SMALL SIZED MHD ACCRETION DISC FOR HIGH MAGNETIC FIELD AROUND SUPERMASSIVE BLACK HOLES : DOES IT CONTRADICT OBSERVED GALACTIC ACCRETIONS?

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Abstract

Thin accretion flow, i.e., geometrically thin accretion disc was first studied by Shakura and Sunyaev. Relativistic fluid flows around a black hole produce enormous energy on the cost of permanent lost of the gravitational potential due to the fall into a infinitely sloped gravitational well or to be specific, into a space time singularity. This energy is actually observed in different wavelengths and we specify the source as Active Galactic Nuclei, quasars, Gamma-ray burst sources etc. Eventually, two popular kind of accretion disc models are there. The first one is advection dominated, known as geometrically thin optically thick accretion disc. The other is geometrically thick but optically thin as it does not capture photons inside! The jets formed by accretion phenomena are still not well explained. Size of the accretion disc, power of the jets can be powered by magnetic fields generated by the ionized particles of the accretion flow. We show the exact dependency of the disc size upon the magnetic field present along with the quantity of the central gravitating mass.

The thin accretion disc dynamics was first illustrated by Shakura and Sunyaev [**?**]. When we try to confine our accretion study, taking the central massive object as a black hole (BH) only, as the event horizon is not a hard surface like neutron stars or white dwarfs, the matter flow turns supersonic very near to the BH. Matter speed at the BH horizon reaches the speed of light, while, by causality condition, sound speed can be at the most $c/\sqrt{3}$. Therefore, there must be a location where matter speed crosses local sound speed making the flow transonic. The role of the sonic points in accretion disc resembles with that of the critical points in simple harmonic oscillators[1].

The observation of ultra luminous X-ray sources(compact sources) in the nearby star forming galaxies points out some unusual classes which are supposed to be optically thick, radiation pressure dominated systems with strong advection and matter is strongly ejected out in the form of outflows/jets(may be driven by strong radiation pressure). Different optically thick Keplerian disc failed to explain the jets from the advective regions in these cases. Again except than the nature of the disc itself, the size of BH is observatioanally proved to be a stronger catalyst to form the jets. Supermassive BHs are found to be centering at the AGNs rather than the stellar mas binary system BHs. The spin of the central object is also been treated as an important factor giving rise to the jets.

The origin of the turbulence and then the viscosity parameter α was supposed to come into existence due to the magneto rotational instability [2] in hot Keplerian disc.

Super massive BHs are supposed to be there in quasars and AGNs. Accretion discs around BHs are thought to be the jet ejector. A large-scale magnetic field (of, for example, the interstellar medium) would be dragged inward and greatly compressed by the accreting plasma.

In this paper for all the calculations we will consider GM/c^2 as the unit of length, M, being the mass of the compact object, G, the gravitational constant and c, the speed of light. Moreover, c and GM/c will be treated as the unit of velocity and angular momentum respectively. Taking x and $z = h(x)$ to be the radial distance and the height of the disc; v and λ to be radial velocity and angular momentum resp.;B_x, B_ϕ and B_z to be radial, azimuthal and vertical magnetic field components resp.; P and ρ to be pressure and density; Γ_1 and Γ_3 to be two functions of the polytropic index γ; α and f to be Shakura-Sunyaev parameter and the viscous heat coefficient resp. Then the equations governing the flow are given as (derived from the equation of continuity,

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Radial, Azimuthal and vertical momentum balance equations, entropy equation, Electro-Magnetic Induction Equations, Magnetic Monopole Equation):

$$\frac{dB_x}{dx} = -\frac{B_x}{x} \tag{1}$$

$$\left.\begin{aligned} \tfrac{dv}{dx} &= \tfrac{N}{D} \\ N = v\mathcal{S}\left[\tfrac{\Gamma_1+1}{\Gamma_3-1}\mathcal{A} + \tfrac{2\Gamma_1}{\Gamma_3-1}\tfrac{P}{\rho}\mathcal{H} - \tfrac{3f}{4x}\tfrac{B^2}{4\pi\rho}\right] - \tfrac{2f\alpha^2 v\mathcal{W}}{\rho}\left\{\mathcal{A} + \tfrac{P\mathcal{H}}{\rho} - \left(\mathcal{H} - \tfrac{2}{r}\right)\tfrac{\mathcal{W}}{4\rho}\right\} - \tfrac{f\alpha v\mathcal{W}}{\rho}\mathcal{G} + \tfrac{2f\alpha\lambda\mathcal{W}}{\rho x^2}\tfrac{B_x^2}{4\pi\rho}\ , \\ D = \mathcal{S}\left(\tfrac{\Gamma_1+1}{\Gamma_3-1}v^2 - \tfrac{P}{\rho}\tfrac{2\Gamma_1}{\Gamma_3-1}\right) - f\alpha\tfrac{B_xB_\phi}{4\pi\rho}\tfrac{\mathcal{W}}{\rho} - f\alpha^2\left(v^4 - \tfrac{P^2}{\rho^2}\right) \quad , \\ \mathcal{A} = \tfrac{\lambda^2}{x^3} - F_{Gx} - \tfrac{1}{4\pi\rho}\left(\tfrac{B_x^2}{x} + \tfrac{B_\phi^2}{x} - B_z\tfrac{B_x - B_x|_{z=0}}{h}\right) \quad , \quad \mathcal{H} = \left(\tfrac{3}{2x} - \tfrac{1}{2F}\tfrac{dF}{dx}\right) \quad , \\ \mathcal{S} = \left(v^2 - \tfrac{B_x^2}{4\pi\rho}\right) \quad , \quad \mathcal{G} = \tfrac{1}{4\pi\rho}\left(\tfrac{B_\phi B_x}{x} + B_z\tfrac{B_\phi - B_\phi|_{z=0}}{h}\right) \quad , \quad \mathcal{W} = \left(P + \rho v^2\right) \end{aligned}\right\} \tag{2}$$

And one by one we can determine the other parameter's derivatives in terms of $\frac{dv}{dx}$

$$\frac{dP}{dx} = \rho\left[\mathcal{A} - v\frac{dv}{dx}\right] \quad , \tag{3}$$

$$\frac{d\rho}{dx} = -2\rho\left\{\mathcal{H} + \frac{\rho\mathcal{A}}{2P} + \frac{1}{v}\left(1 - \frac{\rho}{2P}v^2\right)\frac{dv}{dx}\right\} \quad , \tag{4}$$

$$\frac{dB_z}{dx} = B_z\left(\frac{1}{x} - \frac{1}{v}\frac{dv}{dx}\right) \quad , \tag{5}$$

$$\begin{aligned} \frac{d\lambda}{dx} = x\Bigg(\Bigg[\alpha v\left\{2\left(v^2 - \frac{\mathcal{W}}{2\rho}\right)\left(\mathcal{H} + \frac{\rho\mathcal{A}}{2P}\right) - \left(1 + \frac{\mathcal{W}}{2P}\right)\mathcal{A} - \frac{\mathcal{W}}{\rho}\left(\frac{1}{x} + \frac{\mathcal{H}}{2}\right)\right\} + \left\{\frac{2\lambda B_x^2}{4\pi\rho x^2} - v\mathcal{G}\right\}\Bigg] \\ + \left[\frac{B_xB_\phi}{4\pi\rho} + \alpha v^2\left\{\frac{\rho v^2}{2P} + \frac{\mathcal{W}}{2\rho v^2} - 1 - \frac{\mathcal{W}}{4P}\right\}\right]\frac{dv}{dx}\Bigg) \Big/ \left(v^2 - \frac{B_x^2}{4\pi\rho}\right) \quad , \end{aligned} \tag{6}$$

$$\begin{aligned} \frac{dB_\phi}{dx} = \Bigg(B_x\Bigg[\alpha\left\{2\left(v^2 - \frac{\mathcal{W}}{2\rho}\right)\left(\mathcal{H} + \frac{\rho\mathcal{A}}{2P}\right) - \left(1 + \frac{\mathcal{W}}{2P}\right)\mathcal{A} - \frac{\mathcal{W}}{\rho}\left(\frac{1}{x} + \frac{\mathcal{H}}{2}\right)\right\} + \left\{\frac{2\lambda v}{x^2} - \mathcal{G}\right\}\Bigg] \\ + v\left[B_\phi + \alpha B_x\left\{\frac{\rho v^2}{2P} + \frac{\mathcal{W}}{2\rho v^2} - 1 - \frac{\mathcal{W}}{4P}\right\}\right]\frac{dv}{dx}\Bigg) \Big/ \left(v^2 - \frac{B_x^2}{4\pi\rho}\right) \quad , \end{aligned} \tag{7}$$

and the disc height in every step will be determined from

$$h(x) = \frac{1}{2F_{Gx}}\left(\frac{xB_x}{4\pi\rho}\frac{dB_z}{dx} + \sqrt{\left(\frac{xB_x}{4\pi\rho}\frac{dB_z}{dx}\right)^2 + \frac{4xPF_{Gx}}{\rho}}\right) \quad . \tag{8}$$

In this work we have eight variables dependent on the radial distance, x, as the only independent variable. Though we have eight coupled differential equations too, we can not use all of them to determine the distinct differentiations of the eight variables. This is due to the fact that inclusion of vertical momentum balance equation forces to determine a very simple form of the dv/dx, which on its particular case does not match with the previous works where magnetic fields are not been considered. So we have changed our system in a bit tricky way. Now we will iterate our system with the equations (1) to (8). Now the first thing we need is to determine the initial conditions for the variables. If we locate the point $D = 0$ by sonic point then the relation between the sonic point radial velocity and the sound speed can be derived as

$$v_c = c_{sc}\left(\frac{-b + \sqrt{b^2 - 4ac}}{2a}\right)^{\frac{1}{2}} \tag{9}$$

where a, b and c are defined as

$$a = \frac{\Gamma_1+1}{\Gamma_3-1} - f\alpha^2 \quad , b = -\frac{f\alpha}{3n^2} - \frac{1}{\Gamma_3-1}\left(\frac{\Gamma_1+1}{3n^2} + \frac{2\Gamma_1}{\gamma}\right) \quad and\ c = \frac{2\Gamma_1}{3\gamma n^2(\Gamma_3-1)} - \frac{f\alpha}{3\gamma n^2} + \frac{f\alpha^2}{\gamma^2} \quad .$$

The continuity of the flow requires the N to be equal to zero at the sonic point. Now if we want to determine the energy of the accreting particle properly then we have to supply a critical point radius x_c which reduces the equation $N = 0$ as an equation of v_c, B_{xc}, $B_{\phi c}$, B_{zc} , λ_c , ρ_c, P_c and h_c. Here to reduce so many variables for the critical point only we will make some assumptions : (i) The P_c is assumed to take the form $\rho_c c_{sc}^2/\gamma$, (ii) The magnetic fields are assumed to be symmetric with each other and the Alfvenic speed, $B^2/4\pi\rho$, at the sonic point, is a fraction of the magnetosonic speed, namely c_{sc}^2/n^2. These conditions determine the magnetic fields at sonic point as $B_{xc} = B_{\phi c} = B_{zc} = \sqrt{4\pi\rho_c c_{sc}^2/3n^2}$. These two assumptions shows that only providing ρ_c will be enough, (iii) λ_c will be provided manually, (iv) h_c will be given as $c_{sc}\sqrt{F_{Gx_c}/\gamma x_c}$, which however will be required to minimize the $N = 0$ equation as an equation of v_c only as from the next step h will be determined from algebraic equation only which will not use the last step h value.

With all these assumptions we can determine v_c at r_c and also the other variables at critical points as well. Now as at the critical point we have $dv/dx = 0/0$, so using l'Hospital's rule we can have a differential equaton of order one degree two giving two different branches to proceed with (while finding our solution from critical point.). One of these will give the accretion solution whereas the other will construct the wind.

Fig 1a-f shows if no magnetic field is present the accretion disc length is larger. The flow is subkeplerian. But as we increase the magnetic field slowly, the radial length for which a physical accretion to be found reduces. Only at 25 Gauss we get very small accretion phenomenon. For magnetised accretion even we can see that the flow turns to be superkeplerian at a finite distance where we take the end of the disc. This is the point where all the azimuthal velocity/ angular momentum changes to vertical velocity which we are not considering here. This is very clearly a indication of jet formation. This is again supported by the increment of z/x as we increase magnetic field. The disc becomes more geometrically thick as the magnetic field is increased. The temperature however is not increased much to point out any advection. Apparently, though it can be seen as some thing not observed, as the galactic accretion around a supermassive black hole is highly magnetised and large. But we should think that as the field is the flux flowing through an unit area, as we take larger mass black hole, the accrtion disc starts from a larger radius giving large area perpendicular to the magnetic field. Ultimately small field even signifies high magnetic flux.

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*Fig.*1*a* *Fig.*1*b* *Fig.*1*c* *Fig.*1*d* *Fig.*1*e* *Fig.*1*f*

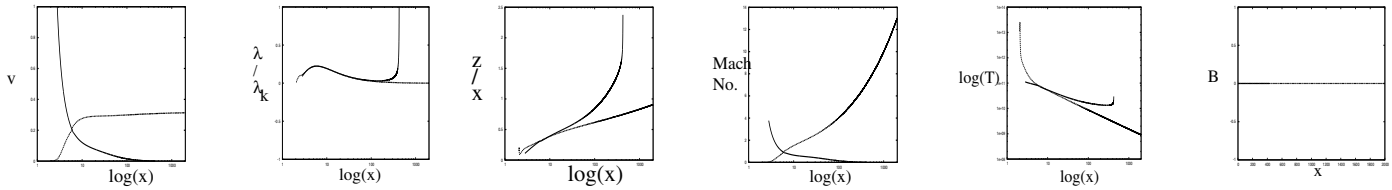

Fig 1a-f describes the graphs of v, λ/λ_k, z/x, $Mach\ no$, T and B^2 with respect to x, the radial distance. The central compact object has been considered to have $10^7 M_\odot$ as its mass and no magnetic field is considered to be produced in the accretion disc.

*Fig.*2*a* *Fig.*2*b* *Fig.*2*c* *Fig.*2*d* *Fig.*2*e* *Fig.*2*f*

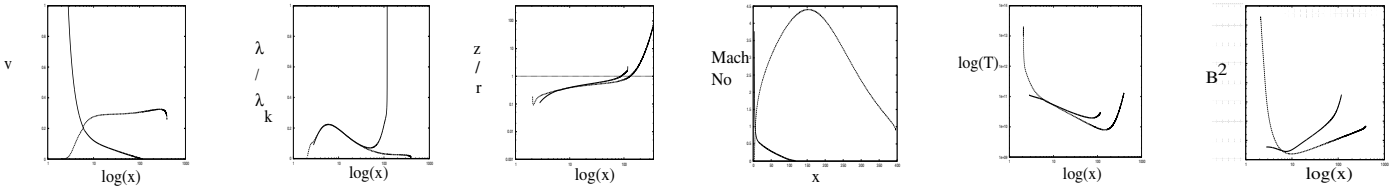

Fig 2a-f describes the graphs of v, λ/λ_k, z/x, $Mach\ no$, T and B^2 with respect to x, the radial distance. The central compact object has been considered to have $10^7 M_\odot$ as its mass and the magnetic field produced in the accretion disc at the critical point is taken to be equal to 1 *Gauss*.

*Fig.*3*a* *Fig.*3*b* *Fig.*3*c* *Fig.*3*d* *Fig.*3*e* *Fig.*3*f*

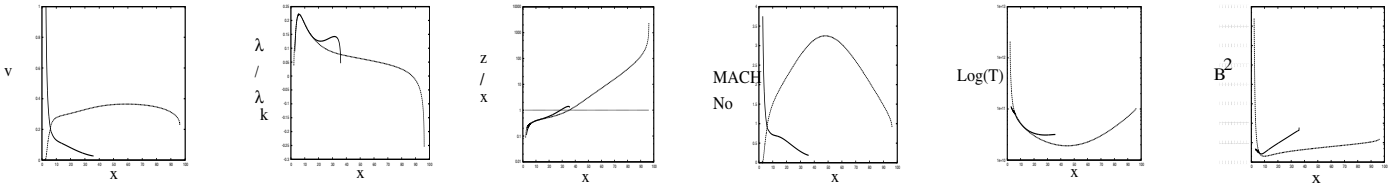

Fig 3a-f describes the graphs of v, λ/λ_k, z/x, $Mach\ no$, T and B^2 with respect to x, the radial distance. The central compact object has been considered to have $10^7 M_\odot$ as its mass and the magnetic field produced in the accretion disc at the critical point is taken to be equal to 10 *Gauss*.

*Fig.*4*a* *Fig.*4*b* *Fig.*4*c* *Fig.*4*d* *Fig.*4*e* *Fig.*4*f*

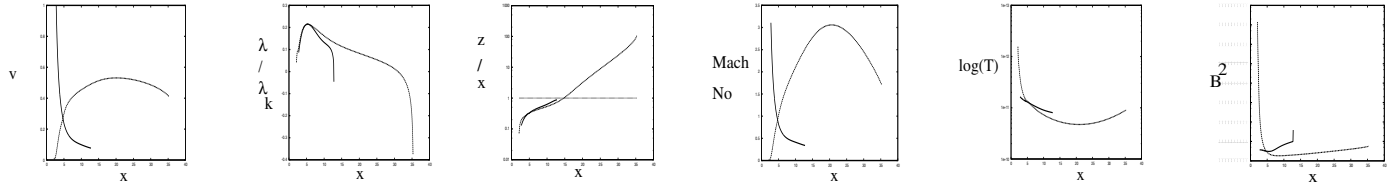

Fig 4a-f describes the graphs of v, λ/λ_k, z/x, $Mach\ no$, T and B^2 with respect to x, the radial distance. The central compact object has been considered to have $10^7 M_\odot$ as its mass and the magnetic field produced in the accretion disc at the critical point is taken to be equal to 25 *Gauss*.