On the Spatial Scale of Streamers

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Abstract

We calculate the streamer velocity, which is determined by the streamer mechanisms which include electron drift, electron diffusion and photoionization, in a 1D model that also includes the curvature of the ionization front. In particular, we show that the electron drift may contribute only to the propagation of negative streamers, and the effect of photoionization on the streamer velocity is mostly determined by the photoionization length. The results are used to modify the fractal model of [1]. The transverse size of streamers, as well as the fractal dimension of the streamer structure emerges from the front velocity dependence on electric field and the front curvature. In particular, when the photoionization is the main mechanism which determines the streamer propagation, the transverse size of the streamers is of the order of the photoionization length.

1. Introduction

Fractal nature of dielectric breakdown was investigated by [1] for the velocity of the ionization front v (represented in their simulation as the growth probability P) depending on the electric field as $v \propto E^\eta$**.** They pointed out that in the case $\eta = 1$ the streamer growth is described by the same equations as the diffusion-limited aggregation (DLA) system [2] which produces a branching fractal structure. A considerable effort has gone into modelling of streamers from the first physical principles using quasi-electrostatic equations. In particular, the branching of streamers in the presence of only the electron drift mechanism in a two-dimensional cylindrically-symmetric system was demonstrated by [3], whose work was expanded by [4] with inclusion of diffusion and photoionization mechanisms. Currently, there are also other extensive three-dimensional streamer branching modeling efforts [5].

2. Quasi-electrostatic (QES) equations and streamer mechanisms

In the QES equations of [6], let us assume constant electron mobility $\mu < 0$, use electron conductivity $\sigma = e\mu N$ instead of electron density N, rescaled electron conductivity $\sigma = \sigma_{SI}/\varepsilon_0$ and density $\rho = \rho_{SI}/\varepsilon_0$ (for convenience) and the net ionization rate $\nu(E) = \nu_i - \nu_a$. We get

$$\left.\begin{array}{rcl} \mathbf{E} &=& -\nabla\varphi \\ \nabla\cdot\mathbf{E} &=& \rho \\ \dot{\rho} &=& -\nabla\cdot(\sigma\mathbf{E}) \\ \dot{\sigma} &=& \nu(|\mathbf{E}|)\sigma \end{array}\right\} \quad (1)$$

It may be demonstrated by the invariance of these equations in respect to spatial coordinate rescaling that this system does not possess a spatial scale and therefore cannot describe streamer propagation.

The streamer mechanisms must be included by the following modifications to the last equation which describes the continuity of σ:

1. Electron drift, which adds $\nabla\cdot(\mu\mathbf{E}\sigma)$ to the left-hand side (LHS);
2. Electron diffusion, which adds $D\nabla^2\sigma$ to the right-hand side (RHS);
3. Photoionization, which adds an extra source p to the RHS. The photoionization is a non-local mechanism, proportional to the "regular" ionization $S_i = \nu\sigma$ at a distance and given as $p(\mathbf{r}) = \int S_i(\mathbf{r}')F(\mathbf{r}-\mathbf{r}')d^3\mathbf{r}'$. In this paper, we use a model for p which makes use of a Helmholtz equation solution which was suggested by

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[7], namely $(1-\Lambda^2\nabla^2)p = AS_i$ which assumes $F(r) = (A/\Lambda^2)e^{-r/\Lambda}/(4\pi r)$, where Λ has the meaning of the photoionization length and the dimensionless number A may be called the ionization strength. Usually, $A \ll 1$.

3. Ionization front velocity in 1D

Let us assume that the spatial variation is only in *x*-direction and denote for brevity $\partial \equiv \partial/\partial x$, and the translational symmetry surface *(yz)* has total curvature $\kappa \approx const$. Then $\mathbf{E} \parallel \hat{x}$ and the first three equations (1) may be combined into

$$\dot{E} = -\sigma E \qquad (2)$$

which together with the continuity equation for σ confirms our earlier result that there is no spatial scale in system (1) because the spatial derivatives ∂ drop out. Let us find a solution which propagates with a constant velocity $v > 0$ in *x*-direction from the ionized region at $x = -\infty$ into the neutral region with $E = E_0$ and $\sigma = 0$ at $x = +\infty$. We require that the solution is finite and physically correct (i.e., $\sigma, p > 0$) and satisfies the above boundary condition at $x = +\infty$. We find, in particular, that in the presence of electron drift and photoionization the velocity must satisfy inequality $v > v_s$, where

$$v_s \approx v_{d,max} + \Lambda \nu_{max} f\left(\frac{\kappa\Lambda}{2}\right)\left[1 + O\left(\sqrt{A}\right)\right], \text{ with } f(q) = \sqrt{1+q^2} - q \qquad (3)$$

We observe that (1) the maximum electron drift velocity $v_{d,max} = \mu E_0$ is the front velocity in the absence of photoionization; (2) if the velocity is determined by photoionization, then (for a flat front) it is a product of photoionization length Λ and the maximum net ionization rate $\nu_{max} = \nu(E_0)$; (3) the curvature-dependent factor $f(q)$ lowers the velocity of a convex front; and (4) the photoionization strength A plays only a minor role in determining the velocity which may lead to large fluctuation in streamer velocity at $A \to 0$ and indefinite Λ. At $v = v_s$, the perturbation of the medium in the front of the streamer is minimal, so it is the most physically correct solution. We call this requirement the "minimal advanced ionization" condition (MAI).

4. Modifications to the fractal model and results

We modify the square grid fractal calculations of [1] by including the curvature of the ionization front according to the factor $f(q)$ of equation (3) in determining the probability $P \propto v$ of the cluster growth at a given location. Due to the discreteness of the grid, the curvature expression is not very accurate, but still reflects the "roughness" of the boundary between the ionized region ($\varphi = const$) and the neutral region ($\sigma = 0$). The results of the fractal simulations are presented in Figure 1. In particular, they confirm that at the small spatial scale, we may resolve the transverse size of the fractal branches which is of the order of Λ (which is given in the Figure in terms of the grid step), while at the large spatial scale the fractal structure has the same dimension as in [1] and coincides with the general results of the studies of DLA systems, namely $D \approx 1.67 - 1.71$ [2].

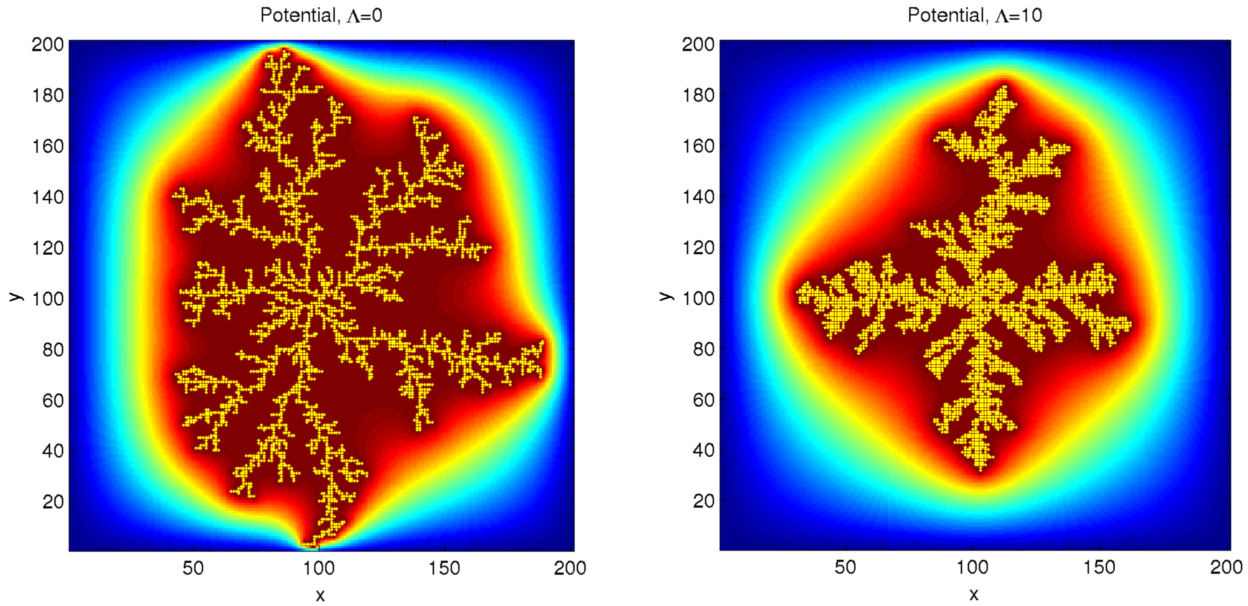

Figure 1. The fractal structures and electrostatic potential calculated for $\Lambda = 0$ (the case of [1]) and $\Lambda = 10$.

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6. References

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