On methods to estimate bandwidth performance for array antennas with ground plane

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Abstract

Array antenna impedance bandwidth performance is a critical factor for antenna design, in particular for arrays in radar application and more recently for array antennas in communication systems. A priori methods to quantify bandwidth include stored energy-based estimates through the quality factor antenna Q, and sum-rule based estimates based on the antenna as a scattering object. A recently introduced, *array figure of merit*, uses a square window or wall-type estimate on the return-loss to obtain bounds in terms of return-loss level, bandwidth, scan-range and unit-cell specific geometrical and material information, it is based on a fundamental physics result, linking bandwidth to essentially the thickness of the array. An alternative approach is based on the Bode-Fano theory, resulting in bandwidth measures in terms of one or several quality factors. Both methods apply both to single and multi-band array antennas. Here we present these methods and make a comparison between the two approaches.

1. Introduction

Electromagnetic bandwidth is limited, both from a physical point of view in antennas, but also in the assigned communication spectrum. Sharp limitations for communication systems on allowed bandwidth usage, and a world-wide variation of assigned communication bands, demands high requirements on bandwidth performance for base-station antennas. Proposed solutions include multi-band arrays but also ultra-wide band array antennas. Recent efforts towards the determination of a physically upper bound on the impedance bandwidth have been investigated in [1] where a minimum antenna-Q was derived for cylindrical source with and without a ground plane. A similar approach were investigated in [2,3], based on an explicit specialized unit-cell Greens function were investigated to obtain respectively the impedance and the stored energy toward limits on available bandwidth.

An alternative approach to bandwidth limitations is based on scattering properties. It focus on the return-loss and a unit-cell is here seen as a scattering problem of a passive, time-invariant and linear system. These properties imply that the return-loss and the reflection coefficient are *scattering passive*, [4] and satisfy a Bode-Fano type identity. This approach was recently applied to arrays in [5] and extended to include multiband and scanning in [6], in the latter, we introduced an array figure of merit, as a measure of antenna performance. The above two methods to obtain bandwidth limitations results in a sum-rule based upper bounds [5,6] or in energy based bounds [1,2], often expressed in terms of antenna Q. For small antennas there has been recent progress in bounds on bandwidth using both methods, see e.g. [7-9]. Both the energy-based quality-factor and investigation of the scattering cross section through sum-rules gives implicit information on the bandwidth. For multi-resonant arrays there is currently no direct relation between bandwidth at a certain return-loss level, and the corresponding quality-factors. Instead we obtain certain natural combination of desired quantities limiting the behavior of the array.

In this paper we investigate how different sum-rule approaches depend on the choice of goal return-loss or antenna Q-description. The weighted integral of the return-loss and its models are either bounded by the material-geometrical information in the unit-cell or alternatively on a certain sum of products of resonance frequencies and Q-factors. It is clear that the array performance estimates depend on the used measure of bandwidth.

2. The array figure of merit

The array figure of merit is derived using a sum-rule applied to the unit-cell lowest order mode of the co-polarized reflection coefficient. Using that the system can be seen as a reciprocal two-port system, results in an identity relate the reflection coefficient of an loss-less unit-cell array antenna over a ground-plane below the grating lobe onset to the reflection coefficient, Γ, at the array excitation port see [5]. The result is based on the low-pass scattering derivation [10] of the unit-cell which quantify the geometrical and size-information in the unit-cell through the polarization and angle dependent quantity q:

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$$q(\theta)=\frac{\pi d}{c}(1+\tilde{\gamma}(\theta))\cos\theta \le \frac{\pi d \mu_s \cos^p\theta}{c}, \quad \tilde{\gamma}(\theta)=\begin{cases} \gamma_m, & TE \\ (\gamma_m+\gamma_e \sin^2\theta)/\cos^2\theta, & TM \end{cases}. \tag{1}$$

Here, d, is the thickness of the array above the ground plane, c, is the speed of light, θ, is the scan-angle from the normal of the array, μ_s, is the maximal value of the static relative permeability. Geometrical and material properties of the unit cell are partly encoded in the unit-cell volume normalized transverse magnetic polarizability, γ_m, and electric γ_e polarizability in the normal direction, e.g. if the normal is in the z-direction, then γ_e represent the zz-element of the corresponding normalized electric polarizability matrix. The number, p, is ±1 (+1 for TE and p=-1 for theTM-case).

A starting point for bandwidth estimates and the array figure of merit is the sum-rule identity, see e.g. [6]

$$I(\theta)=\int_0^{\omega_G} \omega^{-2}\ln(|\Gamma(\omega,\theta)|^{-1})\,\mathrm{d}\,\omega = q(\theta)-r(\theta) \ . \tag{2}$$

The quantity, r, is a rest term with only positive quantities, it contain terms like a possible high-frequency term of the reflection coefficient, as well as zero's in the upper complex plane of the reflection coefficient due to a Blaschke product, see e.g. [4], as well as the remaining contribution of the integral above the grating lobe angular frequency, ω_G. The array figure of merit, η, for wide-band antennas comes from estimates of the ratio $I/(q-r)$, over one or several working band(s) and over a range of scan-angles θ. The figure of merit is derived for wide-band arrays using a wall-type estimate on the reflection, i.e that Γ is below the level or multiple levels $\{\Gamma_m\}_{m=1}^M$ in the M square windows, for the respective working bands and all scan-angles in a given range. We find [6]

$$0 \le \eta = \frac{\sum_{m=1}^M \ln(|\Gamma_m|^{-1})(\lambda_{m,+}-\lambda_{m,-})}{2\pi^2 \mu_s d \cos^p \tilde{\theta}} \le 1, \tag{3}$$

where Γ_m represents the maximal reflection coefficient in the m:th bandwidth interval out of M working bands, and $\lambda_{m,\pm}$, are the upper and lower bandwidths corresponding to the working band upper and lower frequencies in interval m. Here $\tilde{\theta}$ is the selected to optimize the angular function over the desired scan-range and depends on p, see [6].

The figure of merit is clearly a number in-between [0,1]. The largest known value of the figure of merit for one working band, M=1, is 0.64, [11]. Note that (3) indicates that the bandwidth in this estimate is expressed in terms of thickness normalized wave-length bandwidth. Recall also that we express return-loss as RL=-20 $\log_{10}(|\Gamma_m|)$. Note that the product of wavelength differences and maximal return loss is bounded above in terms of scan-angle and thickness. Several wide-band and wide-scan arrays were investigated in [6]. In the associate talk we will illustrate how the array figure of merit provides a method to predict a-priori information about possible performance of an array antenna given a particular size. Below in the paper we consider the broad-side case only.

3. Sum-rule for array reflection coefficient

In the derivation of (3) from (2) we measure bandwidth with a sequence of square-windows, one for each working band. The analytical properties of the reflection coefficient ensure that such an estimate of the integral area in (2) will never be perfect. An associated picture of this is the comparison between the square window band-pass filter and causal filters; causal filters can naturally not perfectly represent a square window.

An alternative method to represent/measure bandwidth is based on the antenna Q-factor through one or several resonances. The Bode-Fano theory states that the integral of the logarithm of the array feeding port reflection coefficient satisfy the relation [4]

$$\lim_{\omega'\to 0^+}\lim_{\varepsilon\to 0}\frac{2}{\pi}\int_\varepsilon^\infty \ln(|\Gamma(\mathrm{j}\omega+\omega')|^{-1})\omega^{-2}\,\mathrm{d}\,\omega = a_1-b_1+2\sum_m \mathrm{Im}\,\omega_{z,m}^{-1} \tag{4}$$

The right-hand side sum, is over zeros, $\omega_{z,m}$, of the reflection coefficient in the upper complex half-plane, where a_1 and b_1 are the high and low frequency behavior of the reflection coefficient as ω approaches zero or infinity respectively.

To quantify the bandwidth of a given reflection-coefficient we assume that the reflection coefficient can be described as a sequence of resonances. Towards this end let *z(s)* be the normalized impedance and let the impedance be approximated by a sequence of *N* second order resonance:

$$z(s) = \sum_{n=1}^{N} \frac{z_n}{1 + Q_n(\mathrm{s}/\omega_n + \omega_n/\mathrm{s})} \;, \tag{5}$$

where ω_n is the resonance frequencies, $s = \omega' + \mathrm{j}\omega$, is the Laplace parameter. Here j=-i is the negative complex unit and Q_n is the parameter controlling the quality factor of the resonance, and z_n is a real-valued amplitude of the resonance. From the equivalent circuit perspective, we see that this model corresponds to a sequence of *N* parallel RCL-circuits coupled in series. This basic model has the advantage that it is easy to fit/place the resonances position at desired locations. Through the relation $\Gamma=(z-1)/(z+1)$ we have hence a flexible approximation of the reflection coefficient. We note that for this model we have $b_1 = 0$ in (4) and clearly $\mathrm{Im}\,\omega_{z,m}^{-1}<0$, thus we obtain the identity

$$J(\theta) = \frac{2}{\pi}\int_0^{\infty} -\ln(|\Gamma(\omega)|)\omega^{-2}\mathrm{d}\omega = 2\sum_{n=1}^{N}\frac{z_n}{Q_n\omega_n} - R \;, \tag{6}$$

where R>0 and decreasing with the distance between the resonances. Thus we have a tool to compare our wall estimate of the integral (2) with a working-band resonance model for the antenna. For the special case when N=1, R=0, and for N=2 with $z_1 = z_2$, we determine that $R = 2(\omega_1\omega_2)^{-1/2}\,\mathrm{Re}\left\{[(\mathrm{Q}_1\mathrm{Q}_2)^{-1} - (\omega_1-\omega_2)^2/(\omega_1\omega_2)]^{1/2}\right\}$. For larger values of N we find that the values of R get more complex. Thus (6) is a parallel and in some sense equivalent to (2) and is in itself a representation of bandwidth limitation of the reflection coefficient. Using this type of bandwidth measure, we are interested in comparing the cost in area loss associated with the estimates, here in particular the wall-type estimate leading to (3). In order for us to compare (6) with the wall estimate applied in (2) we restrict the placement of the resonances ω_n well below the grating-lobe limit, so that (6) and (2) are close, since here the integrand in $\omega^{-2}\ln|\Gamma|^{-1}$ in (6) above ω_G hence assumed to be smaller than the rest-term, *r*, in (2). This is commonly is possible since $\Gamma \to -1$ as $\omega \to \infty$ and the integral contribution in this region will be small.

6. Results

To illustrate how the choice of bandwidth representation affects the integral in (2) and (6) we use a three resonance model in (5). In (2) we use the wall-estimate with return-loss levels (5, 10, 15) dB and compare this with (6). For antennas with working-bands well approximated with the resonance model and for multi-band antennas, there is a substantial loss of area in the approximation of the integral (2), see Fig 1b. A higher array figure of merit is obtained by representing such antennas by the above resonance model. Note that the ratio between the integral and the wall estimate vary non-linearly as we sweep the resonance frequency ω_3. The integral estimate through the wall-shape is continuous in resonance position, but not differentiable. The relation between bandwidth in terms of Q-factors and square-window reference levels like the 10 dB return loss are hence not expected to be analytic see also [8]. This, imply that a fixed level return loss bandwidth may can change without changes in the integral $J(\theta)$. As an example see the point indicated in red in Fig 1a and 1b.

For well separated resonances, we see that the rest term *R*, becomes small. Thus an upper bound of the integral $J(\theta)$ is analytically known for this model, indicating an upper bound on multi-band performance.

7. Conclusion

We compare two methods to estimate bandwidth for array antennas. Analytical results on the upper bound are given both through unit-cell geometric information, and through antenna-Q factors. A comparison between the two models is shown.

8. Acknowledgments

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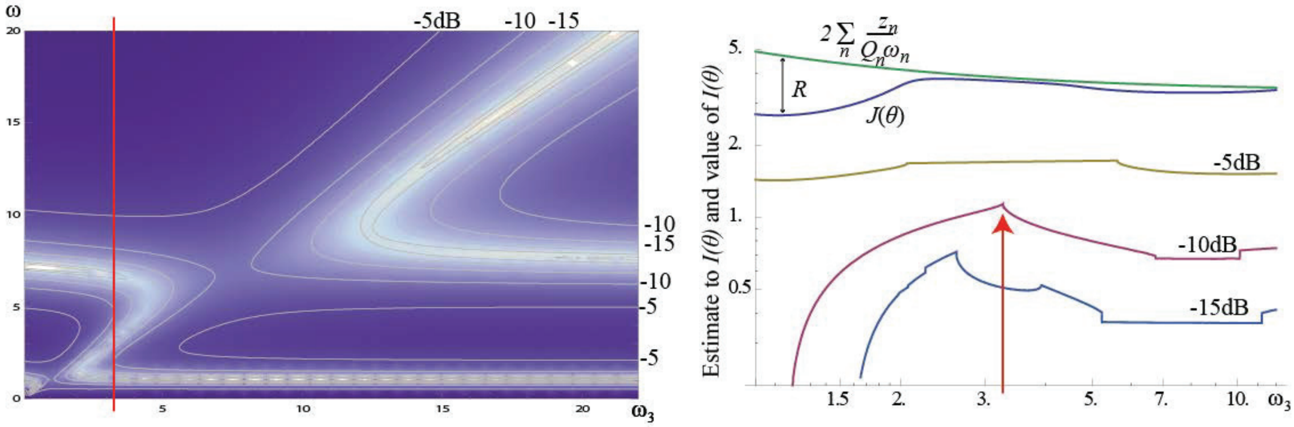

Fig. 1. Left: (a) A contour plot of reflection coefficient for the case with $(\omega_1 = 1, Q_1 = 1); (\omega_2 = 7.5, Q_1 = 2)$; $(\omega_3, Q_1 = 2), z_n = 1$, for n=1,2,3, i.e. $\omega, \omega_2, \omega_3$ are normalized in terms of ω_1. Right: (b) Estimates of the integral in (2) and (6) for a reflection coefficient that is modeled with a 3-resonance impedance, using the rectangular wall estimate for one/multiple working-bands of reflection coefficient level -5,-10 and -15 dB. We also show the value of estimate of the integral in (6) in green. The red vertical line indicate where the -10dB corner is located on both-figures, it is also a passage from two -10dB working bands to one such band see the vertical axis to the left.

9. References

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