Analysis of Terrain Effects on Radio Wave Propagation Through a Full Solution of an Integral Equation

Emanoel Costa* and Markus Liniger

1 Centro de Estudos de Telecomunicações, Pontifícia Universidade Católica do Rio de Janeiro, Rua Marquês de São Vicente 225, 22451-900 Rio de Janeiro RJ Brazil, e-mail: epoc@cetuc.puc-rio.br

2 LiniKomm GmbH, CH-3012 Bern, Switzerland, e-mail: markus_liniger@bluewin.ch

Abstract

The present contribution describes algorithms for the numerical solution of an integral equation which models diffraction effects due to irregular terrain on the propagation of radio waves. An algorithm based on the Method of Moments which takes full consideration of backscattering effects will be proposed. Corresponding results from the described algorithms will be presented and compared among themselves, as well as with associated experimental data.

1. Introduction

Many models are available to predict diffraction effects on the propagation of radio waves over irregular terrain in the VHF and UHF bands that are or will be used by digital TV applications. These effects are generally estimated using: (i) the classical prediction models proposed by Bullington, Epstein-Peterson, Deygout and others [1-3]; (ii) the ones described by the most recent versions of Recommendations ITU-R P.526 [4] and ITU-R P.1546 [5]; (iii) the Longley-Rice model [6]; (iv) computationally-intensive method based on ray tracing and the numerical solutions of parabolic or integral equations [7, 8]. However, results indicate that, in spite of the continuous progress obtained from work performed over decades, there still is a sizeable margin for improvements in the prediction capability of the effects of diffraction due to irregular terrain on propagation of radio waves in the VHF and UHF bands. As an extreme example of still-existing difficulties in the current prediction capability, Figure 1 sketches the vertical terrain profile of the 38.71-km link operating at 647.3 MHz between BNTG (47N58.73, 7E31.78) and Balsthal2 (47N18.68, 7E40.91) in Switzerland and compares field-strength measurements with calculations by different models for different heights above the local terrain at the receiver. Note that several examples of good agreement between predictions and measurements could have presented as well. However, the standard deviation of errors between predictions and measurements, estimated using results from over 9400 links, is larger than approximately 10 dB.

Motivated by the need for improvements on the agreement between predictions and measurements, the present contribution will describe an algorithm for the numerical solution of an integral equation which models diffraction effects due to irregular terrain on the propagation of radio waves. The proposed algorithm is based on the Method of Moments, takes full consideration of backscattering effects and will be applied to selected examples from the above data set.

Figure 1. (Left) Vertical terrain profile of the 38.71-km link operating at 647.3 MHz between BNTG (47N58.73, 7E31.78) and Balsthal2 (47N18.68, 7E40.91) in Switzerland and (right) comparison between field-strength measurements and predictions by different models.
2. The Integral Equation and its Numerical Solution (no Backscatter)

Assume that a two-dimensional irregular terrain profile \( C \), characterized by the continuous function \( h(x) \), is illuminated by a vertical electric dipole located at the point \((0, h_{Tx})\), as illustrated in Figure 2. The integral equation model adopted in this work to represent propagation of radio waves in the presence of irregular terrain, originally developed by Hviid et al. [8], relates the magnetic current density on a smooth perfect magnetic conductor to the source electric field. For vertical polarization, the assumption of perfect magnetic conductivity corresponds to a reflection coefficient of \(-1\), which is a good approximation to this coefficient for relatively high frequencies and for grazing incidence over real ground. Representing the magnetic current density and the vertical component of the electric field as products of complex amplitudes by elementary cylindrical waves, the following equation was obtained by Akorli and Costa [9]:

\[
m_c(x) - \frac{e^{\frac{\pi}{4}}}{\sqrt{\lambda_o}} \int_{0}^{d} K_c(x, x') m_c(x') \, dx' = 2m_c(x),
\]

where

\[
m_{c1}(x) = \frac{\hat{n} \cdot \hat{R}_o}{\sqrt{R_o}} \left( \hat{x} \cdot \hat{R}_o \right)
\]

and

\[
K_c(x, x') = \frac{\hat{n} \cdot \hat{R}_z}{\sqrt{R_z}} \left( \frac{R_o}{R_1 + R_2} \right) e^{\frac{j}{2} \left[ \left( R_o - R_1 \right) \right]} \sec \theta'
\]

The variables \( x \) and \( x' \) measure the distances from the projection of the source onto the horizontal axis to those of the observation point \( P \) and the arbitrary point \( P' \). The functions \( m_c(x) \) and \( m_c(x') \) are the complex amplitudes of the magnetic current density at the corresponding points. Further, \( \hat{x} \) is the horizontal unit vector and \( \hat{n} \) is the (essentially upward) unit vector perpendicular to the terrain at the observation point \( P \). The distances \( R_o, R_1 \) and \( R_2 \) can also be seen in Figure 2. The unit vectors \( \hat{R}_o \) and \( \hat{R}_z \) point from the source to \( P \) and from \( P' \) to \( P \), respectively. It should be observed that \( R_o \) and \( \hat{n} \) depend only on \( x \), \( R_1 \) depends only on \( x' \), and that \( R_2 \) depends on both \( x \) and \( x' \). The angle between the tangent to the terrain profile at \( P' \) and the horizontal direction is represented by \( \theta' \). Finally, \( \lambda_o \) and \( k = \frac{2\pi}{\lambda_o} \) are the wavelength and the wavenumber, respectively.

The original scheme [8] for the solution of equation (1) initially divides the terrain profile into a large number \( N_E \) of elements with lengths smaller than \( \lambda_o/2 \) and assumes that \( m_c(x) \) remains constant within each element. Next, it neglects backscattering effects to determine the values of \( m_{c1}(x) \) at the centers of all the elements \( n = 1, ..., N_E \). That is, it is initialized by \( m_{c1} = m_{c1}(x_n) \) for the first element. Then, it recursively determines the value of \( m_{c1}(x) \) at the center of element \( n \) by the application of equation (1) to the known value of \( m_{c1}(x_n) \) and the previously-determined values \( m_{c1}(x_m) \), where \( 1 \leq m < n \). Akorli and Costa [9] proposed accelerating procedures to solve equation (1) faster. One possible limitation of these procedures is that backscattering effects may be important in some cases (for example, in deep-shadow regions). On the other hand, full consideration of backscattering effects with basis on the Method of Moments [9, 10] in combination with the original scheme leads to a system of linear equations of large order \( N_E \). For example, \( N_E \) would be greater than 20,000 for a 10-km link operating at 300 MHz. The solution of such system would be extremely slow by conventional methods.
3. Proposed Formulation and its Numerical Solution (with Backscatter)

To develop an algorithm which takes full consideration of backscattering effects with basis on the Method of Moments, equation (1) will be applied to determine the magnetic current density $m_L(x)$ along the linear terrain profile $L$, also illustrated in Figure 2. The linear profile can be obtained from the least-square fit of a straight line to the irregular terrain profile $C$. Since the scalar product in equation (3) is always zero along $L$, it is immediate that $m_L(x) = 2m_i(x)$. Therefore, equation (1) can be rearranged in the forms:

$$m_c(x) - 2m_L(x) = 2\left[ m_c(x) - m_L(x) + \frac{e^{j\frac{\pi}{4}}}{{\sqrt{4\pi}}} K_C(x,x') m_L(x') \ dx' \right] +$$

$$+ \frac{e^{j\frac{\pi}{4}}}{{\sqrt{4\pi}}} K_C(x,x') \left[ m_c(x') - 2m_L(x') \right] \ dx'$$

and

$$\delta m(x) - \frac{e^{j\frac{\pi}{4}}}{{\sqrt{4\pi}}} K_C(x,x') \delta m(x') \ dx' = 2\delta m_l(x).$$

Note that: (i) $\delta m(x) = m_C(x) - 2m_L(x)$; (ii) $\delta m(x)$ is defined by the known term between square bracket in equation (4); and (iii) equations (1) and (5) are structurally the same.

In radio wave propagation applications, the irregular terrain $C$ is usually represented by a vector of altitudes at uniformly-spaced distances from the base of the transmitting antenna. The difference between this vector and the one provided by the linear terrain profile $L$ can be Fourier-analyzed and only the strongest components that contribute to a large fraction of the total power will be retained. For example, the previous terrain profile, characterized by $N_{PRF} = 1931$ points with a resolution $\Delta x \approx 20 m$ and displayed by the blue curve in the left-hand side of Figure 3, has been Fourier-analyzed ($N_{FFT} = 2048$). The corresponding power spectral density is shown in the right-hand side of the same Figure and the lowest $M = 106$ Fourier components concentrate 99.5 % of the total power. The inverse Fourier transform of these components is represented by the red curve in the left-hand side of Figure 3, showing a very good agreement with the original terrain profile. Note that the determination of $m_C(x)$ by the full solution of equation (1) by the Method of Moments (considering backscatter) would lead to a system of linear equations of order greater than $N_E \approx 167,000$. In this particular example, $M$ is much smaller than $N_{PRF}$ and $N_E$.

![Figure 3](image-url) (Left) Original vertical terrain profile (blue) and inverse Fourier transform of the only the lowest Fourier components that concentrate 99.5 % of the total power (red). (Right) Normalized power spectral density of the original vertical terrain profile.

The radiated field is strongly dependent on the vertical terrain profile, which can be characterized to a very good approximation by its strongest Fourier components. Thus, it seems reasonable to represent $\delta m(x)$ by a linear combination of the same Fourier components:

$$\delta m(x) = \sum_{u=1}^{M} c_u e^{\frac{2\pi i u x}{N_{PRF}}}. \ N_{FFT} = \sum_{u=1}^{M} c_u f_u(x).$$
In the nomenclature of the Method of Moments, the present algorithm is considered a meshless scheme and \( \delta m(x) \) is represented by a linear combination of \( M \) global expansion functions. Substituting the right-hand side of equation (6) for \( \delta m(x) \) in equation (5), one gets

\[
\sum_{u=1}^{M} \left[ e^{\frac{2\pi i u x}{N_{xy}}} - e^{\frac{2\pi i u x'}{N_{xy}}} \right] K_{e}(x, x') e^{\frac{2\pi i u x'}{N_{xy}}} dx' \right] c_u = 2\delta \eta_l(x) \rightarrow \sum_{u=1}^{M} g_u(x) c_u = 2\delta \eta_l(x). \tag{7}
\]

After the proper selection of testing functions \( w_v(x) \), where \( v = 1, \ldots, M \), followed by scalar multiplications of equation (7) by each testing function, a system of linear equations of order \( M \) is obtained. Note that this order may be much smaller than \( N_e \). Line \( v (v = 1, \ldots, M) \) of this system is characterized by

\[
\sum_{u=1}^{M} \langle g_u, w_v \rangle c_u = 2\langle \delta \eta_l, w_v \rangle \quad \text{where} \quad \langle g_u, w_v \rangle = \int_0^d g_u(x) w_v^*(x) \, dx. \tag{8}
\]

The pair of angular brackets and the asterisk in equation (8) represent the scalar product and the complex conjugate operations, respectively. Different selections of the testing functions lead to particular versions of the Method of Moments. For example, the collocation, Galerkin and least-square methods assume \( w_v(x) = \delta(x-x_v) \), \( w_v(x) = f_v(x) \), and \( w_v(x) = g_v(x) \), respectively.

Once the magnetic current density \( m_C(x) \) is determined by any of the above methods, the vertical component of the electric field at an arbitrary observation point in the air can be obtained using an expression similar to equation (1).

4. Future Work

Corresponding results from the algorithms described in Sections 2 and 3 will be presented and compared among themselves, as well as with associated experimental data.

5. References


