Numerical Simulations of Weakly Collisional Plasmas in $E \times B$ Fields

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Abstract

We study the stability of weakly collisional, magnetized plasmas in the presence of the $\boldsymbol{E} \times \boldsymbol{B}$ fields with self-consistent particle-in-cell simulations. Collisional interactions with a neutral background can lead to distorted, unstable velocity distributions. In the case of charge-exchange collisions, we observe the formation of a ring-shape velocity distribution for ions. For elastic ion-neutral collisions the ion velocity distribution is distorted and broadened. In both cases, after one gyroperiod, we observe a significant increase in the fluctuations in potential, which can be related to the instability growth. In the saturated frequency-wavenumber spectrum, nonlinear three-wave interaction is observed.

1. Introduction

In the presence of external electric and magnetic fields, the plasma will drift with the $\boldsymbol{E} \times \boldsymbol{B}/B^2$ velocity. In the collisionless case, electrons and ions will drift with the same velocity in the same direction. Thus, there will be no net current through the plasma. However, the situation can change significantly, when one or both of the species experience collisions with neutrals. If one of the species becomes unmagnetized due to collisions, a net current will be present. Above a certain threshold, this can lead to instabilities and an exponential growth of fluctuations in potential and electric fields [1]. In the ionosphere, enhanced fluctuation levels can have implications for the scintillations of for instance Global Navigation Satellite System signals, such as GPS signals. Thus their understanding is also important for the assessment of space weather effects [2]. A typical example of an unstable plasma configuration for ionospheric plasmas is the well known Farley-Buneman instability, for which the ions are effectively unmagnetized [3]. In the present work, we focus on a different regime, in which the ions are magnetized, but the collisions are frequent enough to induce significant changes in the ion velocity distributions.

It is known that in the ionospheric E region, a collisional regime exists where $\omega_{ce} >> \nu_{en}$, while $\Omega_{ci} \leq \nu_{in}$, in terms of electron and ion cyclotron frequencies ω_{ce}, Ω_{ci}, and collision frequencies ν_{en}, ν_{in} between charged particles and neutrals. This regime is relevant for the Farley-Buneman instability, which has been studied extensively in the past [4]. At higher altitudes, however, the regime may exist where $\Omega_{ci} \geq \nu_{in}$, while electrons are still collisionless. Under these conditions, it has been shown that a ring-shaped velocity distribution may form for ions with charge-exchange collisions [5]. Such distribution will generally be unstable, but other instabilities may also be excited [1,6], before the fully shaped ring velocity distribution is even established. Thus, the nonlinear situation will be a competition between several instabilities, which are characterized by different ranges in frequencies and wave numbers. Studies of such a regime may be important for comprehensive knowledge of ionospheric processes at different altitudes. Of particular interest is the role of different collision types for the dynamics of the system. While charge-exchange collisions could give rise to unstable ring-shaped velocity distributions, the relative importance of charge exchange and elastic ion collisions is not clear.

Weakly collisional plasmas in the $\boldsymbol{E} \times \boldsymbol{B}$ fields can form highly nonlinear systems, and thus analytical studies can be difficult [6]. A natural choice for studies of such systems will be by using first principle numerical simulations that account for nonlinear plasma dynamics. In this study, we employ the particle-in-cell (PIC) numerical code to carry out self-consistent simulations and address the problem of the role of collisions for the plasma stability in weakly collisional plasmas.

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2. Numerical simulation

Our analysis is carried out with a three-dimensional, electrostatic PIC code. The dynamics of electrons and ions is simulated in self-consistent electrostatic fields in a periodic system. We account for external electric and magnetic fields, and collisions with a neutral background gas. This new code is based on our previous PIC codes, with details of numerical implementation given in earlier works [7,8]. The underlying principles of the code are standard, with the grid being used for field calculations to reduce the numerical complexity of simulations of a large number of plasma particles, and the leap-frog method and Boris algorithm used for advancing particle trajectories [9]. Collisions between plasma particles and neutral background are implemented with the null-collision method [10]. This method is an efficient algorithm for plasma-neutral collisions, and it allows for arbitrary collision cross-sections σ. In particular we can simulate real, energy dependent collision cross-sections. However, to have a control over the dynamics of the system and to allow for systematic studies, in the present work we choose to use a constant collision frequency ν. In our simulations, we allow for charge exchange collisions for ions, and elastic collisions for electrons and for ions.

We use the box size of 0.5 m in each direction, with the spatial grid resolution of 0.7 cm. The plasma density in the simulation is $n = 10^{12}$ m^{-3}, with the initial electron temperature $T_e = 8.6$ eV, and the electron to ion temperature ratio $T_e/T_i = 4$. External magnetic and electric fields are $B_0 = 0.005$ T and $E_0 = 550$ Vm^{-1}, respectively. For these parameters, the $\boldsymbol{E}\times\boldsymbol{B}/B^2$ drift will be supersonic, i.e., $\mathrm{v}_d = 2\sqrt{T_e/M}$, where M is the ion mass. The ion collision frequency is $\nu_{in} = 3.52\cdot10^5$ s^{-1}. This frequency is lower than the ion gyrofrequency, $\nu_{in} = \Omega_{ci}/5$, while the electron collision frequency is given by $\nu_{en} = \nu_{in}\sqrt{M/m}$, where m is the electron mass. To speed-up the simulations we simulate the reduced mass ratio $M/m = 500$, while keeping the electron mass realistic. We set mass of neutral species $m_n = M$, and assume neutrals to be cold, $T_n = 0$. The plasma parameters can be related to the scaled conditions in the upper parts of E-region of ionosphere. However, while the respective ratios of the parameters are within the relevant range, the one-to-one correspondence is not maintained. Thus, while we expect that the main phenomena will be present in the simulations, some of ionospheric processes might be not well represented.

With a timestep of $\Delta t \approx 0.1\cdot 2\pi/\Omega_{ce}$ we can resolve also the electron gyromotion. With these parameters, we can also resolve the smallest scales in the system, i.e., the Debye length and electron gyroperiod, which is necessary for the stability and reliability of the simulations [9]. We typically run the code for about $t = 95000\Delta t$ which corresponds to 19 ion gyroperiods, and simulate 10^7 plasma particles using the Message-Passing-Interface (MPI) for the parallel computing environment.

3. Results

During the course of the simulation we observe the effects of collisions on the ion velocity distribution function. The initially Maxwellian distribution (see Figure 1a) rotates in the u_y-u_z velocity phase space for velocity components perpendicular to the magnetic field direction, where $\boldsymbol{B} = B_0\hat{x}$. For the charge exchange collisions, the gyrorotating bi-Maxwellian cone becomes narrower in time, as the colliding ions move to the origin in the velocity phase space. Thus, ions that have collided, give rise to the ring shape of the velocity distribution function. After one ion gyroperiod the full ring distribution is formed. We observe that after several ion gyroperiods the ring distribution fills in, indicating the saturation of the instability. The resulting distribution is characterized by a peak close to the origin, which is due to charge exchange collisions, see Figure 1(b). On the other hand, ion-neutral elastic collisions lead to diffusion in the velocity phase-space and give rise to a broad distribution function, which center is shifted by the drift velocity, as shown in Figure 1(c).

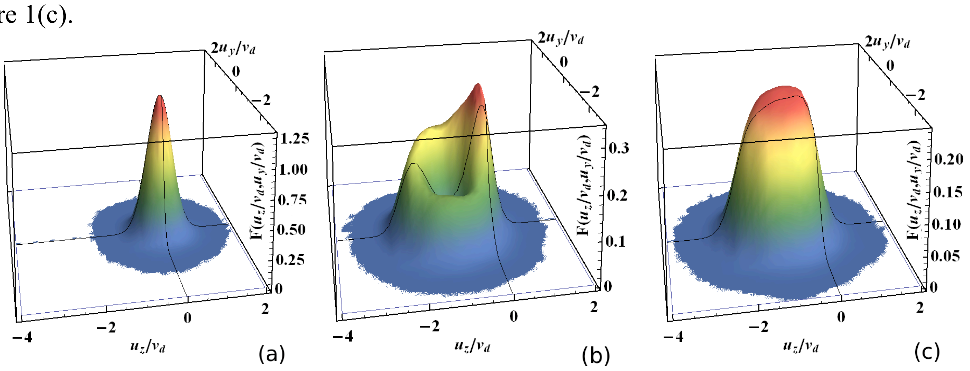

Figure 1. Ion velocity distributions for the velocity components perpendicular to the magnetic field direction at the beginning of simulations (a), and after four ion gyroperiods for charge exchange collisions (b) and elastic collisions (c).

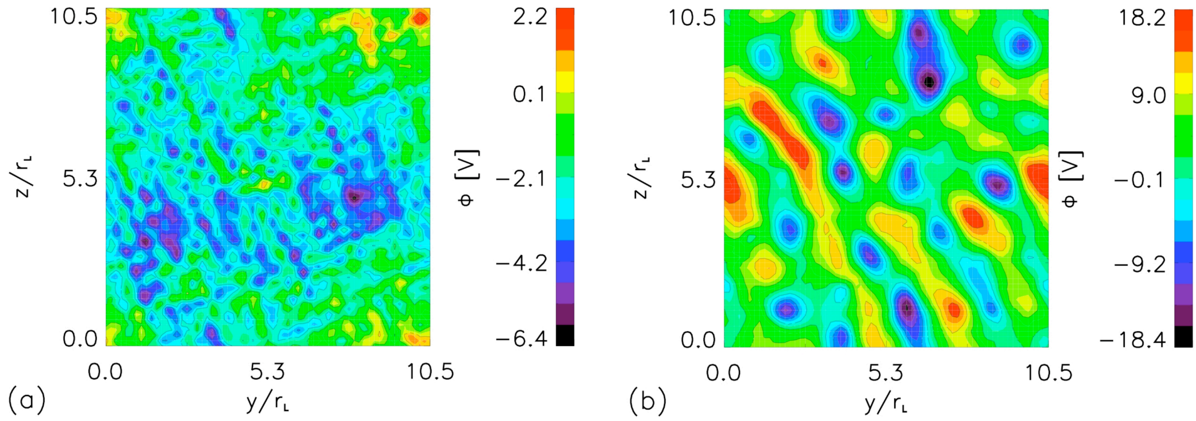

Figure 2: Electrostatic potential distribution in the plane perpendicular to the magnetic field direction for charge-exchange collisions, at time $t=1.2\cdot 2\pi/\Omega_{ci}$ in the growing phase of the instability (a), and for the saturated stage at $t=3\cdot 2\pi/\Omega_{ci}$ (b).

The evolution in the ion velocity distribution function is accompanied by the growth of the amplitude of the potential and electric field fluctuations. The fluctuation level growth occurs after the first ion gyroperiod, and saturates after several gyroperiods. The growth is by approximately one order of magnitude. There is little difference between the growth rate for the elastic and charge exchange collisions. However the saturation level is higher for the charge exchange collisions. In this case, the fluctuation level remains high at the saturation stage, suggesting strong nonlinear interactions between different wave modes generated in the system. For elastic collisions, the saturation is lower by a factor of two for the considered parameters. The evolution of the potential distribution in the plane perpendicular to the magnetic field is illustrated in Figure 2 for the case with charge exchange collisions, where one can observe a development of coherent structures during the growth phase (Figure 2a), and propagation of such structures in the $\boldsymbol{E} \times \boldsymbol{B}$ direction the at the saturated stage (Figure 2b).

With the frequency-wavenumber spectral analysis of the electrostatic potential fluctuations, we observe that the number of harmonics increases during the growing phase of the instability. Because of the assumed vanishing neutral temperature and distorted velocity distributions, we have a rapid initial growth of wave amplitudes, with amplitudes increasing by a factor 2 within approximately one ion gyroperiod after the onset of instability. The dominant component of the fluctuations propagates at an angle to the $\boldsymbol{E} \times \boldsymbol{B}$ direction and $\omega/k_z \approx E/B$, while $k_x \approx 0$. We find the dominant wave-numbers in the saturated spectrum to be $|k| \approx \Omega_{ci} /C_s$ within a factor 2. At the saturated stage, the growth of modes at low frequencies is observed, and their relations indicate nonlinear three-wave interactions. For charge-exchange collisions we also observe the presence of half-harmonics of the ion cyclotron frequency. Furthermore, in the spectra we identify possible resonances between the wave and electrons corresponding to their thermal velocities.

4. Conclusions

With numerical simulations we have studied the relative role of charge exchange and elastic ion collisions with neutrals on the evolution of the weakly collisional plasma in the $\boldsymbol{E} \times \boldsymbol{B}$ fields. We have observed significant distortions of the ion velocity distributions that highly depend on the type of collisions. Such distorted distributions may be kinetically unstable. Indeed, we observe diffusion in the velocity phase space in the case of the charge exchange collisions. The simulations considered cold neutral background, but in reality, neutrals will be thermalized in the charge exchange events, and the ring-shaped distributions for ions will be slightly more diffuse already in the initial phase. For both cases, there is a growth of amplitude of the field fluctuations, and larger coherent structures propagating in the $\boldsymbol{E} \times \boldsymbol{B}$ direction are observed. In the frequency-wavenumber spectra we observe a number of harmonics during the growth phase, including modes at half-harmonics of the cyclotrone frequency. Note, that narrow frequency-band oscillations near half of the cyclotron frequency of hydrogen have at times been observed in the lower parts of the ionospheric F-region [11]. In the saturated phase, we see the growth of three main modes, reflecting the nonlinear three-wave interaction [12]. Since we expect that in this nonlinear regime there is a competition between several instabilities, further detailed studies are required to indentify different competing and dominant processes in the evolution of this system.

4. Acknowledgments

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