Computational Approach to Phase Detection in Frequency-Modulation Interferometry

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Abstract

We present a new approach to the interference phase detection in the frequency-modulation interferometry. Using a frequency-tunable laser beam and several computational steps we show that the interference phase can be detected with a sub-nanometer resolution from a single interference signal coming from a non-polarizing interferometer sampled by a single photo-detector. This represents significant simplification of the necessary optical setup in comparison to the conventional homodyne interferometry that requires several polarizing optical elements to produce two interference signals in quadrature. The experimental results show that our method achieves comparable accuracy as the homodyne detection which makes it usable in applications where the interferometric setup simplicity is necessary.

1. Introduction

The laser interferometry represents the most precise methods of measuring geometric distances with sub-nanometer resolution. The principle lies in observation of interfering laser beams and detection of their mutual phase difference $\phi(t)$. There are two basic branches referred to as the homodyne and the heterodyne interferometry that employ the laser beams of the same and the different constant frequencies, respectively. Besides these, there is a rapidly developing branch referred to as the frequency-modulation interferometry that employs frequency-tunable laser beams [1].

In this paper we deal with the frequency-modulation interferometry and present a novel approach to the interference phase detection. Using the frequency-tunable laser beams and several computational steps we show that a pair of signals in quadrature suitable for the phase detection with a sub-nanometer resolution can be derived from a single interference signal coming from a non-polarizing interferometer sampled by a single photo-detector. This represents significant simplification of the necessary optical setup in comparison to the homodyne interferometry that requires several polarizing optical elements. The experimental results show that our method achieves comparable accuracy as the homodyne detection which makes it usable in applications where the optical setup simplicity is advantageous for the cost reduction.

2. Methods

A typical homodyne interferometer with a single-frequency laser source contains a polarizing beam splitter to produce the measurement and reference beams with a different polarization plane (see Figure 1). As a result, there is a complex light beam compound from the waves with mutually perpendicular polarization on the output of the interferometer rather than a simple interference. The compound beam is further processed in two detection branches involving another polarizing optical elements that produce a pair of interference signals I_x and I_y in quadrature, i.e., mutually phase shifted by $\pi/2$

$$\begin{aligned} I_x(t) &= \cos(\phi(t)) \\ I_y(t) &= \sin(\phi(t)). \end{aligned} \tag{1}$$

The immediate value of the interference phase $\phi(t)$ is extracted by a quadrature detection, i.e., it is calculated using an inverse function

$$\phi(t) = \arctan\left(\frac{I_y(t)}{I_x(t)}\right). \tag{2}$$

To reduce complexity of the optical system, we have developed a novel technique for interference phase detection which employs digital processing of detected signals rather than optical manipulations of the output laser beams [2, 3]. The interference phase is still calculated from Equation (2) but the required pair of signals in quadrature is derived from a single interference signal by means of several computational steps. For this purpose we utilize a frequency-tunable laser source characterized with sinusoidal waveform of the angular frequency

$$\omega(t) = \omega_s + \omega_m \sin(\Omega t), \tag{3}$$

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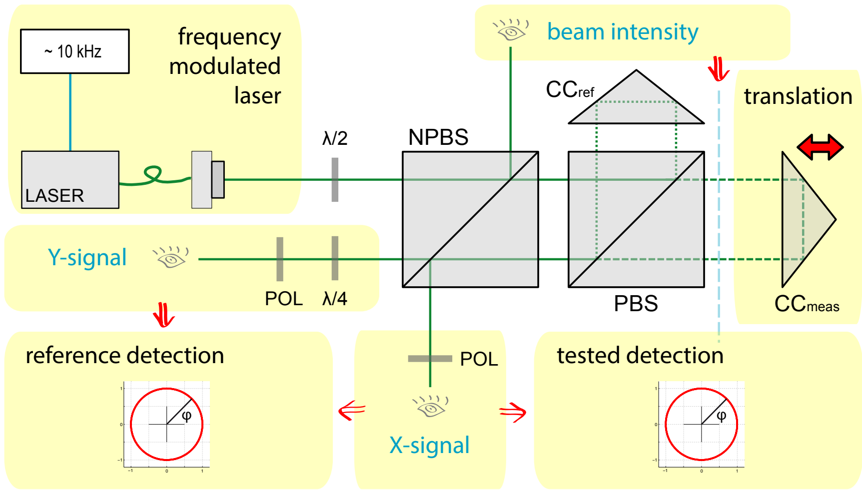

Figure 1: Experimental setup: The interferometer consists of the polarizing beam splitter PBS, the reference corner cube CC_{ref} and the measurement corner cube CC_{meas}. The interferometer output is split by the non-polarizing beam splitter NPBS into two detection branches that employ the quarter-wave plate $\lambda/4$, two polarizers POL and two photo-detectors PD and produce the quadrature signals I_x and I_y as an output. The non-polarizing beam splitter also splits part of the input beam to a photo-detector that monitors the beam intensity.

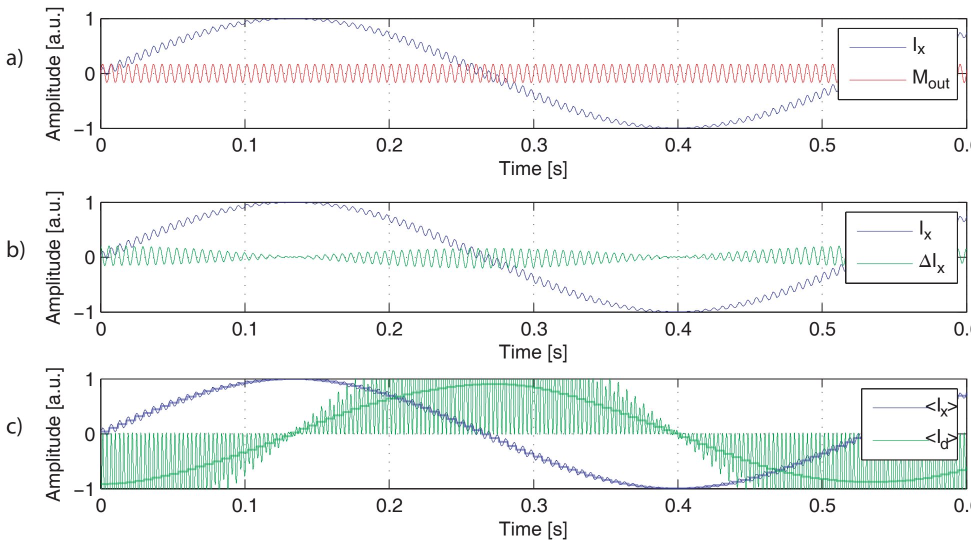

Figure 2: Principle of digital computation of quadrature signals: (a) the frequency modulation driven by a modulation signal (red) causes a modulation effects on the interferometer output (blue). (b) The modulation effect demodulated (green) from the interference signal (blue). (c) The modulation effect mixed with a phase-shifted copy of the original modulation signal (green) and averaged. The resulting signal (bold green) coincides with the original signal (blue) with filtered modulation effects (bold blue) with exception of $\pi/2$ phase shift.

where ω_s is the mean value of angular frequency, ω_m the amplitude of the sinusoidal modulation and Ω is modulation angular frequency. Denoting the time delay between the reference and measurement beams by τ and assuming that $\omega_m\tau \ll 1$, we get the normalized interference signals in the form

$$\begin{aligned}\tilde{I}_x(\tau,t) &= \cos(\omega_s\tau) - \omega_m\tau\sin(\Omega t)\sin(\omega_s\tau)\\ \tilde{I}_y(\tau,t) &= \sin(\omega_s\tau) + \omega_m\tau\sin(\Omega t)\cos(\omega_s\tau).\end{aligned} \tag{4}$$

The time-averaged intensities are given by the functions

$$\begin{aligned}\langle\tilde{I}_x(\tau,t)\rangle &= \cos(\omega_s\tau)\\ \langle\tilde{I}_y(\tau,t)\rangle &= \sin(\omega_s\tau),\end{aligned} \tag{5}$$

which are suitable for the interference phase detection by means of the quadrature detection due to their formal similarity to the functions given by Equation (1), see also Figure 2.

The key step of our method lies in derivation of the mean intensity $\langle \tilde{I}_y \rangle$ from the observed intensity $\tilde{I}_x$ by means of numeric processing. Subtracting the intensity $\tilde{I}_x$ at the neighboring times t and $t - \Delta t$, multiplying the result by $\cos(\Omega t)$ (note that it is a phase-shifted copy of modulation signal, originally $\sin(\Omega t)$) and calculating the time average, we get the derived signal

$$\begin{aligned} I_d(\tau, t) &= \langle [\tilde{I}_x(\tau, t) - \tilde{I}_x(\tau, t - \Delta t)] \cos(\Omega t) \rangle \\ &= -\frac{1}{2} \omega_m \tau \sin(\Omega \Delta t) \langle \tilde{I}_y \rangle. \end{aligned} \tag{6}$$

Since $I_d(\tau, t)$ is proportional to the mean intensity $\langle \tilde{I}_y \rangle$, it is clear that we are able to infer $\langle \tilde{I}_y \rangle$ from $\tilde{I}_x$. This fact allows for significant reduction of the interferometer setup – respectively, the polarizing beam splitter and two detection branches can be replaced by a non-polarizing beam splitter and a single photo-detector which produces the interference signal $\tilde{I}_x$ as an output.

Besides the above computation of the quadrature signals, the scale linearization techniques[4, 5] are used for an additional suppression of optical setup imperfections, noise and the residual amplitude modulation (referred to as RAM) caused by the laser modulation (for details see [2]). We have also added a monitoring of the laser intensity that serves two purposes: it allows us to mitigate the effects of residual amplitude modulation and can be used in place of the modulation signal information that is used for the I_d reconstruction.

3. Experimentation

The experimental setup assembled in order to verify capabilities of the proposed method is sketched in Figure 1 and corresponds to a traditional scheme of a polarizing homodyne interferometer. It consists of a laser source, a Michelson interferometer, two detection branches and an auxiliary detection of the laser beam intensity. As a frequency-tunable laser source we employed a DFB diode based laser module operating at 1530.25 nm with 10 mW output power (Rio Orion) with injection current modulation possibilities (±50 MHz at 10 kHz). The interferometer consists of a polarizing beam splitter and two cube-corner retroreflectors – the one in the measurement arm is placed on a 100 mm long translation stage. A non-polarizing beam splitter is used to both feed part of the laser beam intensity to a photo-detector that monitors the intensity as well as to split the interferometer output to the X- and Y- detection branches.

The output wave of the polarizing interferometer consists of two components from two interferometer's arms characterized by mutually perpendicular plane of polarization. Therefore, both detection branches involve a linear polarizer that changes the polarization state of the components into a single plane so that the interference actually occurs at the photo-detector. The Y-branch also includes the zero-order retarder plate that shifts the phase of a single component by $\pi/2$ so that the resulting interference signal is also phase shifted and the X- and Y-branch form a quadrature output that serves for the reference homodyne detection.

To evaluate the performance of the proposed method we have measured the interference phase by both methods for various path differences between the reference and the measurement beam. The retroreflector in the measurement arm was successively placed into 21 different equidistant measurement points within the operating range of $\Delta L \in (0 - 100)$ mm. At each measurement point the measurement arm was successively displaced between 32 equidistant sub-points with spacing of 0.1 μm which cover the distance of several interference fringes ($\approx 10\,\mu$m). At each sub-point we obtained 200 signal readings; in total we measured ≈ 135000 phase readings.

The beam frequency was modulated by a free running sine wave at 10 kHz; the output signals were digitized at 32 kHz and stored to a control PC. The acquired data were processed off-line and evaluated. At each sub-point the RAM was compensated and the phases were computed. All sub-points that belong to a single measurement were aggregated and the scale linearization correction was applied. At the end we evaluated the coincidence of the two phase reading methods depending on the measurement position. The coincidence is referred to as a phase determination error PDE defined by

$$\mathrm{PDE} = P_{ref} - P_{tst}, \tag{7}$$

where P_{ref} is the reference phase and P_{tst} is the phase read by the tested method.

4. Results and Discussion

The experimental results are shown in Figure 3 that displays overall dependence of the mean PDE on the measurement distance. The overall PDE keeps below ±0.5 nm and the standard deviation keeps below ±0.75 nm. The trends indicate worse results towards the near end that are caused by insufficient modulation depth that lead to the increased influence of noise and the RAM. Considering the range $50 - 100$ mm, the PDE keeps below 0.21 ± 0.22 nm.

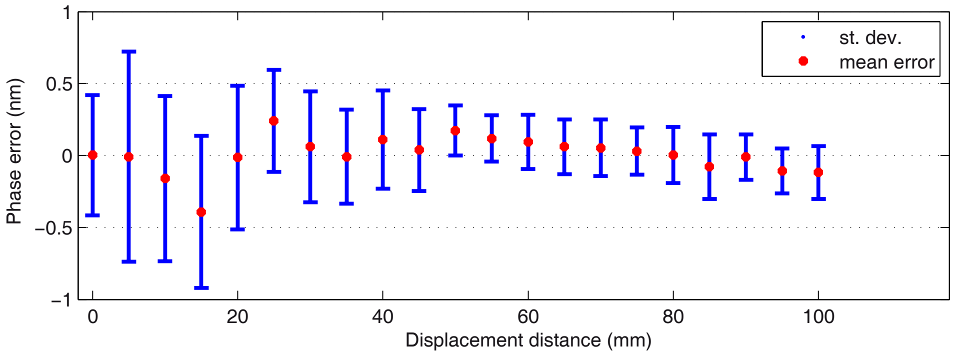

Figure 3: Experimental results: the red points show the dependence of the mean phase determination error on the path length difference; the blue error bars indicate the interval of radius of standard deviation around the mean PDE at each corresponding point i ($n_i = 6400$).

The results indicate that the proposed method is a suitable candidate for a cost-effective laser interferometry based measurement systems. The comparison with reference interferometer indicate a sub-nanometer level of coincidence over the entire operating range. The lower accuracy in the region where the path lengths of the interferometer arms are nearly balanced arises from the simple fact that in the balanced state the frequency modulation does not affect the interferometric output signal and thus makes the Y- signal reconstruction impossible. The workaround is to introduce a dead length in order to move the measurement arm out of balance, but it will also worsen the impact of the refractive index fluctuations.

The higher demands posed on the laser source are compensated by the fact that the method also scales well when several measurement axes are used. The reason is that the added complexity per each added axis is minimal in comparison to the traditional means as it has been already demonstrated in multi-axis geometric measurements [6].

5. Conclusion

We have proposed a novel method of the interference phase detection within the frame of frequency-modulation interferometry. This allows for significant reduction of the optical setup in comparison to the classical homodyne interferometry. The experimental evaluation has shown that the proposed method achieves comparable accuracy as the homodyne detection and therefore it is a promising concept for the subsequent application.

6. Acknowledgments

The theoretical part of the method was funded by Grant Agency of CR project GP102/10/1813. The infrastructure of the research was supported by the European Commission and Ministry of Education, Youth, and Sports of the Czech Republic projects CZ.1.05/2.1.00/01 and LO1212.

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