSs generated from relays can constitute a codeword of the corresponding linear code. To our knowledge, there is few work which constructs the PNC based on the linear block codes. Our numerical results show that our proposed design provides $m+1$-th order diversity when there are m relays. Moreover, the proposed design provides a significant sum-rate enhancement over the over the orthogonal multiple access with network coding in [1].

II. System Model and Motivations

In this section, we describe the system model of the proposed design, i.e., the multiple-user multiple-relay (MUMR) wireless networks. Fig. 1 shows a simple model of MUMR network, which consists of k sources ($S_i, 1 \leq i \leq k$), m relays ($R_\ell, 1 \leq_\ell \leq m$) and a destination ($D$). The transmissions in MUMR network are naturally divided into two stages: 1) the stage that S_i transmits to R_ℓ and D, i.e., the S-RD stage; and 2) the stage that R_ℓ transmits to D, i.e., the R-D stage.

III. General Design and Problem Formulation

A. S-RD Stage

In the S-RD stage, all sources simultaneously transmit their signals to relays and the destination. We assume that all sources employ the same constellation mapper, denoted by $\mathcal{M}_U(\cdot)$. The electromagnetic signals superimposed at R_ℓ and D are given by

$$y_\ell = \sum_{i=1}^{k} h_{i,\ell} x_i + n_\ell, \tag{1}$$

and

$$y_D = \sum_{i=1}^{k} h_{i,D} x_i + n_D, \tag{2}$$

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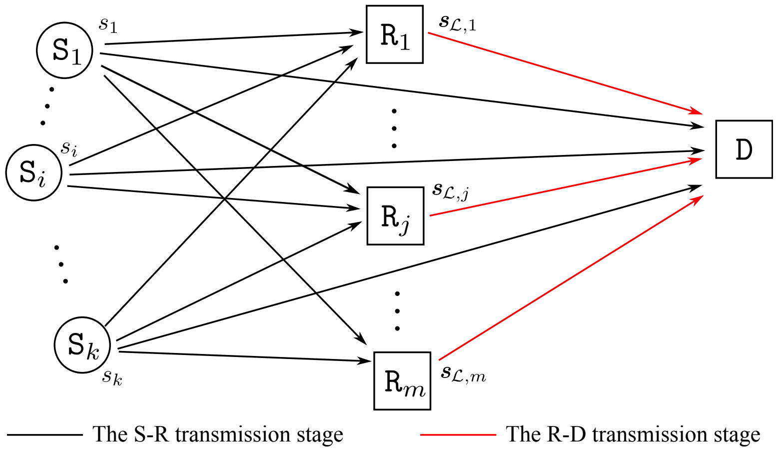

Fig. 1. The system model of MUMR.

respectively, where $x_i = \mathcal{M}_U(s_i)$, $i \in \{1, 2, ..., k\}$ is the modulated symbol of the i-th source and s_i is the source symbol. The quantities $h_{i,\ell}$ and $h_{i,D}$ represent the channel gains between S_i and R_ℓ, and between S_i and D, respectively. The quantity n_ℓ is the complex additive white Gaussian noise (AWGN) with variance σ_w^2 per complex dimension. We refer to

$$x_{ss,\ell} \triangleq \sum_{i=1}^{k} h_{i,\ell} x_i, \tag{3}$$

as the noiseless *superimposed signal (SS)* at R_ℓ. In the proposed design, D adopts the maximum likelihood (ML) rule to estimate the source symbol s_i while R_ℓ adopts the LB-PNC to decode the linear combination of s_i.

B. *Linear Mapping at Relay*

The LB-PNC allows the relays to perform the linear mapping, denoted by $\mathcal{L}$, over the Galois field $\mathbb{F}_q$, given by

$$\mathcal{L} : x_{ss,\ell} \mapsto s_{\mathcal{L},\ell}, \tag{4}$$

where $s_{\mathcal{L},\ell}$ denotes the NCS generated at R_ℓ, taking the form:

$$s_{\mathcal{L},\ell} = p_{1,\ell} \otimes s_1 \oplus p_{2,\ell} s_2 \cdots \oplus p_{i,\ell} \otimes s_i \cdots \oplus p_{k,\ell} \otimes s_k, \tag{5}$$

where $p_{i,\ell} \in \mathbb{F}_q$ and the operations $\oplus$ and $\otimes$ in (4) are the addition and multiplication, defined over $\mathbb{F}_q$. Please refer to Chapter 2 in [4] for details. Based on these, we have: $s_{\mathcal{L},i} \in \mathbb{F}_q$. We note that if $p_{i,\ell} = 1$, $\forall i \in \{1, 2, ..., k\}$, the linear combination in (4) is simplified to the eXclusive-OR mapping as in [3].

Integrating the linear mapping function (3) into the maximum likelihood (ML) detection, we have

$$\widetilde{s}_{\mathcal{L},\ell} = \arg\max_{s_{\mathcal{L},\ell}} p\left(y_\ell | s_{\mathcal{L},\ell}\right) = \arg\max_{s_{\mathcal{L},\ell}} \sum_{\forall x_i \text{ s.t. } \mathcal{L}: x_{ss} \to s_{\mathcal{L},\ell}} \prod_{i=1}^{k} P\left(x_i\right) p\left(y_\ell | x_{ss,\ell}\right), \tag{6}$$

where we note that the summation includes all x_i such that $\mathcal{L} : x_{ss,\ell} \mapsto s_{\mathcal{L},\ell}$. The conditional probability density function (PDF) $p\left(y_\ell | x_{ss,\ell}\right)$ follows the Gaussian distribution, given by

$$p\left(y_\ell | x_{ss,\ell}\right) = \frac{1}{2\pi\sigma_w^2} \exp\left(-\frac{\left|y_M - x_{ss,\ell}\right|^2}{2\sigma_w^2}\right). \tag{7}$$

C. *R-D Stage*

In the R-D phase, R_ℓ maps the NCSs into the modulated symbol, giving $x_\ell = \mathcal{M}_R(s_{\mathcal{L},\ell})$, where $\mathcal{M}_R(\cdot)$ is the relay constellation mapper. Each relay forwards its modulated NCS to D and the received superimposed signal is estimated via the ML detection.

IV. The Linear Block Coded Physical-layer Network Coding

In this section, we first provide the code construction of the proposed LBC-PNC. Then, we investigate the unambiguous decodability of linear mapping of the proposed LBC-PNC.

A. Code Construction

In the proposed design, all relays map k source symbols into m NCSs to form a rate k/n linear block code, where $n=m+k$. We know that for a rate k/n linear block code, the minimum Hamming distance is upper bounded by the Singleton bound [4]

$$d_{\min} \le n-k+1. \tag{8}$$

The class of codes that achieve the Singleton bound, say, $d_{\min}=n-k+1$, is called the maximum distance separable (MDS) code. To this end, the vector of NCSs generated by all relays, denoted by $\mathbf{s}_{\mathcal{L}}=\left[\begin{array}{ccccc} s_{\mathcal{L},1} & \cdots & s_{\mathcal{L},\ell} & \cdots & s_{\mathcal{L},m}\end{array}\right]$, is a valid codeword of this rate k/n MDS code $\mathcal{C}:\mathbb{F}_q^k \mapsto \mathbb{F}_q^n$. As an important and widely used class of q-ary linear block codes, the Reed-Solomon (RS) codes are a good candidate of MDS code. It is well known that a t-symbol-error-correcting RS code has the following characteristics: 1) the number of party-check symbols is $2t$; 2) the minimum Hamming distance is $d_{\min}=2t+1$; and 3) the erasure correction capability is $2t$.

Let $\mathbf{s}\triangleq\left[\begin{array}{ccccccc} s_1 & s_2 & \cdots & s_i & \cdots & s_k\end{array}\right]$ denote the source symbol vector. Let $\mathbf{P}\triangleq\left[\begin{array}{ccccc} \mathbf{p}_1 & \cdots & \mathbf{p}_\ell & \cdots & \mathbf{p}_m\end{array}\right]$ denote the coefficient matrix of NCSs, whose entry is $\mathbf{p}_{i,\ell}$ as given in (4). Let $\mathbf{G}$ denote the *transfer matrix* of the whole network. In order to achieve the expected $m+1$-th order diversity, the network transfer matrix is also treated as the generator matrix of the RS code, taking the form:

$$\mathbf{G}=\left[\begin{array}{cccccccc} 1 & 0 & \cdots & 0 & p_{1,1} & p_{1,2} & \cdots & p_{1,n-k} \\ 0 & 1 & \cdots & 0 & p_{2,1} & p_{2,2} & \cdots & p_{2,n-k} \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 & p_{k,1} & p_{k,2} & \cdots & p_{k,n-k}\end{array}\right]=\left[\begin{array}{cccccc} \mathbf{I}_k & \mathbf{p}_1 & \cdots & \mathbf{p}_\ell & \cdots & \mathbf{p}_m\end{array}\right]=\left[\mathbf{I}_k\,|\mathbf{P}_{k\times m}\right], \tag{9}$$

where the coefficient matrix $\mathbf{P}_{k\times m}$ is constructed from the parity check matrix of RS code such that the resulting NCS $\mathbf{s}_{\mathcal{L},\ell}$ is in fact the ℓ-th parity check symbol. The k–dimensional identity matrix $\mathbf{I}_k$ is the message part of this symmetric RS code, which indicates the direct transmissions from sources to D. Based on these, we have the following mapping relationship between $\mathbf{s}$ and $\mathbf{s}_{\mathcal{L}}$:

$$\mathbf{s}\otimes\mathbf{G}=\mathbf{s}_{\mathcal{L}}=\left(\begin{array}{ccccc} s_{\mathcal{L},1} & \cdots & s_{\mathcal{L},\ell} & \cdots & s_{\mathcal{L},n}\end{array}\right). \tag{10}$$

We note that if any link from source or relay to D is in outage, D can observe such event and pass a flag to the RS decoder, indicating that the symbol is an erasure, not a valid code symbol. A system outage event occurs when there are more than $2t$ links are in outage.

B. Unambiguous Decodability

The linear mapping $\mathcal{L}(\cdot)$ in (3) needs to ensure that the resulting NCS $s_{\mathcal{L}}$ is uniquely determined by source symbol vector s. In other words, the mapping function $\mathcal{L}(\cdot)$ should be injective. This is referred to as the *unambiguous decodability*.

Definition 1: The unambiguous decodability for k-user PNC is given as

$$\begin{aligned}
&\mathcal{L}(s_1,s_2,s_3,\cdots,s_k)\neq\mathcal{L}(s_1,s_2',s_3',\cdots,s_k'),\ \forall(s_2,s_3,\cdots,s_k)\neq(s_2',s_3',\cdots,s_k')\\
&\mathcal{L}(s_1,s_2,s_3,\cdots,s_k)\neq\mathcal{L}(s_1',s_2,s_3',\cdots,s_k'),\ \forall(s_1,s_3,\cdots,s_k)\neq(s_1',s_3',\cdots,s_k')\\
&\mathcal{L}(s_1,s_2,s_3,\cdots,s_k)\neq\mathcal{L}(s_1',s_2',s_3,\cdots,s_k'),\ \forall(s_1,s_2,\cdots,s_k)\neq(s_1',s_2',s_4'\cdots,s_k')\\
&\vdots\\
&\mathcal{L}(s_1,s_2,s_3,\cdots,s_k)\neq\mathcal{L}(s_1',s_2',s_3',\cdots,s_k),\ \forall(s_1,s_2,\cdots,s_{k-1})\neq\left(s_1',s_2',\cdots,s_{k-1}'\right),
\end{aligned} \tag{11}$$

which is in fact a generalized *exclusive law* [5] for multi-user network.

Theorem 1: The proposed LBC-PNC automatically ensures the unambiguous decodability given in (11).

Proof: Since the generated NCS vector $\mathbf{s}_{\mathcal{L}}$ in (10) belongs to a (n,k) linear block code over $\mathbb{F}_q$, q^k NCSs generated by q^k source symbol vectors form a k-dimensional subspace of the vector space of all the k-tuple over $\mathbb{F}_q$. In other words, q^k NCSs for q^k distinct source symbol vectors are distinct. Based on this, each NCS can be uniquely determined by the corresponding source symbol vector. The proof of *Theorem 1* is thus completed. ■

C. Hierarchical Decode-and-forward for LB-PNC

The linear mapping in (5) indicates that if a linear code $\mathcal{C}_s$ over $\mathbb{F}_q$ is employed by each source, the resulting NCS after mapping is still a linear codeword, which facilitates the hierarchical-decode-forward (HDF) strategy, i.e.,:

$$\mathbf{s}_{\mathcal{L},\ell}=p_{1,\ell}\otimes\mathbf{s}_1\oplus p_{2,\ell}\mathbf{s}_2\cdots\oplus p_{i,\ell}\otimes\mathbf{s}_i\cdots\oplus p_{k,\ell}\otimes\mathbf{s}_k\ =\bigoplus_{i=1}^{k}p_{1,\ell}\otimes(\mathbf{d}_i\otimes\mathbf{G}_s)=\left(\bigoplus_{i=1}^{k}p_{1,\ell}\otimes\mathbf{d}_i\right)\otimes\mathbf{G}_s, \tag{12}$$

where $\mathbf{s}_i$ is the coded symbol sequence over $\mathbb{F}_q$, given as $\mathbf{s}_i=\mathbf{d}_i\otimes\mathbf{G}_s$; $\mathbf{d}_i$ is the original uncoded message over $\mathbb{F}_q$; and $\mathbf{G}_s$ denotes the generator matrix of linear code $\mathcal{C}_s$. Based on the HDF principle, the resulting NCS is then feed into the decoder of

$\mathcal{C}_s$. The decoder outputs a combination of the original messages, given as $\mathbf{d}_{\mathcal{L},\ell} \triangleq \left(\bigoplus_{i=1}^{k} p_{1,\ell} \otimes \mathbf{d}_i\right) \otimes \mathbf{G}_s$. The relay re-encodes $\mathbf{d}_{\mathcal{L},\ell}$ and modulate it for transmission.

V. Outage Probability Analysis

In this section, we analyze the overall outage probability for the proposed LB-PNC in MUMR network. The overall probability characterizes the probability of a system outage event.

Let $\mathcal{G} = \{S_i\} \bigcup \{R_\ell\}, i \in \{1, \ldots, k\}, \ell \in \{1, \ldots, m\}$ denote the subset of sources and relays whose symbols or NCSs are in outage. Clearly, a system outage event, denoted by X, occurs when $|\mathcal{G}| > m+1$, where $|\mathcal{G}|$ represents the total number of source symbols and NCSs are in outage and $m+1$ indicates the erasure correction capability of the selected RS code. Let $p \triangleq |\{S_i\}|$ denote the number of sources whose symbols are in outage. Let $w \triangleq |\{R_\ell\}|$ denote the number of relays whose transmitted NCSs are in outage. Given this, the set of ordered pairs (p, w) can be used to define the system outage event, given by

$$\mathcal{X} = \{(p, w) \,|(p + w) > (n - k), 1 \le p \le k, 1 \le w \le n - k\}. \tag{13}$$

Clearly, the system outage probability is characterized as

$$P_{\mathrm{LB-PNC}}^{\mathrm{Out}} = \Pr(\mathcal{X}). \tag{14}$$

Example 1: When $k = 3, m = 4$, the network transfer matrix is equivalent to the generator matrix of a $(7, 3)$ RS code. Based on this, the outage events are given as

$$\mathcal{X} = \{(2,3), (2,4), (3,2), (3,3), (3,4)\}, \tag{15}$$

where we note that a $(7, 3)$ RS code can correct at most 4 symbol erasures. As such, for example, if 2 source symbols and 3 NCSs are decoded erroneously, a system outage event occurs.

Let $P_{\mathrm{SD},i}^{\mathrm{Out}}$ denote the individual outage probability for the link of the i–th source to D. It is clear that after averaging over channels, we have $\mathbb{E}_{\mathbf{H}}\left(P_{\mathrm{SD},1}^{\mathrm{Out}}\right) = \cdots = \mathbb{E}_{\mathbf{H}}\left(P_{\mathrm{SD},i}^{\mathrm{Out}}\right) \cdots = \mathbb{E}_{\mathbf{H}}\left(P_{\mathrm{SD},k}^{\mathrm{Out}}\right) \triangleq \overline{P}_{\mathrm{SD}}^{\mathrm{Out}}$, where $\mathbb{E}_{\mathbf{H}}(\cdot)$ denotes the empirical mean over channels and $\mathbf{H}$ denotes the channel matrix of MUMR network; $\overline{P}_{\mathrm{SD}}^{\mathrm{Out}}$ is used to define the outage probability for transmission from sources to destination, averaging over channels.

Let $P_{\mathrm{SR},\ell}^{\mathrm{Out}}$ denote the individual outage probability for the link of i–th source to the ℓ–th relay. Let $P_{\mathrm{RD},\ell}^{\mathrm{Out}}$ denote the individual outage probability for the link of the ℓ–th relay to D. Then end-to-end outage probability of the source-relay-destination link, denoted by $P_{\mathrm{SRD},\ell}^{\mathrm{Out}}$ is given by

$$P_{\mathrm{SRD},\ell}^{\mathrm{Out}} = P_{\mathrm{SR},\ell}^{\mathrm{Out}} + \left(1 - P_{\mathrm{SR},\ell}^{\mathrm{Out}}\right) \cdot P_{\mathrm{RD},\ell}^{\mathrm{Out}}. \tag{16}$$

Similarly, we define $\overline{P}_{\mathrm{SRD}}^{\mathrm{Out}}$ as the average outage probability for transmission over the source-relay-destination link. Based on $\overline{P}_{\mathrm{SD}}^{\mathrm{Out}}$ and $\overline{P}_{\mathrm{SRD}}^{\mathrm{Out}}$, the defined system outage probability in (11) can be calculated as:

$$P_{\mathrm{LB-PNC}}^{\mathrm{Out}} = \Pr(\mathcal{X}) = \sum_{p:p\in\mathcal{X}} \binom{k}{p} \left(\overline{P}_{\mathrm{SD}}^{\mathrm{Out}}\right)^p \left(1 - \overline{P}_{\mathrm{SD}}^{\mathrm{Out}}\right)^{k-p} \cdot \sum_{q:q\in\mathcal{X}|p} \binom{m}{q} \left(\overline{P}_{\mathrm{SRD}}^{\mathrm{Out}}\right)^q \left(1 - \overline{P}_{\mathrm{SRD}}^{\mathrm{Out}}\right)^{m-q}. \tag{17}$$

The details of individual outage probability in the MAC channel can be found in [6]. This applies to the calculation of $P_{\mathrm{SD},i}^{\mathrm{Out}}$ and $P_{\mathrm{RD},\ell}^{\mathrm{Out}}$ for the transmissions from sources to destination and relays to destination. However, we know that when using PNC, the rate region of all users in the S-R stage is different from the conventional MAC channels. To this end, we pay particular attention to calculating $P_{\mathrm{SR},\ell}^{\mathrm{Out}}$. We know that the rate region in the S-R stage satisfies

$$\mathcal{R}_{\mathrm{SR}} = \{(R_1, \ldots, R_k) : R_i \le I(y_\ell; s_{\mathcal{L},\ell}), \forall i \in \{1, \ldots, k\}\}, \tag{18}$$

where $I(y_\ell; s_{\mathcal{L},\ell})$ is the mutual information between y_ℓ and $s_{\mathcal{L},\ell}$, which is calculated as

$$(y_\ell; s_{\mathcal{L},\ell}) = H(s_{\mathcal{L},\ell}) - H(s_{\mathcal{L},\ell}|y_\ell), \tag{19}$$

where the entropy $H(s_{\mathcal{L},\ell})$ is calculated as

$$H(s_{\mathcal{L},\ell}) = -\sum_{s_{\mathcal{L},\ell}} P(s_{\mathcal{L},\ell}) \log_2(P(s_{\mathcal{L},\ell})), \tag{20}$$

and the conditional entropy $H(s_{\mathcal{L},\ell}|y_M)$ is calculated as in (20),

$$H(s_{\mathcal{L},\ell}|y_\ell) = -\sum_{s_{\mathcal{L},\ell}} P(s_{\mathcal{L},\ell}) \int_{y_\ell\in\mathbb{C}} p(y_\ell\,|s_{\mathcal{L},\ell}) \log_2\left[\frac{p(y_\ell\,|s_{\mathcal{L},\ell})\,P(s_{\mathcal{L},\ell})}{\sum_{s'_{\mathcal{L},\ell}} P\left(s'_{\mathcal{L},\ell}\right) p\left(y_\ell\,\middle|s'_{\mathcal{L},\ell}\right)}\right] dy_\ell, \tag{21}$$

where we note that conditional PDF $p(y_\ell\,|s_{\mathcal{L},\ell})$ is given in (6).

As (20) and (21) cannot be directly measured, we use Monte-Carlo integration instead, given by

$$I(y_\ell; s_{\mathcal{L},\ell}) = -\sum_{s_{\mathcal{L},\ell}} P(s_{\mathcal{L},\ell})\log_2\left(P(s_{\mathcal{L},\ell})\right) + \mathbb{E}_{p(y_\ell|s_{\mathcal{L},\ell})}\left\{\log_2\left[\frac{p\left(y_\ell\left|s_{\mathcal{L},\ell}\right.\right)P\left(s_{\mathcal{L},\ell}\right)}{\sum\limits_{s'_{\mathcal{L},\ell}} P\left(s'_{\mathcal{L},\ell}\right)p\left(y_\ell\left|s'_{\mathcal{L},\ell}\right.\right)}\right]\right\}. \tag{22}$$

Based on the calculated rate region $\mathcal{R}_{\mathrm{SR}}$, we can calculate $P^{\mathrm{Out}}_{\mathrm{SR},\ell}$ as $P^{\mathrm{Out}}_{\mathrm{SR},\ell} = \Pr\left\{R_i > I\left(y_\ell; s_{\mathcal{L},\ell}\right)\right\}$.

VI. Performance Evaluation

In this section, we evaluate the frame error rate (FER) and erogdic sum-rate in terms of end-to-end transmission, for the proposed design. The orthogonal multiple access with network coding (OMAC-NC) in [1], which employs the same type of RS code, is introduced as the benchmark. We assume that all channels experience quasi-static i.i.d. frequency-flat Rayleigh fading with unit variance. The noise variances at each node are equal. The original uncoded data frame length is 360 bits. The uncoded data frames are then encoded by a RS code over $\mathbb{F}_q$ and then mapped into complex symbol sequences for transmission, where each symbol is drawn from the q-ary QAM/PSK alphabet. The simulations are averaged over 10^6 channel realizations for accuracy. For simplicity, a 3-source-4-relay wireless network is assumed. Each source adopts a $(7,3)$ RS code ($\mathbb{F}_8$) to encode the original data sequence, i.e., each group of 3 bits constitutes a symbol over $\mathbb{F}_8$ and each group of 3 symbols are encoded to obtained a 7-symbol RS codeword. Then 8PSK is employed as their modulation scheme. Each relay decodes the NCS using (4) and feeds the NCS into the $(7,3)$ RS decoder to implement HDF. The relay would keep silent if any residual error occurs.

Fig. 2 shows the FER comparison between the proposed LB-PNC and benchmarks. When the underlaying channel code is not employed, we clearly observe that the FER of LB-PNC is superior to that of the multiple accesses without involving relays. The FER of LB-PNC is worse than that of OMAC-NC due to the fact the LB-PNC needs to deal with more interference at relays and the destination. We can also observe that the LB-PNC can achieve the same diversity order as OMAC-NC, i.e, $m+1=5$-th order diversity.

Based on Fig. 2, one may also observe that when SNR is large, the coding gain of LB-PNC with channel code is about 2dB while that of the OMAC-NC with channel code is about 1dB. This is because even though the OMAC-NC avoids interference due to the orthogonal channel use, it needs to deal with more noise compared with LB-PNC and the (7,3) RS code is not powerful enough to correct the errors introduced by the additional noise.

Fig. 3 shows the ergodic sum-rate for the LB-PNC and the benchmark. We observe that the LB-PNC shows a clear sum-rate enhancement over the OMAC-NC. This is because the LB-PNC has the capability to deal with the interference, which improves the spectral efficiency.

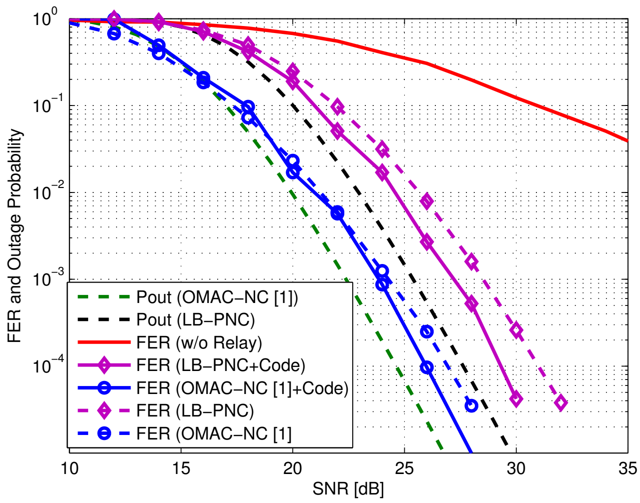

Fig. 2. FER comparison among different strategies

VII. Concluding Remarks

In this paper, we have proposed a novel PNC, called LB-PNC, which is constructed from linear block codes, for the MUMR wireless networks. In the proposed design, the relays compute the linear combination of source symbols, namely, the NCS and forwards it to destination. The destination collects all NCSs plus the original source symbols, which forms a systematic

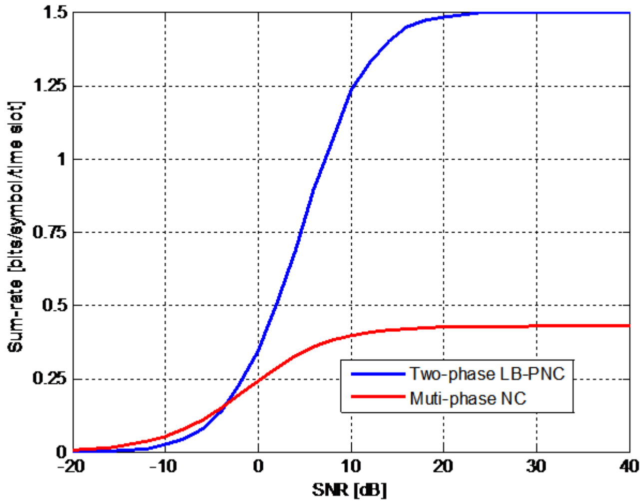

Fig. 3. Sum-rate comparison among different strategies

codeword of the linear code. The resulting codeword can be then decoded to reliably extract the original source symbols. The numerical results show that our proposed design provides $m+1$-th order diversity assuming the number of relays is m. Moreover, the proposed design provides a clear sum-rate enhancement over the over OMAC-NC in [1].

VIII. Acknowledgements

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